Making Decisions Under Uncertainty: A Conceptual Framework[1]

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Abstract

This paper presents a conceptual framework that summarizes existing approaches and theories related to decision-making under uncertainty. The paper focuses on the process of decision-making, how all its parts should be combined, and how they should be reflected in decision rules. Although this topic is not new, techniques for making decisions and managing uncertainty have improved significantly over the past fifty years, alongside the development of probability theory and fuzzy set theory. The paper aims to bring together the main components of the decision-making system, including the concept of uncertainty, different ways of thinking, creating models, and approaches to decision-making. These issues are examined in their dialectical relationship. Instead of providing detailed explanations of each technique, the presentation seeks to explain their essence and practical applicability. In addition to data-driven decision-making, the presentation also explores non-quantitative methods.

Keywords: risk, probability, Bayesian, fuzzy sets, decision theory, prospect theory

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1. Introduction

We believe that the purpose of human activity, including entrepreneurship, is to make logical decisions and implement them. Fortunately, we do not have to make decisions every day. During most activities, there are times when we must make decisions and other times when we simply carry out what we have decided. The field of decision theories focuses on the first type of human activity. A comprehensive examination of decision-making, particularly in business, is provided in (**Buchanan, O'Connell, 2006**). The authors trace the history of decision theory from ancient Greek philosophers to the present. Modern approaches to decision-making emerged in the mid-20th century through collaborative efforts across various academic disciplines.

Decision theory, as a whole, is a discipline that applies universally across different subject areas. Economists, statisticians, psychologists, political scientists, sociologists, philosophers, and entrepreneurs all successfully utilize it. Economists, in particular, are intrigued by the methodology of decision-making, which is not surprising as many of them have made significant contributions to this field while developing economic theories.

It is crucial to emphasize that most decision-making techniques primarily rely on quantitative analysis and require appropriate mathematical models. Developing these models is a skill typically held by individuals with analytical and mathematical mindsets. However, it is not always the case that the person making the decision possesses this ability. Nevertheless, the decision needs to be made, and others should be convinced of its accuracy. What should be done in such situations? Often, decision-making practices exist as a means of persuading the decision-maker and their surroundings using non-quantitative models. These models frequently employ narratives and metaphors to convey their message. Noteworthy is the synthesis of this approach presented by (**Lehner J., 2002).** We refer to these models as semantic models. The correctness of a decision, of course, should ultimately be validated through practice. Nonetheless, decision-making based on a semantic model can instill a strong sense of conviction within the decision-maker and their team, which is no less valuable than the correctness achieved through a quantitative approach. The confidence in a decision's correctness often surpasses the belief stemming from using quantitative methods. We are striving to establish a suitable form within the decision-making methodology and will embark on this journey with this publication.

It is especially worth noting that most decisions are not momentary. They require time to comprehend the event or phenomenon with which the person is in contact with the decision being made. Moreover, the decision-making process is very complicated and requires a particularized and systemic view, which we will do in the paper.

2. Decision-Making Conceptual Framework

Many authors have examined the topic of creating a conceptual framework to build a cohesive methodology of decision-making. They have used different principles to accomplish this. Here are some exciting approaches.

(Holsapple C.W., Moskowitz H. 1980) suggested theoretical concepts and an organized overall framework for studying the decision-making process. They proposed a set of factors that are used to construct various hypotheses. Additionally, they provided empirical evidence that supports these hypotheses and argues for the feasibility of the proposed conceptual framework. The suggested framework can be seen as an extension of earlier research in this field. It identifies the underlying abilities that influence behavior and provides a structure for developing alternative models to describe behavior. These abilities are referred to as the aspects of decision-making. The authors introduced three main aspects and four related attributes of decision-making. The first three aspects include (1) power, which is the ability to govern, (2) information collection, and (3) the ability to formulate a plan or model. The next three aspects involve the ongoing adjustments between (4) information collection and design, (5) power and information collection, and (6) between the model and power. The final ability relates to adaptation, which refers to the ongoing adjustments among the other six facets.

We can find some frameworks for generalization in the field of political decision-making. According to **(Kindon, 1984),** the policy entrepreneur is conceptualized through three variables: Agendas, Alternatives, and Public Policies. These three variables represent the Multiple Streams Framework, which provides opportunities for the policy entrepreneur to identify and solve problems in the political context. **(Lindblom, 1959, 1979)** suggests that decision-making is typically formalized as a means-to-an-end relationship using two variables: means and goals. The means should be evaluated and chosen based on the selected goals, which are determined independently before selecting the means. **(Sabatier and Mazmanian, 1980)** propose multiple premises related to three main factors that can be seen as conceptual variables. These factors include (1) the treatability of a problem, (2) the ability of laws to structure implementation, and (3) non-statutory variables that influence implementation. These conceptual variables are then broken down into several sub-variables and become a power tool for making political decisions.

(Snowden and Boone, 2007) present a framework that classifies decision-making contexts based on the degree of connection between cause and effect. The framework includes four conceptual situations: simple, complicated, complex, and chaotic. In the simple case, clear cause-and-effect relationships are evident to everyone, and there are definite right answers. Cause and effect relationships exist in the complicated case but may not be immediately obvious. Expert diagnosis is required, and more than one right answer is possible. The complex situation assumes that there are no right answers, but patterns can be observed in retrospect. Efforts need to be made to explore the situation and understand what is happening to identify the relationship patterns. Finally, in a chaotic state, the relationships

between cause and effect are constantly shifting, and no manageable patterns exist. In this case, directive leadership is necessary to establish order.

In this paper, the approach to developing a conceptual framework for decision-making deals with the process of understanding the situation in which a person needs to make a decision and subsequently take action. It is based on the assumption that decisions must be made in uncertain situations. Therefore, the first step is to identify the type of uncertainty being faced. Then, the individual must learn to think effectively about this uncertainty, or in other words, choose an appropriate way of thinking. Before making and carrying out a decision, the person must create a model that will guide their subsequent actions. Finally, once the model is established, the individual can make their decision. However, this is not an easy task as it requires choosing the criteria and method for making the decision, which can be challenging. This choice is complicated because there are many different approaches to decision-making, and selecting a specific approach is often not straightforward.

Let "D" represent a decision made in order to achieve a desired outcome. We will consider it as a function of four conceptual variables, as described below.

$$D = f(U, P, M, T).$$

These conceptual variables have the following meanings:

$U \in \mathfrak{U}(U_1, U_2, \ldots, U_K)$ represents "UNCERTAINTY", which refers to the type of uncertainty that a decision-maker takes into account when finding a solution to a problem;

$P \in \mathfrak{R}(P_1, P_2, \ldots, P_N)$ stands for "PARADIGM", which refers to the approach (the way of thinking) about uncertainty;

$M \in \mathcal{M}(M_1, M_2, \ldots, M_L)$ represents "MODEL", which is the tool that a decision-maker can use to describe a phenomenon or situation being considered;

$T \in \mathcal{T}(T_1, T_2, \ldots, T_S)$ introduces ”THEORY", a set of methods for making a final decision based on the selected criteria.

The paper briefly explains each conceptual variable and tries to establish their relationship. It emphasizes that it is pointless to consider these components separately because they are strongly interconnected in a dialectical way. Consequently, the paper consistently refers to other parts while describing each variable.

As shown in the above function, uncertainty is the main conceptual variable and serves as the foundation of the entire system. It is recognized as an inherent state in nature and the world. Before understanding how to manage behavior in the face of uncertainty, one must first grasp how uncertainty operates.

Once we acknowledge that uncertainty is unavoidable, we need to accept it. After accepting it, we must develop a positive attitude towards it. We should not view uncertainty solely as a negative factor in our lives. It is crucial to understand that uncertainty brings new opportunities, meanings, and values. If we desire, we can embrace uncertainty and hope for a mutual understanding with it. We can achieve this by learning to think effectively in uncertain situations.

The probabilistic paradigm is the most common approach to assessing uncertainty, which involves accepting future events' ambiguity and evaluating their likelihood. However, this is not the only method. Another helpful approach is fuzzy sets, which is a more constructive way of thinking. The decision-maker must choose their preferred method of thinking to form an idea of what may occur in the future, in other words, they must build a model.

In intelligent human activity, the model always comes before decision-making. The model formalizes the decision-maker's way of thinking into a specific image that prompts action. A common model for decision-making is mathematical, created with symbols. As mentioned previously, there are other methods of modeling, such as semantic models. There may be other techniques, such as graphics. Ultimately, works of art can also be viewed as modelling a process or occurrence, ultimately sparking a desire to act. Beethoven's Ninth Symphony is a prime example of this.

Making a decision is the final step in the general system. We are prepared to make a decision, but there must be established rules for doing so. By creating a model - such as using probabilistic representations - the decision-maker can choose the one that best suits him/her from various options. It's crucial to have a specific criterion for decision-making and use it to make the best possible choice. With mathematical models, the process is relatively simple. Once a criterion is selected, an algorithm immediately calculates the best option. Consequently, the accountability for making the decision is shifted partly to the algorithm. However, using non-quantitative methods implies that the responsibility remains with the decision-maker, increasing the likelihood of making a mistake.

So, once we realize that uncertainty is inevitable and learn to think adequately and create a model, we must complete the process and make a decision. We will dwell into more details on each of these components of the system.

3. Uncertainty

Uncertainty has fascinated thinkers, researchers, entrepreneurs, and ordinary people throughout much of human history. Generally, uncertainty is viewed as the opposite of certainty and complete predictability. However, this section does not aim to provide a definitive and clear definition of uncertainty. Instead, it aims to showcase the variety of perspectives and theories on the subject to help the readers form their own attitudes.

To grasp the meaning and substance of uncertainty, it is advisable, to begin with philosophical concepts and abstracts. Exploring the underlying essence of the concept will aid in determining its practical application. Therefore, let us dedicate some time to this endeavour. This is how the phenomenon of uncertainty has been perceived throughout history.

Since ancient Greek philosophy, the attitude to uncertainty has been radical, i.e., determinism has been excluded. Let's first trace the dynamics of comprehension of the phenomenon of uncertainty with a bunch of **Socrates, Plato, and Aristotle**. It’s a well-known **Socrates** statement: "As for me, all I know is that I don't know anything." Like a

paradox, there is a belief that uncertainty, or admitting that one does not know something, is necessary for true wisdom. Two conclusions immediately follow from the teachings of Socrates. Firstly, those who claimed to have all the answers were often the ones who were most ignorant, as they closed themselves off to new ideas and perspectives. And then, by embracing uncertainty and remaining curious and humble, one can continue to expand their knowledge and understanding of the world.

Plato attempts to structure the attitude towards uncertainty. Here is his framework. He introduces the chorus as the "perceiving principle" or a mother, explaining the world's origin, and the eidos as the idea model or a "father" of all things generated. Beginning with Plato, humans started contemplating semantic uncertainty. This uncertainty arises due to a vague or ambiguous understanding of the language used to describe our world. His main concern was that it is necessary to use words in their reliable sense: ”The fire constantly reveals itself to one or the other”. It is necessary to speak not of “this” but of “such” fire. Also, water should not be called "this" but "such”. Indeed, it is not necessary to ascribe to all such things the stability expressed by the words "this" and "this", by means of which we designate something definite ... [because] they elude us... They are born, passing in a circle from one to another”.

Later **Aristotle** attempted to build a structure of uncertainty. He argues that uncertainty is a natural part of life and cannot be avoided; it arises from our limited knowledge and understanding of the world and the unpredictable nature of the world itself. People can use reason and logic to reduce uncertainty and make informed decisions by gathering information, analysing evidence, and considering all possible outcomes when faced with uncertainty. In other words, uncertainty is a challenge to be overcome through careful thought and action rather than as an insurmountable obstacle.

Prominent philosophers of the later period continued to develop the theme of uncertainty. **Hegel’s,** dialectical method of reasoning acknowledges the role of uncertainty and contradiction in the process of arriving at the truth. Truth is not a fixed and static concept but rather a dynamic and evolving one that emerges from the interplay of opposing forces and ideas. Uncertainty and contradiction are necessary components of the dialectical process, as they help reveal the limitations and contradictions of existing ideas and open up new possibilities for understanding and knowledge. From the point of view of practical behaviour, the attitude of **(Heidegger, 1962)** is noteworthy. According to his teaching, uncertainty is an inherent part of human existence, and we must confront this uncertainty to understand our place in the world. Moreover, the humane tendency to seek certainty and stability can lead us away from an authentic experience of being. In his philosophy, Heidegger emphasizes the importance of being present at the moment rather than getting caught up in our fears and anxieties about the future. He encourages us to embrace the ambiguity and mystery of existence rather than trying to impose rigid categories and definitions onto the world around us.

We could continue discussing various philosophical views regarding uncertainty, but now it's time to focus on decision-making. However, before we do, it is important to point out that

attitudes towards uncertainty are often expressed in artistic ways. As **Voltaire** once said, "Uncertainty is an uncomfortable position, but certainty is absurd".

In ancient times, people faced the uncertainty of the future and its associated risks but could not counter them with anything rational to predict it to some extent. Therefore, in their projection of the future, they relied on oracles, soothsayers, shamans, and other fortune tellers.

Much later, humanity discovered the law of physics, proving that at the heart of matter, everything material in this world consists of, is uncertainty. This law was called the "uncertainty principle" and was first formulated by the outstanding German physicist **Werner Heisenberg**. The uncertainty principle postulates that it is impossible to determine a microparticle's position and momentum with the same accuracy. The boundaries set by this principle cannot be overcome by improving the means of measurement. At least for the time being, the uncertainty principle is considered a fundamental proposition of quantum mechanics, according to which every object in the universe behaves both as a particle and a wave.

Aside from the material interpretation of the world, it can be argued that some absolute force created the world uncertain. In other words, uncertainty is an intrinsic property of all things.

One of the first to attempt to structure the decision-making process in the face of uncertainty was the Scottish philosopher and economist (**David Hume, 1772).** According to his framework, there are three forms of human reason: Knowledge, Proofs, and Probabilities. By knowledge, he means the assurance arising from the comparison of ideas, by proofs, those arguments, which are derived from the relation of cause and effect, and which are entirely free from doubt and uncertainty, and finally, by probability, that evidence, which is still attended with uncertainty.

The most cited economist from the past **(Adam Smith, 1776),** recognized that those making economic decisions often faced incomplete knowledge. He believed economic knowledge was not precise or clear but uncertain, vague, or even non-existent. Additionally, he questioned the value and feasibility of probabilistic knowledge. **Smith** placed more importance on relying on sound human reasoning and moral principles when making decisions in uncertain situations rather than relying on probability calculations. His final verdict read, that Human knowledge is limited, and economic events do not happen randomly. Consequently, probability calculus is of little relevance to economics. Indeed, the Smithian interpretation of uncertainty did not become dominant in economic thinking. Instead of virtue ethics, probabilistic reasoning gained importance in the economic discourse on the imperfection of human knowledge.

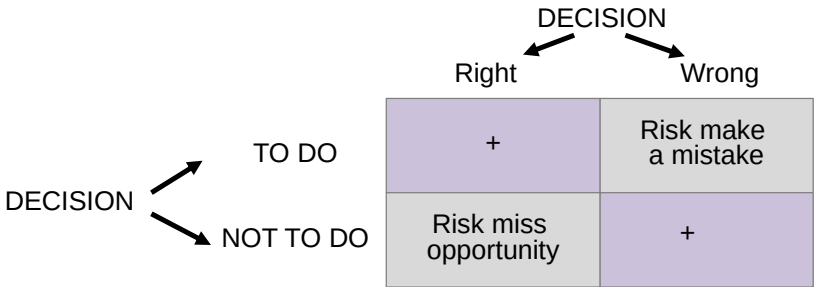
First, let's understand the sources of the origin of uncertainty. The generally accepted point of view defines two kinds: aleatoric uncertainty and epistemic uncertainty. And we consider them in some detail. But there is a solid intention to introduce one more type of uncertainty, which we will call semantic. So, let's devote some attention to the kinds of uncertainty regarding origin.

First, we encounter uncertainty when surrounded by an unstable, ever-changing physical environment. This kind of uncertainty is called aleatoric uncertainty and reflects the world's stochastic nature. The second manifestation of uncertainty is the lack of knowledge or understanding of events and phenomena. Such uncertainty is called epistemic uncertainty. It is secondary to physical uncertainty. When people try to guess the roll of a die, the sex of a child, and the outcome of a horse race, they deal with aleatoric uncertainty. Aleatoric uncertainty might transfer into epistemic. Here is a simple example: in tossing a coin before the coin flip, we experience aleatory uncertainty; if we flip the coin and hide the result, our psychology switches to epistemic uncertainty. Usually, people experience epistemic uncertainty as more aversive. At the same time, people most often face the manifestation of epistemic uncertainty.

Two approaches dominate management to understanding risk. The first approach understands risk only as losses and dangers that await the company. In Merriam-Webster's Collegiate Dictionary, we will find a definition of risk as the “possibility of loss or injury”, and to take a risk means “to expose to hazard or danger”. This is how most people understand it, i.e. risk is determined mainly negatively. But this is correct only in relation to natural and man-made disasters, but not in the case of economics and management. The keywords in this approach are stability and security. This is not what management does. Management should ensure growth associated with the maximum increase in its value. Stability in a competitive environment always means stagnation which leads to crisis. A guaranteed result is the opposite of growth.

We can formalise such a situation by means of the error matrix presented in Fig. 1.

Figure.1. Project Error Matrix



Source: Author’s suggestion

According to the scheme, there are two risks: a) rejecting a project if it is correct and b) accepting a project if it is wrong. Every person faces such risks. Most people are afraid to make the wrong decision. Fewer people experience the risk of missing out on an opportunity to become more affluent. In business, it's more complicated than that. You can't look at the opportunities (good) and the threats (harmful) separately. If you focus only on threats without considering the positive opportunities, the business must be closed immediately. The fight against threats cannot be the goal of the company. The company exists in the hope of success but knows there are failures.

There is another significant source of uncertainty, which we call Semantic Uncertainty. People constantly experience the negative impacts of ambiguous interpretations and understanding particular concepts and terms. The uncertainty of the interpretation of words creates uncertainty in understanding the text. Frequently this leads to misunderstanding in the transmission of meanings and, as a result, to wrong choices.

To decompose this kind of uncertainty, let us start with (Nalimov,1989). In his Probabilistic theory of meanings, he assumes that consciousness is a triad of meaning, texts, and language. The text reveals the meaning through the sign system of the language. The main assumptions of the approach are the following: (1) every word has a variety of meanings; (2) a text consisting of words also has a variety of meanings; (3) the variety of meanings is modelled in the form of the probability distribution.

General theoretical standpoints are needed to study the phenomenon of uncertainty.

We start with the approaches to understanding uncertainty advanced by Keynes and Knight. According to (**Keynes, 1921**), uncertainty is ontological (especially concerning social phenomena), and probabilistic logic is more adequate to our world of uncertainty than ordinary logic. Probability refers not to the characterisation of an event but to our assessment of the truth of a statement that a given event will occur (under certain conditions).

Wanting to find a place for uncertainty in the overall decision-making system **(Frank Knight, 1921)** suggested differing risk and uncertainty. This distinction is based on an analysis of probability situations. Situations of 'risk' are ones in which it is possible to determine numerically definite probabilities (usually statistical frequencies), whereas situations of 'uncertainty' are characterised as impossible to do so. This juxtaposition of uncertainty and risk seems more semantic (i.e., relating to meaning in language and logic) than meaningful in relation to decision-making. Knight's distinction between risk and uncertainty is theoretically meaningless and practically irrelevant to making decisions.

Along with the above descriptions of the phenomenon of uncertainty, this presentation proposes a more narrow-ranging classification and considers its implications. Our task is to offer the most straightforward possible idea of the decision-making system, focusing on applied aspects. First, we note that uncertainty should be considered between two extremes. The first extreme is characterized by a state of complete certainty or determinacy when the consequences of actions are known. It is not attractive from a decision-making standpoint since there is no alternative. The opposite extreme can be characterised as absolute uncertainty (ignorance). It seems appropriate here to use the notion of "chaos" and to formalize it accordingly. We consider uncertainty a state of nature or our understanding of nature. And it is supposed to be done for all three origins of uncertainty: aleatory, epistemic, and semantic. And we consider the risk as a primary consequence of the uncertainty. If we compare the three states of nature, we can figure out the following differences:

- In a state of chaos, we can neither model nor assess the risks and opportunities.
- Under uncertainty, we can assess risks, and there are approaches for this (which we will briefly touch upon below),
- In a state of complete order, such a task is simply meaningless.

Moreover, another distinctive feature of the state of uncertainty is the decision maker's responsibility for the results of his activities.

4. The Ways of Thinking

Turning to the second component of the system, we immediately discover the variety of ways of thinking regarding uncertainty.

According to (**John Locke, 1690**), "the probability is nothing but the appearance of such an agreement or disagreement by the intervention of proofs, whose connexon is not constant and immutable, or at least is not perceived to be so, but is, or appears, for the most part, to be so, and is enough to induce the mind to judge the proposition to be true or false, rather than the contrary".

(**Ludwig von Mises, 1964**) turned to the problems of uncertainty and probability, setting himself on creating a universal deductive science of human behaviour, known as praxeology, and its particular part - the theory of market behaviour, or catallactics. Mises considers uncertainty (albeit related to the limitations of our knowledge) to be a condition for freedom of choice. "The most that can be achieved relative to reality is a probability".

(**George Edward Moore, 1903**) proceeded from the need (albeit ideally) to assess the probability of all possible consequences of our actions and to determine the expected value (positive or negative) to which these actions can lead. In the appeal to probability, understood traditionally, he saw the manifestation of rationality. "We can, in all likelihood, only claim to consider the consequences of actions during the so-called 'near' future …

At one time, the founder of axiomatic probability theory, **Andrew Kolmogorov** said: "We have at least one severe advantage – we own probabilistic thinking (**see Nalimov, 1989**)." Probabilistic thinking occupies an intermediate position between deductive logic and intuition. Central to probabilistic thinking is "assumption": a person assumes that an event is about to happen but is unsure. They must assess the chances that this event will occur. Based on this assessment, he must make a specific decision, for example, to bet on a particular horse in a race.

And now, let's try to understand what probabilistic thinking is. It is generally accepted that there are three ways to think probabilistically: classical (logical), frequency and subjective. The first two are objective because they are independent of the decision-makers opinion. At the same time, the objective understanding of probability has limited application in practice since, in most practical cases, the decision-maker does not encounter situations where there is a symmetry of outcomes or a sufficient number of experiments can be conducted.

In the most common sense, subjective probability is a way of stating our belief in the validity of a random event. A subjective probability is anyone's opinion of the chance for an event when he/she has no sample of trials (so he cannot use relative frequency) and no theory (so he cannot use theoretical probability).

The subjective theory of probability was developed by (**Frank Ramsey 1931**), (**Bruno de Finetti, 1975**) and (**Leonard Savage, 1954**). Their versions of the theory are broadly similar.

Let's consider the subjective probabilities as betting quotients. Ramsey – de Finetti's first general idea was to measure degrees of belief by betting. This was made precise by introducing betting quotients. The logical conclusion of this chain was that for betting quotients to be coherent, they must satisfy the axioms of probability and thus can be regarded as probabilities.

Savage's contribution to the subjective probability way of thinking is that he paved a bridge between subjective probabilities and making decisions, i.e. made the subjective probability theory more practical. Savage thus ties together the idea of subjective probability advocated by Ramsey and de Finetti with the concept of expected utility derived by (von Neumann and Morgenstern, 1947).

Now we stress one more critical issue of subjective interpretation of the probabilistic way of thinking. From the classical point of view, a subjective probability of any event is a matter of personal preferences. But another judgment exists (**Richard Jeffreys, 2004**), which argues that there is a unique admissible probability assignment, given the totality of information you can access. And this judgement brings us to the Bayesian approach to modelling uncertainty. (**Tomas Bayes, 1763**) outlined a method known as Bayes' rule for updating probabilities considering new information. Bayes' method does not specify how the prior probabilities to be updated are determined. However, if we combine the interpretation of subjective information as a degree of belief with factual data, we can reduce the ambiguity regarding the assignment of subjective information and use it to make decisions.

Let's look at ideas and approaches that are very close to the subjective probabilities considered, if not coincide with them completely. This will allow us to enrich the overall picture of the description of uncertainty by subjective probabilities.

The imperfection of the process of assigning subjective probabilities was discovered in the works (**Tversky and Kahneman, 1982**), which proved that biases in judgments reveal some heuristics of thinking under uncertainty. They are (i) representativeness, (ii) availability, and (iii) adjustment and anchoring.

There are two more exciting ways of thinking. The First is suggested by (**Karl Popper, 1995**), who argued that the essence of probability is propensity. Propensities are the ultimate ontological reality and the basis of the entanglement between subject and object. Nevertheless, he found the link with not-subjective probabilities, arguing that propensities can generate empirically observed relative frequencies and can be measured by a frequentist concept of probability. The second way was generated by (**Patrick Suppes, 1984**). He uses probability concepts to deal with metaphysical and epistemological matters, according to which causality is probabilistic, not deterministic.

There exists one more approach to thinking uncertainty which we call radical subjectivism. This approach was proposed by (**Shackle, 2010**). According to his theory, under uncertainty, a person sees things as he would like and then imagines the consequences of his future actions; these consequences may seem extremely favorable or not to him. Because of these submissions, he will make a decision in the future. Shackle contends that

individual choices are made between alternatives, subjective representations of alternative future sequels to action; choices are not between future alternatives.

While we understand the theoretical infallibility and practical utility of the probabilistic way of thinking, we must nevertheless understand that it is not unique. There are other ways of thinking about understanding uncertainty. If we allow uncertainty to be treated as fuzziness, it becomes possible to formalize this representation with the help of so-called fuzzy sets or sets with fuzzy boundaries. This approach was proposed by (**Lotfi Zadeh, 1996**). Fuzzy logic is a calculus of compatibility. Unlike probability, which is based on frequency distributions in a random population, fuzzy logic describes the characteristics of properties. According to this approach, for example, uncertainty about the reliability of your partner can be represented using his function of belonging to a set of reliable people. The membership function is introduced to assess the degree of belonging of an object to a fuzzy set. It is fundamentally different from the idea of the characteristic property of elements, which was used earlier to construct classical sets. If we want to compare the probabilistic way of thinking and the fuzzy one, firstly, we can figure out that, unlike probability, fuzziness does not dissipate with time. Fuzziness is an intrinsic property of an event or object.

In addition to the approaches described above, there are attempts to comprehend uncertainty using representations that go beyond the probabilistic and fuzzy representations. Uncertainty can be represented as a set of options without attributing probability to them. This way of thinking of uncertainty appeared as the simplest one from the point of view of deep penetration in the essence of uncertainty. The idea is elementary: since I am not sure I understand uncertainty correctly, let me abstract from these complexities and choose from several equally possible alternatives. Then the preferred option will be chosen according to a specific criterion.

5. Modelling

Before making a decision and doing something, any reasonable person will think. This is what the second component of the decision-making system tells us. But oftentimes, it is not enough to think about it. You should do this in a certain orderly way. That is where the concept of a model comes in. From the most general standpoint, a model is a formal structure, represented in semantic images, mathematical formulas, diagrams, and graphs, that helps us understand a process or phenomenon and make an effective decision.

First, let us agree that there is no perfect model. As (**George Box, 1976**) stated, “All models are wrong, but some are useful.” If we want to express the same idea in terms of the general system of decision-making set out here, we could say that we are always dealing with ‘second-order’ uncertainty: being uncertain about our very model of uncertainty. In other words, by creating a model of uncertainty, we increase the degree of uncertainty for decision-making because we are not sure about this model but are trying our best.

Creating a model always occurs as a conversion of inputs to outcomes. The model transforms the initial information, in whatever form it may be, into a specific finite representation, so that with its help, the decision-maker can achieve the goal.

In each case, the decision-maker must choose to determine the further course of events, and the corresponding action will follow this choice. Most often, this situation will soon happen again. It means you will need to make the next decision. This is the content of the life of a reasonable person, no matter who he is a scientist, politician, commander, entrepreneur, etc.

A model is a typical form of knowledge formalization. Models encapsulate knowledge, explain, and predict in various fields of human activity: economic models of market competition, sociological models of networks, geological models of earthquakes, ecological models, psychological models of cognition, etc.

In general, we have two ways of model creation. In the first option, we prepare for the decision based on quantitative criteria (Data-Driven Decision Making). In the second version, the model does not use quantitative categories. In each option, we must trace the model's origin and connection with the way of thinking.

From a formal standpoint, the modelling process can be represented graphically (see Fig 2). This is a graphical model that explains the modelling process's content, which consists of transforming the initial information into outcomes that are useful for decision-making.

Figure. 2. Graphical model of modelling

This image is not exclusively a transformation of quantitative data. Texts and other non-quantitative representations can be used as input to the creation of the model, and meanings appear in the output of the model. Creating a model is more of an art than a science. This art cannot be comprehended by observing it from the outside. Full involvement of decision-makers is needed.

Firstly, we consider data-driven modelling of uncertainty. Models based on quantitative criteria are quite well formalized. Returning to the paragraph «The ways of thinking», we must recognize two ways of modelling quantitative data under uncertainty: probabilistic and fuzzy. One more approach can be used to model uncertainties in a set of alternatives, each of which has no preference over the others. We consider this method of modelling uncertainty as a case of probabilistic, taking the equal probabilities for each alternative.

It should be noted that probabilistic modelling historically appeared before fuzzy and received a more thorough development. As we mentioned above, there are three interpretations of probability: classical, frequency, and subjective. Subjective probability is most common when building decision-making models. There are simple models when the decision-maker assigns these probabilities and then calculates the criterion that will be the

basis for decision-making by mutual analysis. In practice, the most common is the Bayesian approach, which is systematically presented by (**Savchuk, Tsokos, 2011**). According to this approach, a person first assigns prior probabilities and then combines them with observations using the Bayes rule. As a result, he arrives at posterior probabilities that serve as the basis for decision-making.

The following principles summarize the ideas of the Bayesian approach to modelling uncertainty.

1. The Bayesian approach follows probability axioms which are the same as those for classical and frequency probability.

2. The Bayesian decision-maker has a complete set of probabilistic beliefs. In other words, to each proposition, he/she assigns a subjective probability, *P(H).* A Bayesian decision-maker can assign a degree of belief about everything. Therefore, Bayesian decision-making is always decision-making under certainty.

3. When exposed to new information, the event with conditional probability *P(A/H)* (the probability that *A* occurs, given that *H* is true), the Bayesian decision-maker changes his beliefs under new information according to Bayes' rule.

$$P(H/A) = \frac{P(H) \cdot P(A/H)}{P(A)}.$$

The fourth issue of the Bayesian approach closely deals with the decision-making process. **(Savage, 1954)** suggested it. It states that the rational decision-maker chooses the option with the highest expected utility. It will be the topic of the next paragraph. To decompose this fundamental uncertainty, Savage suggests a convenient representation of a decision problem by a matrix of the kind exhibited in the table of Fig. 3.

Figure. 3. Decision Matrix

	States of the world		
Options	S_1	...	S_n
A^1	C_1^1	...	C_n^1
⋮	⋮	⋱	⋮
A^m	C_1^m	...	C_n^m

Savage's model of presenting decision problems shows that in trying to decide what to do, a decision-maker is uncertain about: (i) what states and consequences there are, (ii) what actions are available, (iii) which states of the world are actual and what the consequences are of acting. This model was the basis for choosing the best alternative.

Probabilistic modelling can be produced using random events, variables, and stochastic processes. Returning to Fig. 2, consider when random variables will be used as inputs. The random variable is modelled using a probability density function (pdf), namely $f(x_i), i = 1,2, \dots, n$.

The outcomes $Y_1, Y_2, \dots, Y_m$ are random variables as well because each outcome depends on the inputs through the function: $Y_k = \varphi_k(X_1, X_2, \dots X_n) \; \forall \; k = 1,2, \dots, m$

The problem is to find out the pdf of each Y_k given the pdf for each $k=1,2,\dots,m$.

In general, the 'inputs to outcomes' conversion models are very complicated, which makes it difficult to solve this problem analytically. This is where the Monte Carlo simulation technique comes in. This technique is universal. That is, it has no limitation in terms of the content and complexity of the model. In essence, the Monte-Carlo method is an approach to modelling random variables with a given pdf utilizing the generation of the pseudo-random variables by special software.

The fuzzy way of thinking also has a strict mathematical formalization. The basis for creating a fuzzy model is the so-called membership function. It establishes a correspondence between the elements of the universal set U $(u_1, u_2, \dots u_n)$ and the numerical values of their degrees of belonging to some target set A. Going back to the example of human height (see paragraph 3) set U is the set of all men and set A is the set of tall people. The membership function $\mu_A(u)$ takes values in the interval [0,1]. The value of the membership function $\mu_A(u)$ for an element $u \in U$ shows to what extent that element belongs to the A. If the degree of membership takes only two values of 0 or 1, then set A has unfuzzy bounders; otherwise, this set is fuzzy (the degrees of membership can take any value on the interval [0,1], for example, 0.2 or 0.8).

Now let's come to non-data-driven modelling. Recently, a new notion appeared, "mental models" as cognitive constructs that describe a person's understanding of the real world. A "mental model" is usually a semantic model: people understand the world by forming mental models. The general form of this hypothesis is not new: Even Immanuel Kant argued that there is no direct access to "things in themselves". Therefore, it is necessary to build a mental model.

In the practice of building models, mainly three approaches are common (**Page, 2021**)

1) an approach of maximum embodiment, striving for maximum reliability,
2) the method of analogies assumes that it is possible to abstract from concrete reality and use a suitable analogue,
3) the method of an alternate reality does not intentionally represent or reflect reality.

Each approach deserves a detailed study, which will be done further. But now let's talk about what it means to make models effective, that is, convincing for decision-making. At the same time, it is necessary to consider that the decision will be made either by the person who created the model or by others to whom it will be offered. It seems that narratives and metaphors are the most useful for constructing semantic ones. The narrative is the most suitable tool for maximum embodiment and analogies. The metaphor will create vivid images through analogies and alternative reality methods.

A narrative refers to any narrative text whose function is precisely to model a representation of a phenomenon in verbal form. This is a specific type of exposition with a plot, distinguishing it from ordinary verbal descriptions or explanations. The "refined" texts make it possible to make the model bright and as accessible as possible for perception and

subsequent decision-making. The plot presented in the narrative, in a general sense, is a certain degree of reflection on the understanding of reality. And finally, unlike predictive models based on data, the narrative model is built to convince and transform people's mental models by presenting a particular plot.

A metaphor is usually considered a bright artistic image. In creating a model, metaphor has a more essential and, at the same time, more pragmatic purpose. Metaphor allows us to penetrate deeper into the essence of the object of modelling, going beyond our own object. Like a paradox, we better understand the essence of what is happening, going beyond it. This works equally effectively for the analogy method and the alternate reality method. Metaphor is the process by which one entity or state is described in terms initially intended to describe other things. Metaphor is the change of signs that are different in meaning but used in the same semantic contexts. For this reason, metaphor is most effective in building models using the method of alternative reality.

Unlike the traditional combination of concepts, metaphor has one undeniable advantage. Metaphor always combines a concept and a vivid manifestation of emotions. If, when building a model, it is possible to find an apt metaphor, this will allow not only to build an effective model but also to form an emotional field of attraction. The latter will be crucial in the decision-making process.

Semantic models are just one non-data-driven method of modelling. Graphic models can serve as very useful for clear perception. A visual image in the form of a diagram, a canvas, or even a painting can help a decision-maker present the overall picture of the object of decision-making and draw the correct conclusion. If we turn to business applications, we can find as a vivid example the format of the model, which has the form of a canvas on which all the essential factors of building a successful business are structurally located. The Business Model Canvas, suggested by (**Osterwalder and Pigneur, 2010).** is a powerful strategic management tool used to document existing business models and develop new ones? It offers a visual chart with elements describing a firm's or product's value proposition, infrastructure, customers, and finances, assisting businesses in aligning their activities by illustrating potential trade-offs. All these issues are getting together and can allow the decision-maker to develop a powerful strategy.

6. Decision Making

So, being uncertain, having learned to think adequately and create models, we are ready to make decisions. In general, there are two main approaches in decision theory: descriptive decision theory (sometimes called behavioral decision theory) and normative decision theory (sometimes called prescriptive decision theory). The first approach describes how specific people make decisions based on considerations beyond formal logic. Such descriptions may include behavioral patterns or sociological factors relevant to a decision. The second approach prescribes decision-making procedures based on certain formalized logic and

applying quantitative criteria. It is assumed that a person behaves rationally and is unaffected by behavioural patterns or sociological factors.

The descriptive theory is often associated with constructing a mental model, which uses metaphors and narratives in addition to the usual semantic images. And how vivid these metaphors and narratives will cause a person's emotions will depend on the degree of confidence of the person in the correctness of the decision made.

In normative decision theory, the decision is justified by using formalized criteria. The following two approaches are mainly used here. The first approach uses probabilistic models. And this opens a broad palette of possibilities and practical applications in decision-making. We have already discussed probabilistic thinking and modelling, and now it's time to conclude this sequence with specific tools for decision-making based on probabilistic models. There is a special case of the first approach, which considers a situation with a set of alternatives with the same probability. It means that we eliminate probability issues while posing the making decision problem. The problem is choosing the optimal alternative from the set of possibilities for a given set of states of the world. All possible combinations of system states and options are considered, and the one that provides the maximum or minimum value of the assigned criterion is selected. In this point of view, it is possible to proceed without probabilistic modelling, and a simple set of possible options sets uncertainty. No probabilistic techniques are used for decision-making. We call this approach combinatorial.

According to the second approach, making decisions is based on fuzzy sets and logic. In the previous paragraph, we discussed that these approaches pursue the same goal, modelling uncertainty, but by different means.

The Expected Utility Theory (EUT) deserves the most attention regarding applied aspects. Its founder should rightly be considered (**Danial Bernoulli, 1738**). His main idea came from the St. Petersburg Paradox. The principal assumption of EUT is that the increment of Utility U is proportional to this increment of Wealth (*W*) and inversely proportional to the magnitude of the Wealth (W). Simply thinking, individuals gain a correspondingly smaller increase in their satisfaction and happiness as income increases. It immediately follows from this assumption that the Utility Function has the form of a logarithmic function $U= Ln(W)$.

(Von Neumann and Morgenstern, 1947) made a sweeping generalization of this theory. Their excellent task was to lay a rational foundation for decision-making under uncertainty according to expected utility rules. Thus, EUT received its first axiomatic characterization. They state a series of axioms about the individual's preferences over indifference classes of lotteries and offer proof that an individual obeying these axioms will follow the expected utility theory. In the normative interpretation, these axioms are regarded as tenets of rational choice and should be judged by their normative appeal. If an individual does not maximize his expected utility, he violates some precise axiomatic principles, which are rationally binding in his choice. Von Neumann and Morgenstern's expected utility theory has been generally accepted as a normative rational choice model. EUT states that the decisions of a decision-maker conform to an expected utility function of the outcomes. In practice,

individuals should always choose the alternatives that offer them the highest utility under uncertainty, i.e. the alternatives that offer higher earnings (wealth) or the lowest losses ever.

Years after **(Leonard Savage, 1954),** proposed the first complete axiomatic Subjective Expected Utility Theory, focusing on uncertainty. He provided necessary and sufficient conditions for utility, probability's existence, joint uniqueness, and the interpretation of individual choice under uncertainty as expected utility-maximizing behavior. In Savage's approach, the probability is presented broadly, assuming the possibility of refining prior judgments with additional information. Savage subjective expected utility theory and the Bayesian rule for updating the decision-maker's information still represent the orthodoxy in making decisions under uncertainty. This treatment of decision problems reduces the decision maker's fundamental uncertainty concerning what to do with uncertainty regarding the actual state of the world.

In analytical terms, Savage's approach is based on the expected utility of the set of options, i.e. weighted average value of utility for the decision-maker. In other words, if the decision-maker adheres to axioms of rationality, believing an uncertain event has possible outcomes xi, each with a utility of u(xi), the choices of the individual can be explained by this utility function combined with the subjective belief that each outcome is probable, $P(x_i)$. Therefore, the subjective expected utility is the resulting expected value of the utility:

$$E(U) = \sum_{i=1}^{N} u(x_i) \cdot P(x_i).$$

EUT allows determining the relations of decision makers to the risk. Regarding risk, three groups of decision-makers exist: risk-averse, risk-neutral, and risk-seekers. Risk-averse decision-makers would take the expected value of a prospect with certainty rather than gamble on an uncertain outcome. Risk-neutral people are indifferent between a gamble on an uncertain outcome and a prospect with certainty. At last, risk-seekers would rather gamble on the uncertain outcome than take the expected value of a prospect with certainty.

From the point of view of the formal procedure, everything looks quite strict and understandable. But, as the saying goes, "the devil is in the details." It turns out that the result of the decision-making procedure is sensitive to the probabilities assigned by the individual. It is helpful to mention the well-known paradoxes of **(Allais, 1953)** and **(Ellsberg, 1961)**.

In turn, these doubts became the basis for conclusions obtained by **(Tversky Kahneman, 1982)** and concerning biases in judgments revealing some heuristics of thinking under uncertainty. They suggested the Prospect Theory, which describes risk choice theoretically. The Prospect Theory differs in many ways from EUT, where decision-makers determine the value of total wealth. In contrast to some generalizations of the decision theories, Kahneman and Tversky derived their theory of prospects from empirically identified and documented features of the behavior of actual respondents under uncertainty. Based on experimental studies, prospect theory makes a paradoxical conclusion: people tend to take on more risk to avoid losses than to receive an additional premium at high risk. Losses have a more

significant effect than gains of equal size, known as loss aversion. According to this theory, the investor is free to hold stocks that depreciate but sell those that are rising in value. So, in the joke "a strategic investor is an unsuccessful speculator", there is some truth. "I know prices will still jump; then I will sell my shares." Such reasoning is familiar to many.

Prospect theory revealed another feature of decision-making under uncertainty: people inadequately perceive probabilities. Psychologically, the individual overestimate small probabilities and underestimates medium and large ones. What's more, people choose to ignore a priori probabilities in exchange for minor data and analogies. Based on the nonlinear nature of the probabilistic value function used in prospect theory, the authors explain that people's emotional perception of events creates their probabilistic interpretation.

Moreover, according to the prospect theory, the empirical justification has acquired the function of probabilistic values. Fundamentally, it cannot be interpreted as a probabilistic measure since it does not correspond to the axioms of probability. This is manifested in the fact that subjective estimates of probability may not be equal to objective probabilities, being less than one in total.

In real practice, the term "decision-making" implies the choice of a certain decision and its subsequent implementation. At the same time, there are often situations when the decision-maker, having made a decision, still determines if its implementation should be started immediately. A typical example is investment decisions. These decisions involve a long implementation period, which increases the degree of uncertainty and, as a result, increases the risks of loss of invested funds. As part of the probabilistic approach to decision-making, the method of real options can be recommended. Introducing this notion **(Stewart Myers, 1977),** tried to apply the theory of financial options traded in the derivatives market to analyze a company's financial policy using financial leverage. More recently, this approach has been used to assess opportunities that result from strategic decisions that may arise in the future. The main difference between real and financial options is that a real option is not a security. It does not circulate in the derivatives market, where it can be sold or bought. The underlying asset of a real option is the future management decisions that can be made in relation to a specific development project.

From the point of view of the actual practice of strategic management, real options can be interpreted from two positions. The first, light position, involves using real options techniques to solve various problems, primarily related to Capital Budgeting. In this case, real options are treated as an analogue of financial options, which implies the transfer of the theory of financial options to real assets. The second, more comprehensive, position is that real options should create the basis of the company's development strategy. In this case, they are treated as a universal management process, a phenomenon of strategy in dynamics. The company's management should see and evaluate the options when developing a strategy. And this creates a real competitive advantage for the firm. Both interpretations allow us to determine the real options toolkit for making flexible decisions under uncertainty.

Let’s now come to the special case of the probabilistic approach, which we call combinatorial. The decision-making technique is most straightforward and most transparent

when uncertainty is represented as a set of alternatives unrelated to probabilistic. The decision is made by a simple combinatorial search of alternatives, from which one is chosen since it meets the criterion assigned in advance. Thus, the decision made directly depends on the criterion adopted. And there are several such criteria. The basic idea of any criterion is to replace a whole set of values with a single numerical indicator that characterizes this set from a certain point of view. Here is a list of such criteria: Wald's criterion; the "maximax" criterion; Laplace's criterion; Savage's regret criterion; Hurwitz's criterion.

Wald's criterion is the most "cautious": the optimal alternative would be the one that provides the best outcome among all possible alternatives under the worst set of circumstances.

The "maximax" criterion is the opposite of Wald's criterion. If Wald's reflected the view of the ultimate pessimist, then Maximax corresponds to an attitude of extreme optimism. All attention is paid only to the best outcomes.

Laplace's criterion is based on the principle of insufficient justification. Since, within the framework of the information approach in a situation of uncertainty, the probabilities of states are unknown, there is no reason to assert that they are different. Therefore, it can be assumed that they are the same. According to Laplace's criterion, the average outcome value is used to estimate the alternative.

Savage Minimax Regret Criterion is based on the following justification. Alternatives are evaluated based on the so-called "regret matrix". For an arbitrary alternative and a particular state of nature, the value of "regret" equals the difference between what the alternative provides and how much the maximum can be gained in a given state. From an economic point of view, the amount of "regret" can be interpreted as a lost gain compared to the maximum possible in each state of nature. The Savage criterion reflects the largest possible shortfall in winnings for a given alternative, the reason is that the less you can lose, the better.

The classical Hurwitz's criterion considers only the extreme outcomes of each alternative. It can be viewed as a weighted average of the best and the worst uncertainty realizations. It allows considering the decision-maker's subjective attitude by giving these outcomes different "weights". The "optimism coefficient" λ, $0 \leq \lambda \leq 1$ is introduced into the criterion calculation, so if it is close to 1, the decision-maker feels optimistic and pessimistic otherwise, if λ is near zero.

We cannot conclude which criterion is more correct. The decision-maker chooses the criterion by himself. This can be considered a kind of manifestation of democracy in the decision-making theory.

It should be noted that the probabilistic approach, including the combinatorial case is presented in decision theory much more thoroughly compared to the fuzzy approach, as evidenced by the number of techniques and approaches discussed above.

Possibility theory is an uncertainty theory devoted to the handling of incomplete information. It differs from the probability by using a pair of dual set functions (possibility and necessity measures) instead of only one. Besides, it is not additive and makes sense on ordinal structures. The name "Theory of Possibility" was coined by (**Zadeh, 1978**), and

(**Dubois and Pradé, 1998**) later contributed to its development. In Zadeh's view, possibility distributions provided graded semantics to natural language statements. The meaningful interpretation of the bases of the Possibility theory differs significantly from the probabilistic ones. The possibility of an event, in contrast to probability, which estimates the frequency of its occurrence in a regular stochastic experiment, is focused on a relative assessment of the truth of this event and its preference compared with any other. That is, only the relations "more", "less", or "equals" can be interpreted meaningfully. At the same time, the possibility does not have an event-frequency interpretation (unlike probability), which connects it with the experiment. Nevertheless, the theory of possibilities allows a mathematical model of reality based on empirical facts, knowledge, hypotheses, and researchers' judgments. The principle of minimal specificity drives possibility theory. It states that any hypothesis not known to be impossible cannot be ruled out. Human knowledge is often declarative, using statements to which belief degrees are attached. Decisions are made based on criteria connected to a membership function that reflects the decision-maker's attitude in front of uncertainty.

Conclusions

The main aim of this presentation was to show the different approaches and methods used for decision-making in uncertain conditions. In comparison to previous studies, the suggested conceptual framework for decision-making focuses on understanding uncertainty, applying it to modeling, and making well-informed decisions. To gain a better understanding of decision-making under uncertainty, the structure of this process was discussed in detail. It consists of four components: uncertainty itself, ways of thinking about uncertainty, modelling uncertainty, and decision-making techniques. Each component is discussed separately, but their relationships are addressed throughout.

After reviewing the methods studied, it can be concluded that the most advanced and systematic approaches are those that are based on probabilistic models. These approaches heavily rely on subjective probabilities. However, the use of subjective probabilities can introduce biases when looking at it from a rational standpoint, which can result in incorrect decisions. The Bayesian approach provides a solution to this problem by suggesting the utilization of subjective prior probabilities in decision-making, along with factual data that emerges during the decision-making process. The Bayesian approach is well-established, has a formalized structure, and shows an effective practical application.

The paper also noted that decision-making approaches are not limited to data-driven systems. Not all individuals faced with making decisions under uncertainty possess an analytical mindset and the ability to draw conclusions based on data. However, they may be receptive to semantic models presented through narratives, particularly those containing vivid metaphors. A technique based on narratives and metaphors is especially effective when making collective decisions.

However, the predominant approach is data-driven and includes a variety of methods. These methods are explained thoroughly, with a focus on specific techniques to highlight their core purpose. The presentation avoids excessive use of mathematical calculations and models. The main objective of the presentation was to demonstrate the practical application of the methods being studied and evaluate their usefulness in real-world scenarios.

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