Pre-service teachers' reasoning and sense making of growing patterns

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Abstract

In this study, our aim was to present the features of pre-service teachers' reasoning and sense making in the process of exploring tasks related to growing patterns. The study involved 79 pre-service primary teachers in Sweden. Our analysis was grounded in the concept of critical aspects from variation theory, as well as the concepts of assemblage, lines of flight, segmentarity, and rupture, from a post-structuralist philosophical perspective. The results indicate that the concepts of assemblage, lines of flight, rupture, and segmentarity can serve as useful tools for analyzing reasoning and sense making in mathematics. Furthermore, the results demonstrate that in mathematics, the concept of "line of flight" can be observed in moments of creativity and innovation. Additionally, the findings reveal that "rupture" manifests when there is a connection between concepts or ideas that challenge established mathematical structures and modes of thinking. Moreover, the results show that "segmentarity" creates disciplinary boundaries and hinders reasoning and sense-making. Therefore, it is crucial to recognize and dismantle these boundaries to facilitate more creative and innovative reasoning and sense making in mathematics.

Keywords: reasoning, sense making, growing pattern, assemblage, variation theory, difference, repetition

1. Introduction

Reasoning, and particularly algebraic reasoning, holds great importance in building conceptual knowledge within the realm of school mathematics (e.g., Arcavi et al., 2017). Reasoning also underscores the significance of sense making, which involves connecting new concepts with pre-existing knowledge and understanding. In the classroom, teachers can encourage students to approach algebra by nurturing their understanding of algebraic concepts over time, while also motivating them to view mathematics as a coherent and meaningful system of ideas, rather than merely a collection of isolated facts and procedures. Consequently, the reasoning and sense making abilities of pre-service teachers in algebra are of utmost importance in fostering and developing students' algebraic reasoning and sense making skills. To avoid any confusion, in this context, the term "teachers" is used to refer to individuals actively engaged in teaching at the compulsory school level, "pre-service teachers" denotes university students who are studying to become mathematics teachers but have not yet completed their formal teacher training program, and "students" refers to individuals studying in compulsory school.

Algebra is an integral part of the mathematics curriculum in Sweden, and mathematical reasoning is defined as a general ability that can be developed through various mathematical content (Swedish National Agency for Education, 2022). One such content area is related to "how patterns in number sequences and geometrical patterns can be constructed, described, and expressed" (Swedish National Agency for Education, 2022, p. 58). However, results from the national mathematics test in 2018 revealed that only 42 percent of participating students were able to calculate the later stages of a growing pattern, and merely 30 percent of students could deduce the rule that applied to any stage of the growing pattern (Nydahl & Ridderlind, 2018). This indicates that students often encounter difficulties with growing patterns. Therefore, pre-service teachers should be well-prepared to effectively teach algebra when they enter the classroom. They should also possess a strong foundation in algebra to enhance their understanding and teaching capabilities in this subject. However, research has shown that pre-service teachers often have inadequate knowledge and preparation in mathematics, which can negatively impact their ability to teach the subject effectively (e.g., Lau, 2021). To have the opportunity to improve pre-service teachers' knowledge and preparation in mathematics, particularly in algebra, there is a need to gain a better understanding of pre-service teachers' ways of reasoning and sense making with regard to mathematical concepts and their applications. However, the number of studies focusing on this topic is limited. Therefore, the aim of this article is to investigate and present an insight into pre-service teachers' reasoning and sense making in algebra using growing patterns. The following research question served as a guide in the analysis of data: What are the characteristics of pre-service teachers' reasoning and sense making in the process of exploring tasks related to growing patterns?

2. Literature review

Growing patterns, as defined in the literature, are patterns that systematically increase or decrease and establish a functional relationship. They can also be patterns composed of a sequence of figures that change from one term to the next in a predictable manner (e.g., Billings et al., 2007). Huntzinger (2008) specifies that a growing pattern involves a dependent variable (a quantifiable aspect of the pattern) and an independent variable (the position of the figure in the pattern). There are two predominant strategies for generalizing a growing pattern: (1) recursive/co-variational, which emphasizes the variation between successive values in a sequence, and (2) explicit/corresponding/functional, which assesses the relationship between a specific value in the sequence and its position within the sequence (Hourigan & Leavy, 2015; Warren & Cooper, 2008; Wilkie & Clarke, 2014). In mathematics, recursion is often employed to define sequences that depend on preceding terms. Several studies indicate that students tend to use a recursive approach when investigating a growing pattern but find it challenging to derive a general rule using this approach (e.g., Rivera, 2011).

In the field of mathematics education, there is no universally accepted definition of the term “reasoning” (e.g., Ball & Bass, 2003; Duval, 2002; Martin et al., 2009; Olteanu, 2022). For example, Ball and Bass (2003, p. 28) assert that “mathematical reasoning is no less than a fundamental skill,” while Duval (2002) restricts mathematical reasoning to strict proofs. Olteanu (2022), on the other hand, defines reasoning as a process that often involves several steps: selecting, exploring, reconfiguring, filtering, encoding, abstracting, and connecting. In this context, “selecting” refers to the process of searching for similarities and differences in order to make deliberate decisions when choosing tools, such as concrete, pictorial, and symbolic representations. An example of selecting would be choosing a table of values that displays term numbers and terms within a pattern. “Exploring” is the process of searching for similarities and differences to gain deeper insights into concepts, counting strategies, or problem-solving strategies. “Reconfiguring” is also a process of seeking similarities and differences, but in this case, it involves modifying the structure or arrangement of a mathematical object. “Filtering” is the process of selecting or eliminating specific types of information, and “encoding” involves using words, phrases, or mathematical symbols to represent information. “Abstracting” is the process of identifying common and essential features, leading to the creation of new concepts. For example, abstracting might be applied to formulate a rule for determining the number of flowers in a figure without having to draw all preceding diagrams. “Connecting” is the process of highlighting associations and relationships between different pieces of content, concepts, or ideas. For instance, it can involve using a general term to evaluate algebraic expressions and determine the term number in a pattern.

The term “sense making” also has varying interpretations in the literature. For example, Van Velzen (2016) employs sense making to describe the process of deepening one’s understanding by applying mathematical concepts. Weick et al. (2005) use the term to refer to understanding that results from retrospective reflection on the decisions and actions of individuals. In contrast, Klein et al. (2006) and Olteanu (2022) argue that sense making is an ongoing effort to comprehend connections in order to anticipate their trajectories and act effectively. According to Olteanu (2022), these connections are established by: recognizing similar attributes among different objects of learning, such as the use of variables in algebraic expressions or formulas; identifying relationships between concepts, such as those between algebraic expressions and equations; profiling, which involves considering features that influence the selection of aspects of content or variation patterns; comparing, as seen in the examination of variables; explaining the link between concepts; fitting one piece of information to the next, akin to the rungs on a ladder, and verifying.

Researchers have demonstrated that sense making in the classroom environment is facilitated through interaction, negotiation, and discussion. It involves leveraging students’ thinking through verbal interactions and employing tasks that draw out and expand upon students’ ideas while encouraging explanations and justifications of mathematical concepts (Mueller et al., 2014; Schoenfeld & the Teaching for Robust Understanding Project, 2006). Furthermore, research has shown that a strong connection between reasoning and sense making requires specific conditions (Olteanu, 2022). The first condition revolves around the use of differences in repetition. According to Deleuze (1994), each repetition generates something new and unique. The second condition is breaking away from established mathematical procedures and exploring new possibilities and connections between mathematical concepts and methods. The third condition relates to students’ discernment of the critical aspects of a concept. The term “critical” in “critical aspects” signifies a significant difference in how students grasp and acquaint themselves with the object of learning (e.g., Marton, 2015). According to Marton (2015), the “object of learning” pertains to the specific concept that students are expected to learn and understand. In this study, the object of learning is the reasoning and sense making of pre-service teachers as they solve tasks involving growing patterns.

3. Theoretical consideration

In this study, the unit of analysis comprises two dimensions: concepts in algebra and critical aspects. As per Olteanu (2022), reasoning and sense making are intricately intertwined and manifest within a heterogeneous multiplicity of elements, which include both human and non-human entities. These elements are brought together by specific relations and connections. Deleuze and Guattari (1987) employ the concept of “assemblage” to underscore the dynamic and fluid nature of systems, as well as the importance of recognizing the relationships and connections between the constituent elements of those systems. In this study, the concept of assemblage is applied to comprehend the ways in which pre-service teachers approach mathematical concepts from various perspectives. This may involve incorporating fundamental algebraic concepts, such as variables, constants, and coefficients, alongside different forms of representation, such as visual or spatial. Assemblage underscores the connections between these diverse elements and how they interact to form a holistic understanding. These interactions are constantly shifting and changing, occurring through written forms and/or collaboration. The concept of assemblage also encompasses the processes of assembly and disassembly. This process aids in understanding how mathematical concepts can be deconstructed into smaller components and subsequently reassembled in different configurations to devise novel solutions. This process of disassembly and reassembly constitutes a pivotal aspect of reasoning and sense making.

The concept of “assemblage” is closely intertwined with the notions of “lines of flight,” “segmentarity,” and “rupture” (Deleuze & Guattari, 1987). When an assemblage becomes excessively rigid or fixed, it can give rise to “lines of flight,” which can be understood as a form of creative disruption that paves the way for new modes of thinking and action. An assemblage is comprised of distinct segments, each organized according to different rules. These segments serve as a means of differentiation, which imparts order within an assemblage. However, they can also be a source of rigidity and constraint. For example, “segmentarity” is observable in the hierarchical and linear organization of mathematical concepts, which can lead to a rigid and unyielding comprehension of mathematics, thereby constraining creativity and innovation. “Rupture” denotes the breaking or destabilization of an assemblage, which can emerge when lines of flight gain excessive influence or when the segmentary structure becomes overly inflexible. Rupture can exert both positive and negative effects, contingent upon the context. In certain instances, rupture can instigate fresh forms of creativity and innovation, while in others, it may result in chaos or destruction.

The model introduced by Olteanu (2022) facilitates the study of sense making and reasoning by examining rupture, segmentarity, and lines of flight in relation to the concept of critical aspects (e.g., Marton, 2015). From the perspective of variation theory (e.g., Marton, 2015), the moment of rupture is construed as the way in which one comprehends a phenomenon, and this, in turn, is directly linked to the critical aspects that are discerned. The qualitative differences in the ways individuals experience mathematical objects provide an opportunity to identify lines of flight, segmentarity, and rupture through the process of disassembling and reassembling mathematical concepts and methods. In this article, the concept of “difference” is employed in accordance with Deleuze (1994), who posits that everything in the world is inherently distinct from everything else. He argues that difference isn’t merely a matter of comparison or opposition but represents a fundamental aspect of reality itself. Deleuze’s (1994) concept of difference is closely intertwined with his notions of repetition and multiplicity. According to Deleuze (1994), repetition doesn't involve mere copying or mimicry but encompasses the transformation of the original through its repetition. This process of repetition plays a vital role in creative processes and gives rise to something novel and unique.

Variation theory (see, e.g., Marton, 2015) posits that to develop a specific capability, such as reasoning, all critical aspects of the object of learning must be simultaneously focused on. These critical aspects are further categorized into intended, enacted, and lived aspects (Olteanu, 2014). Intended aspects (ICA) refer to the content aspects that teachers plan to present in the classroom or intend to use in teaching or testing situations. On the other hand, enacted aspects (ECA) pertain to the aspects of the content that teachers actually emphasize during the lesson. The lived critical aspects (LCA) are the aspects of the object of learning that learners distinguish during or after a lesson or while working on a task. This perspective also asserts that one can only focus on what is discerned, discern what is experienced as varying, and experience variation if one has previously encountered instances of variation (e.g., Marton, 2015). Variation theory underscores the importance of variation in the learning process, contending that it is through variations in how concepts are presented that students can discern the critical features of those concepts. This variation creates what Deleuze (1994) refers to as "difference in repetition."

Analyzing the features of pre-service teachers' reasoning and sense making involves examining how they approach and solve mathematical tasks and how they make sense of mathematical concepts and ideas. According to Deleuze and Guattari (1987), an assemblage is characterized by its potential for change and transformation, as well as by the connections and relationships among its constituent parts. By analyzing mathematical tasks as assemblages in which critical aspects are identified through a process of difference and repetition, new forms and possibilities for understanding the connections and relationships between reasoning and sense making emerge.

4. Methodology

4.1 Context

This research was conducted with pre-service teachers enrolled in a mathematics teacher education course. This course is part of the first year of a four-year primary teacher education program at a large public university in Sweden. Currently, the mathematics teacher education course encompasses topics such as arithmetic, algebra, geometry, probability, and statistics. Each of these four sections is divided into three parts: theoretical, practical, and teaching practice in schools. In the theoretical part, the emphasis is on equipping pre-service teachers with conceptual tools, which includes concepts and theoretical constructs derived from research in mathematics teacher education. The practical part is dedicated to applying these tools through activities proposed by teacher educators. The teaching practice component involves pre-service teachers actively participating in authentic school activities.

4.2 Data collection

In this study, data were collected as pre-service teachers solved algebra problems. Out of the six tasks employed in the study, this paper specifically focuses on one task. Two reasons underlie the selection of this task: firstly, it involved the use of variables in two distinct situations, and secondly, growing patterns constitute a foundational component of the algebra segment of the course. The data for this article comprises the written solutions provided by the pre-service teachers.

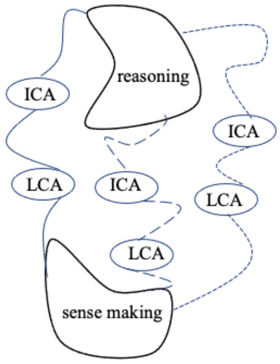
4.3 Participants

This study is a component of ongoing research conducted with pre-service primary teachers enrolled in the aforementioned mathematics teacher education course. The participating pre-service teachers were all aged 19 or older and hailed from diverse socio-economic backgrounds. A total of 79 pre-service primary teachers anonymously participated in a mandatory written exam for the course, where their identities were protected.

4.4 Data analysis procedure

The data analysis employed a qualitative/interpretative approach and was conducted in five steps. In the first step, the tasks were analyzed in relation to the intended critical aspects (ICAs) and their potential for fostering reasoning and sense making. In the second step, lines of flight, segmentarity, and rupture were identified with respect to the ICAs and lived critical aspects (LCAs). In the third step, the written solutions provided by the pre-service teachers were independently analyzed by two researchers using content analysis (Macnamara, 2018). The aim of content analysis was to uncover patterns, themes, and meanings within the data, and to draw inferences and make generalizations based on these findings. The fourth step involved comparing the researchers' analyses to interpret the results, drawing inferences and making generalizations based on the findings. In the fifth and final step, the results were compared to existing theoretical concepts. During this process, the analysis of lines of flight, rupture, or segmentarity commenced by identifying the critical aspects that were being challenged or resisted in the process of reasoning and sense making (as illustrated in Figure 1). Subsequently, the focus of the analysis shifted towards examining rupture or discontinuity, which opens up new possibilities and potentials for change. The analysis also considered the ways in which the line of flight transcended segmentarity by establishing new connections and relationships that cut across established boundaries and structures.

Figure 1: Reasoning, sense making, rupture, segmentarity, and the lines of flight



In this figure, segmentarity is represented as a long line and a dot, the lines of flight as a solid line, and the rupture as dots.

5. Results

One of the tasks solved by the pre-service teachers was:

Examine the number of flowers in the following pattern.

Figure 1 Figure 2 Figure 3

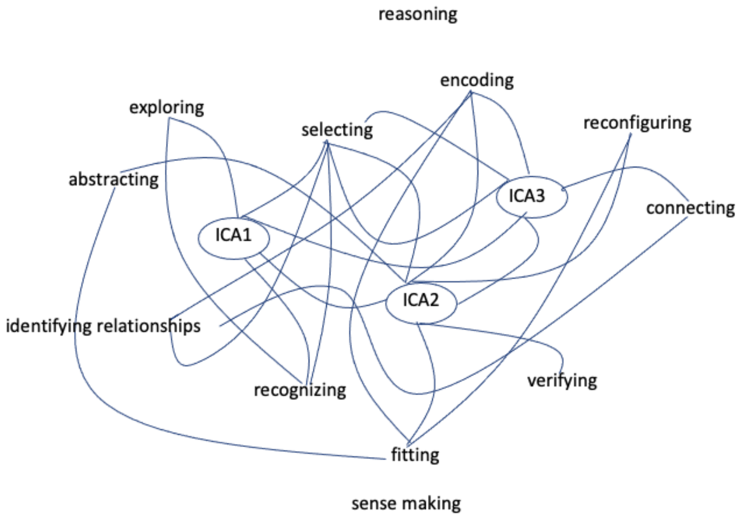
a) How many flowers does Figure 10 have?

b) How many flowers does Figure n have?

c) What is the number of the Figure containing 100 flowers?

The intended critical aspects (ICAs) along with the characteristics of reasoning and sense-making are presented in Figure 2. The task is characterized by three ICAs. The first ICA (ICA1) involves recognizing how the pattern changes, which entails exploring the pattern and selecting a tool that can provide support in understanding how the pattern grows both numerically and geometrically. This differentiation aids in organizing information and fostering an understanding that links the term number to the value of the term, thereby emphasizing the discernment of ICA1. This is not merely a matter of comparison or opposition but rather a fundamental aspect of the pattern. The second ICA (ICA2) is focused on determining the general term of a linear growing pattern using algebraic expressions that involve two operations (multiplication and addition). The reasoning process commences with the selection of the variable “n,” where “n” represents the term number. It involves formulating the numerical rule as an algebraic expression, encoding the algebraic expression by determining the correct expression, and abstracting the common features within the pattern. Sense making in this context entails piecing together bits of information to construct a comprehensive picture and identifying pattern rules for reasonableness and accuracy.

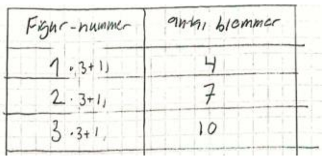
Figure 2: Growing patterns – intended critical aspects (ICAs), reasoning and sense making



The third critical aspect (ICA3) involves establishing connections between solving first-degree equations and determining the term number in a pattern using the general term. At this stage, connecting the algebraic expression or function to the algebraic equation for ascertaining the term number within the pattern extends the task's complexity. It emphasizes the selection of "n" as the unknown variable and the encoding of the equation. Sense making in this context revolves around identifying relationships between concepts. The task assemblage is constructed based on the lines of flight. These lines of flight take various forms and involve a focus on the processes of assembly and disassembly of mathematical concepts, with particular attention to the relationships between different ideas and topics.

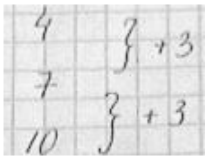
In task 1a, the majority of pre-service teachers (75%) created an input-output table for the first three stages, illustrating the total number of floors used in each stage of the pattern. This indicates that the pre-service teachers sought a tool to discern relationships in growing patterns (selection) and delved into recurring patterns involving reconfigurations. The sense making demonstrated by the students reveals that they recognized the correlation between each term in the growing pattern and its corresponding figure number. For instance, in the sequence 1, 4, 7, 10, …, the first term is 1, the second term is 4, the third term is 7, and so forth. The pre-service teachers also documented the patterns in a table of values (see Figure 3) that displays the figure number and the growth rate (profiling).

Figure 3: Selecting and connecting



These students demonstrated their ability to translate visual information into numerical data and discerned how the pattern changes (lived critical aspect 1 - LCA1).

Figure 4: Recognizing

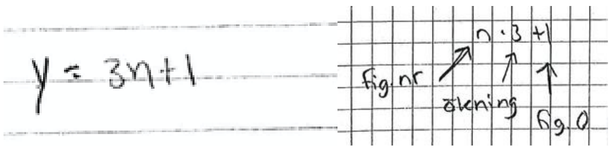


The line of flight emerges in the intersection between the sense making process and the reasoning process, offering an opportunity to break free from the established structure to which they belong. Twenty percent of the students correctly identified that the growth rate is 3 (selection). Their sense making process primarily involved recognizing the common attributes that differentiate the first term from the subsequent terms (as depicted in Figure 4). In this particular case, the students' description of the pattern focused solely on the variation they observed within the pattern.

The students' reasoning was characterized by the selection of numbers to delineate the pattern's classification. In doing so, the pre-service teachers employed a recursive approach during the investigation of the pattern, thereby establishing segmentarity that underscores how arithmetic calculations can potentially resist the utilization of algebra. However, despite this approach, the pre-service teachers were unable to calculate any term within the pattern when provided with the term number. Additionally, five percent of the students did not complete the task.

In Task 1b, 30 percent of the pre-service teachers devised a rule by selecting tools such as letters, symbols, numbers, and a function or an algebraic expression (see Figure 5).

Figure 5: Selecting and explaining

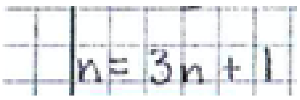


To illustrate their sense making of algebraically expressing the generalization of the pattern, the pre-service teachers opted for: identifying relationships between variables and an algebraic expression; profiling the characteristics to construct the algebraic expression; and comparing variables and explaining the link between variables by fitting one piece of information to the next one.

These students recognized the variable “n” and comprehended the role of variables in constructing an algebraic expression (lived critical aspect 2 - LCA2). A line of flight emerged at the intersection of reasoning and sense making. The students discerned what constitutes the general term of a linear growing pattern using algebraic expressions that involve two operations (multiplication and addition).

Thirty percent of the students selected an equation to represent the general term of the growing pattern (see Figure 6). However, these students did not distinguish the use of variables in the context of growing patterns and did not implicitly grasp the concept of function and equation (recognition). The utilization of the variable “n” to represent both the total number of flowers in the figure and the stage number indicates that these students failed to discern the input and output within the growing pattern (profiling). At this point, a rupture emerged between reasoning and sense making. This rupture refers to moments of disruption or discontinuity in establishing the equation structure.

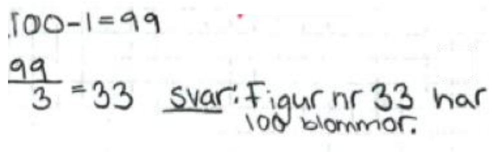
Figure 6: Selecting and explaining



Twenty percent of the students understood that the growth rate is 3 but were unsure about how to incorporate the constant (identifying relationships). Meanwhile, 15 percent attempted to employ proportional reasoning; they believed that if there were four flowers in Figure 1, then 10 multiplied by 4 should equal 40 flowers in Figure 10 (verifying). At this stage, segmentarity and rupture emerged between the reasoning and sense making processes. Five percent of the students did not complete the task.

In Task 1c, 55 percent of the students wrote an equation to determine the figure containing 100 flowers (connecting), while 40 percent used the algebraic expression and substituted the variable (filtering). Approximately half of the students recognized the connections between solving the first-degree equation and determining the term number in a pattern, employing informal methods to solve the equation (lived critical aspect 3 - LCA3) (see Figure 7).

Figure 7: Informal methods to solve the equation

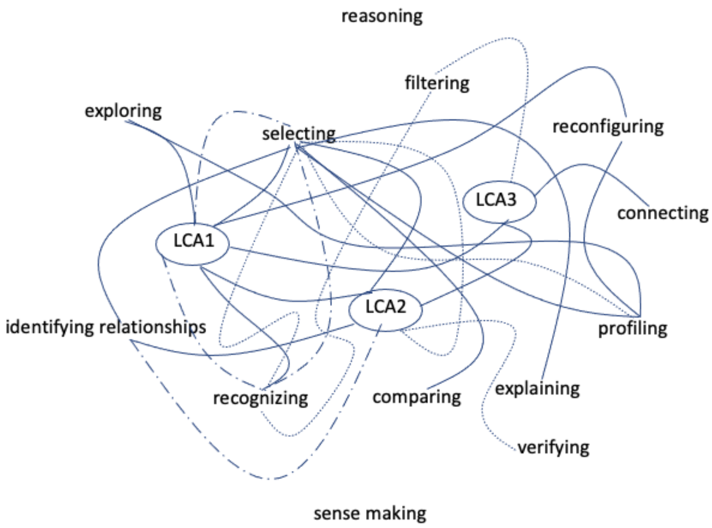


(Translation: Figure 33 has 100 flowers)

The students who replaced the variable with 100 did not distinguish the role of the variable in building an algebraic expression (recognition). Five per cent of the students did not solve the task.

Figure 8 summarizes the characteristics of pre-service teachers' reasoning and sense making.

Figure 8: Growing pattern – lived critical aspect (LCA), reasoning and sense making



The lines of flight, segmentarity, and rupture establish connections between the elements of the assemblage. These connections are established at each lived critical aspect (LCA), and the most common method of reasoning is the selection of tools (both pictorial and symbolic) through deliberate decision-making. If the tools are not chosen carefully, the assemblage is characterized by lines of segmentarity and rupture. The assemblage evolves through the incorporation of associated and linked elements of reasoning and sense making, which are diverse in nature. An assemblage also encompasses a focus on the processes of assembly and disassembly, along with their interactions, contributing to the formation of a comprehensive whole.

6. Discussion

The current study has explored the characteristics of reasoning and sense making exhibited and employed by pre-service mathematics teachers in the process of engaging with growing patterns tasks. Both the task's assemblage and the assemblage of the pre-service teachers' reasoning and sense-making have been analysed.

Researchers have demonstrated that sense making is supported by interaction, negotiation, and discussion (Mueller et al., 2014; Schoenfeld & the Teaching for Robust Understanding Project, 2006). In this study, both reasoning and sense making are supported by the task. By conceptualizing the task as an assemblage in which intended critical aspects (ICAs) link the elements of reasoning and sense making, it is possible to elucidate how difference and multiplicity function to emphasize the key facets of the object of learning. Consequently, this facilitates a deeper understanding of how tasks provide opportunities for pre-service teachers' reasoning and sense making. In this study, the task assemblage involves a repetition of the original growing pattern, leading to what Deleuze (1994) refers to as "difference in repetition," resulting in the emergence of something novel and distinctive. The novelty and distinctiveness in this study pertain to the facets of reasoning, including selecting, exploring, reconfiguring, abstracting, encoding, and connecting, which are pivotal for comprehending the structure within the growth pattern. The encoding process, in particular, proves to be a valuable mechanism for reiterating foundational concepts. These concepts revolve around the distinction between variables as varying quantities and variables as unknowns, a distinction that must be recognized to make sense of a growing pattern. Fitting one piece of information to the next and recognizing relationships between concepts provide the opportunity to assist pre-service teachers in establishing a general rule to describe the nature of the growing pattern, thereby comprehending its generalization. The components of sense making associated with the selected task encompass recognition, identification of relationships, comparison, explanation, and verification. The ICAs create lines of flight that interconnect reasoning and sense making within the task assemblages. These lines of flight underscore the potential for creativity, fostering novel modes of thinking and action.

As noted in previous studies reasoning is a process, and sense making is a continuous effort to understand connections to anticipate their trajectories and act effectively (Klein et al., 2006; Olteanu, 2022). Regarding the characteristics of pre-service teachers' reasoning and sense making, the present study reveals that the elements within the assemblage constitute complex and dynamic systems that continuously evolve and transform through the interplay of lines of flight, segmentarity, and rupture within the assemblage. This highlights the extent of variations in ideas about growing patterns. These variations exemplify how pre-service teachers engaged with the intended critical aspects (ICAs) through processes of selecting, exploring, filtering, reconfiguring, and connecting. Often, reasoning is constrained by existing concepts and categories, which can impede pre-service teachers' capacity for creative thinking. This could be an explanation for way the pre-service teachers often have inadequate knowledge and preparation in mathematics (e.g., Lau, 2021). Numerous lines of flight, segmentarity, and rupture within the assemblage indicate distinctions between ICAs and lived critical aspects (LCAs). This distinction can be understood as differences between various options or ideas for reaching a singular or definitive conclusion or decision regarding a dependent variable (a quantifiable aspect of the pattern) and an independent variable (the position of the figure in the pattern) (Huntzinger, 2008), or about the relationship between a particular value of the sequence and its position in the sequence (Hourigan & Leavy, 2015; Warren & Cooper, 2008; Wilkie & Clarke, 2014). Singularity can also play a role in reasoning, particularly in tasks where a unique approach or solution is required. Understanding the relationship between difference and singularity is crucial for generating new forms by recognizing critical aspects that embrace diversity and difference. It also underscores the potential for creativity and innovation that emerges from this diversity. As for the features within pre-service teachers' sense making of the growing pattern, it is evident that recognition, identification of relationships, profiling, comparison, explanation, and verification are integral elements of the assemblage. These elements are interconnected with LCAs through multiple lines of segmentarity and rupture.

7. Conclusion

The concept of segmentarity indicates that the process of breaking down a complex task into smaller, more manageable parts limits the possibilities for change and transformation among pre-service teachers. The results suggest that segmentarity creates disciplinary boundaries and hinders reasoning and sense-making because segmented thinking tends to be hierarchical and focused on maintaining the status quo. Therefore, it is essential to recognize and break down these boundaries to facilitate more creative and innovative reasoning and sense-making in mathematics. On the other hand, rupture represents moments of potential transformation and change when pre-service teachers break away from the constraints of the past and create new possibilities for the future. It is a transformative process that allows for new lines of flight to emerge. Rupture in pre-service reasoning and sense-making can be seen in the connection of concepts or ideas that challenge established mathematical structures and ways of thinking. This study demonstrates that the concepts of assemblage, line of flight, rupture, and segmentarity can be useful tools for analyzing reasoning and sense making in mathematics and for understanding how mathematical knowledge is created, challenged, and transformed over time. Deleuze and Guattari's (1987) concept of assemblage also provides an opportunity to perceive reasoning within oneself as complex, dynamic systems that are constantly evolving and changing.

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