Modeling, Simulation and Linear Programming: A Case Study for Production Sequencing in the Steel Industry

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Abstract

Steel is one of the most important materials nowadays and it is raw material for several market segments. This article presents a case study about process management, proposing a production sequencing solution and mitigating the bottlenecks found in the analyzed equipment. The analysis was performed in two fronts: Linear Programming (LP) and Modeling, and simulation for the strategic management of the production process. The main contribution of the present research relies on the analysis of the production process of an industry of considerable relevance in the global economy, by using LP tool in a range of 75 products with real data, and supporting a modeling and simulation analysis that achieved 96% of confidence.

Keywords: Operational Research, Simulation, Petri Nets, Sequencing, Processes management.

1. Introduction

The steel industry is an important supplier of raw material for several sectors of the transformation industry, as well as for civil construction. It is an industry characterized by the presence of large companies, which operate the various stages of the production process (Vianna, 2017). It is not new that Brazil has a relevant area of wealth in the steel sector and with a strong presence in the international market, which promotes Brazilian economic development. Currently, the main activities that contribute to the growth of the Gross Domestic Product (GDP) are agriculture, responsible for about 10%; the industrial sector, which represents 25%; and the tertiary sector, which encompasses commerce and services. The year 2020 was marked by the biggest global health crisis in 100 years. The negative impacts of the COVID-19 pandemic have spread across all productive sectors and socioeconomic classes

(Bridi, 2020). According to Instituto Aço Brasil, the fall in productive activity started in March 2020, when the economic activity index of the Brazilian central bank (IBC-BR) dropped 4.8% compared to February. The first quarter GDP in 2020, measured by the IBGE (Brazilian Statistics Institute), fell by 2.2% compared to the previous quarter. Despite the critical moment in the country due to the COVID-19 pandemic, the steel industry has been responsible for keeping the economy growth, contributing to the generation of jobs, foreign exchange, investments, and importation/exportation of goods. According to data released by the Instituto Aço Brasil, the estimate of production in 2020 decreased from 18.8% in April to 6.4% compared to the same period of the previous year. However, with the recovery of the economy, the demand for steel increased to levels observed in January 2020 (63%), that corresponds to a total installed capacity of 51.5 million tons (Callegari, 2021; Ribeiro, 2020). Considering this new scenario of increase in demand, the production sector must adapt to supply the market. With the technological advances that are arising globally, the companies need to maintain a high level of flexibility, agility in production and innovation in its processes.

Managing an organization that is focused on industrial production requires an analysis of a variety of complex factors that includes the market, the technical aspects of production, aspects related to logistics, suppliers and customers. The decision making process in this area needs to consider a series of factors related to the strategic management goals and policies of the company, aiming at quality, benefits and deadlines. Due to the high level of complexity in managing processes, methods and tools have been used to support the understanding and administration (such as BPMN - Business Process Model and Notation), in addition to maximizing productivity and bringing logistical solutions in process management. One of the tools that can be used to support the managing process is the use of operational research (Souza et al, 2019; Corradini et al., 2021).

Operational Research (OP) is a strongly interdisciplinary area of knowledge focused on the development of mathematical models and algorithms for solving complex real problems. Among its aspects are solving linear programming (LP) problems that are commonly described in the literature specially for this type of market (Bernardo et al., 2018; Djordjevic et al., 2019; Zhao et al., 2020). The present research has the general objective to identify the perceptions about Process Management in a steel production line for the automotive sector of a multinational steel industry. Due to the large volume of demand, the company has been facing issues to meet the deliveries within the deadlines. In this context, as specific objectives, it is intended to: understand the productive process of the company in question; propose a modeling of the production system through linear equations to improve the deliveries of the material portfolio of a production line. The main objectives must also meet production restrictions, such as deliveries and internal relocation of raw materials, limitations on finished product stock, material processing speed, among others, and at the same time, it must optimize the productivity and efficiency of the machines. Finally, this work aims to demonstrate a modeling and simulation via Petri nets as an initial proposal for a structure to improve the strategic management. The justification of this work is supported by the need for intelligent and viable solutions to optimize the sequencing production process, which is a requirement for organizations that want to remain in the market, since this improvement can increase the efficiency and performance of the processes and results, increase the relevance of the company on market share, possibility of new business and improve the competitiveness.

2. Methods

This research is configured as a case study and its methodology was divided into three main stages:

- **Evaluation of the straightening and stripping process of bars:** The entire order delivery process was studied, starting from the analysis of the portfolio of the studied production line to the delivery of the order to the customer. The process was illustrated using the BPMN (Business Process Model and Notation) tool in the Bizagi software.
- **Mathematical modeling and evaluation of the optimization process via Simplex**: Mathematical modeling was carried out, paying attention to the entire context of the case studied, aiming to optimize the deliveries of group A, not neglecting to meet the lesser priority categories, such as B and C. In this step, the software used was GUSEK.
- **Analysis of production bottlenecks:** After analyzing the results via linear programming, a study of the bottlenecks of the production process was carried out. To this end, a simulation of Petri nets was proposed, analyzing the case of the production of a certain product that needs to pass through the two production lines to become a finished product. The software used to perform the simulation was HPSim.

3. Results

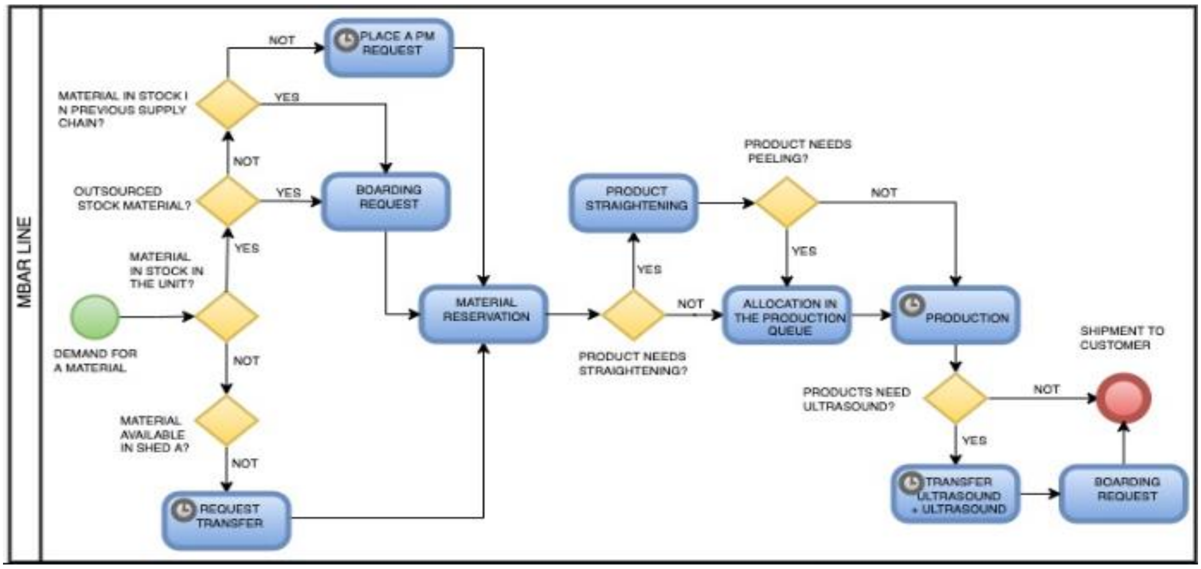
The main process to achieve the results of this research was based on the identification of parameters and variables of the problem, followed by the construction of the mathematical model according to the LP (Linear Programming Problem) structure, then, there was the application of the Simplex algorithm to optimize the problem and, finally, the simulation of Petri nets via HPsim. It is worth mentioning that all the data presented consisted of real values provided by the company.

3.1 Production Process Modeling via BPMN

Each machine in the company studied has an order backlog, that is, the list of orders that need to be delivered. The portfolio is implemented three months before the production of the material, therefore, the production of the materials can be planned within this period of three months. However, there are several limitations that make this sequencing process complex, which impact the rate of deliveries within satisfactory deadlines to customers. The stock is limited, therefore, the lack of raw material can impact production and, consequently, deliveries to customers. There is a problem in the acquisition of raw materials, which is detailed in Figure 1. Figure 1 shows the modeling via BPMN of the described process.

Firstly, the raw material in stock at the factory of the company is evaluated. If the raw material is available in stock, it is necessary to assess its location. If it is available in shed A of factory 2, its production can be requested immediately. If it is not in shed A, the period for internal logistical displacement is three days. Secondly, the raw material not stored in the company is analyzed. When a certain raw material is not stocked at the company, it is necessary to analyze whether there is stock of it in any unit of the group (whether it is outsourced or in-house). The first way to track the stock is through the subcontracted company responsible for stocking raw materials. This company is hired exclusively to stock the material that the factory of the production unit does not support. The delivery period for material from this company to the production factory is five days. If the subcontracted company does not have stock of the raw material, it is necessary to check if there is stock of the same material in the unit that produces it. Shipment orders take eight days to be made available at the factory floor. If the unit mentioned previously does not have the raw material available in stock, it is necessary to order it. Orders can be placed on the 10th day of every month and have a delivery time of forty days (thirty days for manufacturing plus ten days for delivery). The productive path of the materials produced by the machines being studied is not always the same, because each material has different features. Through the analysis of the productive paths is possible to highlight that some materials need to go through a straightening process (1) and then a peeling process (2), that is, they pass through both machines.

Figure 1: BPMN Model



Source: (Authors, 2022)

There are some materials that just need to go through a straightening (1) to become finished products. And there are other materials that just need to be peeled (2). As the process outlined via BPMN, then two analysis processes were developed. The first was based on Operations

Research, with LP fundamentals, for an experimentation of a mathematical modeling to verify the capacity of production scheduling. In its abstraction there were difficulties due to the presence of non-linear elements that were identified. So, a second approach was developed, based on modeling and simulation, as an initial proposal of a structure to verify the productive behavior, with identification of bottlenecks and production gaps.

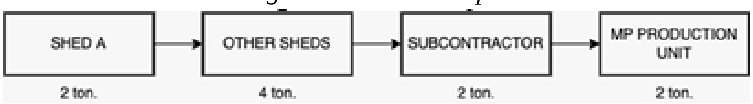
3.2 Optimization Process Via LP

The problem parameters and the necessary variables were implemented in order to achieve the results through the construction of a mathematical modeling using LP.

3.2.1 Equipment workflow

In order to solve the organization problem in the sequencing of the production of the different materials, the mathematical modeling started by analyzing the entire process involving the two machines: The straightening machine (1) and the bar stripping machine (2). It was noted that there was a non-linear event on the modeling of the interaction between the machines, because in certain finished products, the product of one machine become the input of the other machine. Mathematically, because one machines depends on the estimation of the other one, a quadratic function is generated, and this term cannot be solved in a LP problem. In this way, the modeling focused initially only on the bar peeling machine (2), which is the final processing machine of the production. As mentioned before, the production of materials depends on the location of their respective raw materials. In this case, there is a complexity in which there is no standardization of the availability of resources. For instance, supposing a product that requires ten tons of inputs to be able to be manufactured, it is possible to see in Figure 2 the flow chart of the availability distribution. Based on that, a significant increase in the complexity of the modeling was noticed, because the resolution of the problem needs to consider the multiple steps of the logistics of delivery of these inputs.

Figure 2: Stock Example



Source: (Authors, 2022)

The resolution of the problem from that point on, loses the linearity characteristic. To guarantee the linearity of the process, a modeling modification was implemented. In this modification was established that the entire organization of the location of the raw material will be done by the logistics operator of the company, and the analysis of the problem will be carried out considering only the Shed A for the allocation of inputs.

3.2.2 Parameter Identification

The parameters and variables used in the model were established according to the characteristics of the production process. In general, they were related to the productive characteristics, the production days and the features of each product, according to its expected production demand in the month, production time, setup time and delivery date. In table 1, it is possible to identify the name of the products (which are characterized by the junction of gauge, steel and length).

Table 1: Product Identification

x_i	Product	x_i	Product	x_i	Product
i = 1	23,70MM HK60 1534MM	i = 26	44,94MM 5160 1874MM	i = 51	28,57 MM 1045H 8M
i = 2	24,55 MM 5160 1576 MM	i = 27	45,85MM 5160 1744 MM	i = 52	28,57MM 1020 8M
i = 3	24,80MM HK60 1529MM	i = 28	50,00MM 5160 1369MM	i = 53	31,75 MM 1045H 8M
i = 4	25,00MM HK60 1534MM	i = 29	50,16MM 5160 1885 MM	i = 54	34,92 MM 1045H 8M
i = 5	26,00MM HK60 1494 MM	i = 30	50,99MM 5160 1306MM	i = 55	38,10 MM 1045H 8000
i = 6	28,96MM HK60 1387 MM	i = 31	50,99MM 5160 1687MM	i = 56	41,27 MM 1045H 8M
i = 7	29,73MM 5160 1874 MM	i = 32	31,75 MM 1045H 6000 MM	i = 57	41,27MM 1020 8M
i = 8	31,00 MM HK60 1504 MM	i = 33	38,10 MM 1045H 6M	i = 58	41,27MM 1020 8M
i = 9	31,75MM HK60 1715 MM	i = 34	41,27 MM 1045H 6M	i = 59	44,45 MM 1045H 8M
i = 10	31,75MM HK60 1864 MM	i = 35	41,27 MM 1045H 8M	i = 60	34,92 MM 1020 6000MM
i = 11	32,15MM HK60 1619MM	i = 36	41,27MM 1020 8M	i = 61	37,80 MM 1020 6000MM
i = 12	32,16MM HK60 1854 MM	i = 37	44,45 MM 1045H 8M	i = 62	38,10MM 1020 6000 MM
i = 13	33,34MM HK60 1441 MM	i = 38	44,45 MM 1050 6M	i = 63	41,27 MM 1020 6000MM
i = 14	33,34MM HK60 1511 MM	i = 39	45,00 MM 1045H 8M	i = 64	44,45 MM 1020 6000MM
i = 15	33,86MM HK60 1684MM	i = 40	45,70MM 1045H 6M	i = 65	50,80 MM 1020 6000MM
i = 16	34,00MM 5160 1872 MM	i = 41	50,80 MM 1020L 6M	i = 66	57,15 MM 1020 6000MM
i = 17	34,00MM HK60 1569 MM	i = 42	50,80 MM 1045H 6M	i = 67	60,33 MM 1020 6000MM
i = 18	34,15MM HK60 1554MM	i = 43	50,80MM 1050H 6M	i = 68	63,50 MM 1020 6000MM
i = 19	37,86MM 5160 1772 MM	i = 44	60,33 MM 1050 8000 MM	i = 69	35,30MM PL48 5952MM
i = 20	38,00MM 5160 1519 MM	i = 45	63,50MM 1020 6M	i = 70	38,30MM PL48 5960MM
i = 21	38,10MM 5160 1772 MM	i = 46	72,00MM 1020 8000 MM	i = 71	38,30MM 1045 6000 MM
i = 22	39,80MM 5160 1669 MM	i = 47	80,00 MM 1020L 5,6M	i = 72	27,50 MM 1045D 3000 MM
i = 23	39,85MM 5160 1909MM	i = 48	17,50 MM 1045H 8000 MM	i = 73	22,13 MM HK60 1460 MM

i = 24	43,80MM 51601649 MM	i = 49	23,81 MM 42CrMo4 8M	i = 74	38,10MM 1045 6000 MM
i = 25	43,86MM 5160 1404 MM	i = 50	25,40 MM 1045H 8M	i = 75	80,00 MM 1045 6000 MM

Source: (Authors, 2022)

The sequencing comprises 75 products and their organization was done by ordering the priority by customer, in which: products from 1 to 28 are intended for customer 1; products from 2 to 29 are intended for customer 2; products 32 to 47 are intended for customer 3; products 48 through 59 are for customer 4; products 60 to 68 are intended for customer 5; products 69 and 70 are intended for customer 6; product 71 is intended for customer 7; product 72 is intended for customer 8; product 73 is intended for customer 9; products 74 and 75 are intended for customer 10. The name of the products has been changed to guarantee the privacy of the customers. Table 2 refers to the demand in the planning of materials for the analyzed period.

Table 2: Demand for products

x_i	(Demand described in the planning)										
i = 1	18 ton.	i = 14	8 ton.	i = 27	18 ton.	i = 40	5 ton.	i = 53	14 ton.	i = 66	0 ton.
i = 2	38 ton.	i = 15	45 ton.	i = 28	4 ton.	i = 41	4 ton.	i = 54	20 ton.	i = 67	5 ton.
i = 3	60 ton.	i = 16	6 ton.	i = 29	3 ton.	i = 42	30 ton.	i = 55	0 ton.	i = 68	1 ton.
i = 4	4 ton.	i = 17	2 ton.	i = 30	62 ton.	i = 43	15 ton.	i = 56	26 ton.	i = 69	11 ton.
i = 5	8 ton.	i = 18	20 ton.	i = 31	167 ton.	i = 44	0 ton.	i = 57	10 ton.	i = 70	42 ton.
i = 6	0 ton.	i = 19	40 ton.	i = 32	0 ton.	i = 45	39 ton.	i = 58	24 ton.	i = 71	26 ton.
i = 7	4 ton.	i = 20	0 ton.	i = 33	4 ton.	i = 46	20 ton.	i = 59	14 ton.	i = 72	6 ton.
i = 8	14 ton.	i = 21	0 ton.	i = 34	18 ton.	i = 47	20 ton.	i = 60	41 ton.	i = 73	2 ton.
i = 9	0 ton.	i = 22	4 ton.	i = 35	26 ton.	i = 48	10 ton.	i = 61	2 ton.	i = 74	0 ton.
i = 10	4 ton.	i = 23	8 ton.	i = 36	24 ton.	i = 49	20 ton.	i = 62	3 ton.	i = 75	0 ton.
i = 11	22 ton.	i = 24	2 ton.	i = 37	14 ton.	i = 50	100 ton.	i = 63	22 ton.		
i = 12	2 ton.	i = 25	0 ton.	i = 38	30 ton.	i = 51	14 ton.	i = 64	23 ton.		
i = 13	11 ton.	i = 26	25 ton.	i = 39	0 ton.	i = 52	6 ton.	i = 65	5 ton.		

Source: (Authors, 2022)

Table 3 shows the availability for production already considering the planned OEE (Overall Equipment Effectiveness) of the equipment. The acronym refers to the performance indicator that establishes how much time it produces in relation to the time available. The machine runs every day of the month but needs to stop for preventive and corrective maintenance on the second and fourth Monday of the month. The duration of these maintenances corresponds to 12 and 8 planned hours respectively, and this duration is taken from the machine availability time. After that time, maintenance starts counting hours as corrective maintenance.

Table 3: Daily availability

Day	Available time	Day	Available time	Day	Available time
1	504 min.	12	504 min.	23	504 min.
2	504 min.	13	504 min.	24	336 min.
3	504 min.	14	504 min.	25	504 min.
4	504 min.	15	504 min.	26	504 min.
5	504 min.	16	504 min.	27	504 min.
6	504 min.	17	504 min.	28	504 min.
7	504 min.	18	504 min.	29	504 min.
8	504 min.	19	504 min.	30	504 min.
9	504 min.	20	504 min.	31	504 min.
10	252 min.	21	504 min.		
11	504 min.	22	504 min.		

Source: (Authors, 2022)

The total availability of the equipment was 43,440 minutes, however, when applying the planned OEE of 35%, there is a total of 15,204 minutes. When removing the planned corrective maintenance times, the availability on these days is 252 and 336 minutes respectively, and on other days, the availability is equal to 504 planned minutes. Table 4 shows the day of the month agreed for shipment to the customer. Material can be delivered earlier, but delivery later than the agreed date reduces the logistics performance indicator. For the elaboration of this study, an analysis of the technical data of the materials supplied by the company was made, since the speed registered in the equipment is given in meters per minute and the proposed optimization needed the data in tons per minute.

Table 4: Production deadline

x_i	Shipping date										
i = 1	**1**	i = 14	**15**	i = 27	**29**	i = 40	**21**	i = 53	**28**	i = 66	**0**
i = 2	**8**	i = 15	**10**	i = 28	**8**	i = 41	**4**	i = 54	**20**	i = 67	**7**
i = 3	**31**	i = 16	**23**	i = 29	**7**	i = 42	**18**	i = 55	**0**	i = 68	**20**
i = 4	**11**	i = 17	**15**	i = 30	**19**	i = 43	**27**	i = 56	**16**	i = 69	**21**
i = 5	**6**	i = 18	**2**	i = 31	**31**	i = 44	**0**	i = 57	**6**	i = 70	**17**
i = 6	**0**	i = 19	**9**	i = 32	**0**	i = 45	**15**	i = 58	**22**	i = 71	**27**
i = 7	**28**	i = 20	**0**	i = 33	**7**	i = 46	**3**	i = 59	**7**	i = 72	**5**

i = 8	**13**	i = 21	**0**	i = 34	**25**	i = 47	**31**	i = 60	**14**	i = 73	**21**
i = 9	**0**	i = 22	**13**	i = 35	**5**	i = 48	**12**	i = 61	**20**	i = 74	**19**
i = 10	**21**	i = 23	**22**	i = 36	**29**	i = 49	**9**	i = 62	**18**	i = 75	**6**
i = 11	**11**	i = 24	**1**	i = 37	**10**	i = 50	**23**	i = 63	**28**		
i = 12	**4**	i = 25	**0**	i = 38	**4**	i = 51	**24**	i = 64	**3**		
i = 13	**24**	i = 26	**26**	i = 39	**0**	i = 52	**2**	i = 65	**22**		

Source: (Authors, 2022)

In order to solve the difference on the speed metric, the weights per meter of each type of steel were cataloged, taking into account the gauge. With the data of weight (in ton./meter) and speed (in meters/min.) it was possible to multiply the values reaching the unit of ton./min. Table 5 shows to the production time of each product. When there is a need to interrupt the production of a certain material, a series of changes in the equipment is necessary to meet the features of the materials to be processed, and this process is called setup. The setup time of the materials varies according to the previous one. The change between materials with close gauges takes less time than the change of program with more distant gauges. The interaction is 75 × 75, that is, the time relationship of each product to each other. This temporal relationship was implemented in the mathematical modeling.

Table 5: Production Time

x_i	Production Time (ton./min.)										
i = 1	**0,06**	i = 14	**0,149**	i = 27	**0,28**	i = 40	**0,28**	i = 53	**0,149**	i = 66	**0,304**
i = 2	**0,095**	i = 15	**0,149**	i = 28	**0,28**	i = 41	**0,286**	i = 54	**0,196**	i = 67	**0,179**
i = 3	**0,095**	i = 16	**0,173**	i = 29	**0,286**	i = 42	**0,286**	i = 55	**0,238**	i = 68	**0,179**
i = 4	**0,095**	i = 17	**0,173**	i = 30	**0,304**	i = 43	**0,286**	i = 56	**0,256**	i = 69	**0,208**
i = 5	**0,095**	i = 18	**0,173**	i = 31	**0,304**	i = 44	**0,179**	i = 57	**0,256**	i = 70	**0,238**
i = 6	**0,119**	i = 19	**0,238**	i = 32	**0,149**	i = 45	**0,179**	i = 58	**0,256**	i = 71	**0,238**
i = 7	**0,119**	i = 20	**0,238**	i = 33	**0,238**	i = 46	**0,179**	i = 59	**0,268**	i = 72	**0,119**
i = 8	**0,119**	i = 21	**0,238**	i = 34	**0,256**	i = 47	**0,149**	i = 60	**0,196**	i = 73	**0,06**

i = 9	**0,149**	i = 22	**0,238**	i = 35	**0,256**	i = 48	**0,048**	i = 61	**0,238**	i = 74	**0,238**
i = 10	**0,149**	i = 23	**0,25**	i = 36	**0,256**	i = 49	**0,06**	i = 62	**0,238**	i = 75	**0,149**
i = 11	**0,149**	i = 24	**0,262**	i = 37	**0,268**	i = 50	**0,095**	i = 63	**0,256**		
i = 12	**0,149**	i = 25	**0,262**	i = 38	**0,268**	i = 51	**0,119**	i = 64	**0,268**		
i = 13	**0,149**	i = 26	**0,268**	i = 39	**0,28**	i = 52	**0,119**	i = 65	**0,286**		

Source: (Authors, 2022)

3.2.3 Analysis based on the linear programming problem to reduce delays

To meet the objective of developing the mathematical model and its subsequent application, the parameters, variables, problem constraints and the objective function, which is to minimize production delays, were included in the model. The mathematical modeling consists of minimizing the delay *D* of waiting for the fulfillment of a demand for a product *X*, considering a wait in days and production capacity issues. The parameters of the model are described as follows: parameter I: Number of products in the portfolio of the machine; parameter J: number of days of the analyzed period; parameter M: large auxiliary number to force setup if there is a new product; parameter vi_i: parameter that stores the demand in the planning of each product; parameter pi_i: stores the delivery time of each product; parameter kj_j: informs the total productive capacity of each day; parameter tpi_i: informs the delivery time of each product according to the planning; parameter si_{ii}: matrix that stores the setup time from one product to another product; parameter D: matrix that informs the situation (anticipation and delay) of the product on each day of the month. The decision variables were: variable X_{ij}: integer value that informs how much to produce of product I on day J; variable Y_{ij}: binary value informs with the value 1 if there will be setup of product I on day J.

The objective function of the modeling is given by equations (1) to (6):

Minimize $$\left(\sum_{i=1}^{I}\sum_{j=1}^{J} Xij\right).Dij \quad (1)$$

Subject to

$$\sum_{j=1}^{J} X_{ij} = vi_i \qquad \forall_i \ 1..I$$

(2)

$$Y_{ij} \leq X_{ij} \qquad \forall_i \ 1..I, \forall_j \ 1..J \quad (3)$$

$$Y_{ij} * M \geq X_{ij} \qquad \forall_i \ 1..I, \forall_j \ 1..J \quad (4)$$

$$\frac{\sum_{i=1}^{I} x_{ij}}{tpi_i} + \left(\sum_{i=1}^{I} Y_{ij}\right) * si_{ii} \leq kj_j \qquad \forall_i \ 1..I, \forall_j \ 1..J \quad (5)$$

$$X_{ij} \geq 0 \qquad \forall_i \ 1..I, \forall_j \ 1..J \quad (6)$$

where the exact the demand value must be produced (Equation 2); in all products on all days, the number of setups must be less than the number of processes (Equation 3); restriction that forces setup if proceeding with the product (Equation 4); for all production days, the production time of each product, added to the setup time, cannot exceed my total daily production capacity (Equation 5 - considering the products in production and the stored, evaluating their stored time); there will be no negative production (Equation 6).

The mathematical model developed was implemented and executed via Gusek, in order to achieve cost minimization. It was possible to observe that the equipment has days without production, that is, after optimizing the sequencing in order to minimize delays, the machine has idle days, which creates opportunities for the implementation of more materials in the portfolio. Table 6 is a demonstration of the results achieved by the LP model, correlating the produced quantity of each material with its respective demand. There were no production deviations.

Table 6: Demand versus Production

Product	Demand	Production	Product	Demand	Production	Product	Demand	Production
i = 1	18	18	i = 28	4	4	i = 53	14	14
i = 2	38	38	i = 29	3	3	i = 54	20	20
i = 3	60	60	i = 30	62	62	i = 56	26	26
i = 4	4	4	i = 31	167	67	i = 57	10	10
i = 5	8	8	i = 33	4	4	i = 58	24	24
i = 7	4	4	i = 34	18	18	i = 59	14	14
i = 8	14	14	i = 35	26	26	i = 60	41	41
i = 10	4	4	i = 36	24	24	i = 61	2	2
i = 11	22	22	i = 37	14	14	i = 62	3	3
i = 12	2	2	i = 38	30	30	i = 63	22	22
i = 13	11	11	i = 40	5	5	i = 64	23	23
i = 14	8	8	i = 41	4	4	i = 65	5	5
i = 15	45	45	i = 42	30	30	i = 67	5	5
i = 16	6	6	i = 43	15	15	i = 68	1	1
i = 17	2	2	i = 45	39	39	i = 69	11	11
i = 18	20	20	i = 46	20	20	i = 70	42	42
i = 19	40	40	i = 47	20	20	i = 71	26	26

i = 22	4	4	i = 48	10	10	i = 72	6	6
i = 23	8	8	i = 49	20	20	i = 73	2	2
i = 24	2	2	i = 50	100	100			
i = 26	25	25	i = 51	14	14			
i = 27	18	18	i = 52	6	6			

Source: (Authors, 2022)

3.3. Petri Nets Model

To determine the average amount of finished material, including the straightening (1) and bar stripping (2) processes, an analysis was performed using the Petri nets in the HPsim software. In the production simulation, material number 42 was tested. To carry out this simulation, an analysis of how much of this material can be processed monthly by the machines was made. In this analysis required the definition of the average amounts allowed and the exponential average time of each stage of production, in addition to the quantity and time data (Table 7 – process times 1 and 2).

There are two probabilistic variables that were entered from the end of process 1 to the beginning of process 2. The first one corresponds to the amount of inputs that leaves the straightening machine and goes to the debarking machine, and the second corresponds to the finished product of straightening machine.

Based on the demand of the machines, the possibilities were defined for 17% of the material processed by machine 1 to be a finished product and 83% of the same material being an input that goes to machine 2. The software used has limitations regarding values with decimal places, so to represent these probabilities, due to the limitations of the program, a mathematical manipulation was performed to find the substitute numbers as follow:

• It was necessary to round the 83% to 80% (which is the frequency that the product arrives at the machine and the 17% to 20% (which is the frequency that the finished product leaves).

• 1/80=0.13, this value was multiplied by 10 to get close to an integer, which resulted in 1.3. Subsequently, the value of 1.3 was rounded to 1.

• 1/20=0.5, this was then multiplied by 10, which resulted in 5.

It is noted that the frequency of arrival time at the machine is less than that of the finished product and thus it was possible to proceed with the simulation. The same probabilistic analogy had to be made in the final stage of the process, since there is an 80% chance of having finished material in process 2 and 20% of rejects in the same process. From the data in Tables 7, the simulation was prepared and executed in the HPsim software, as shown in Figure 3.

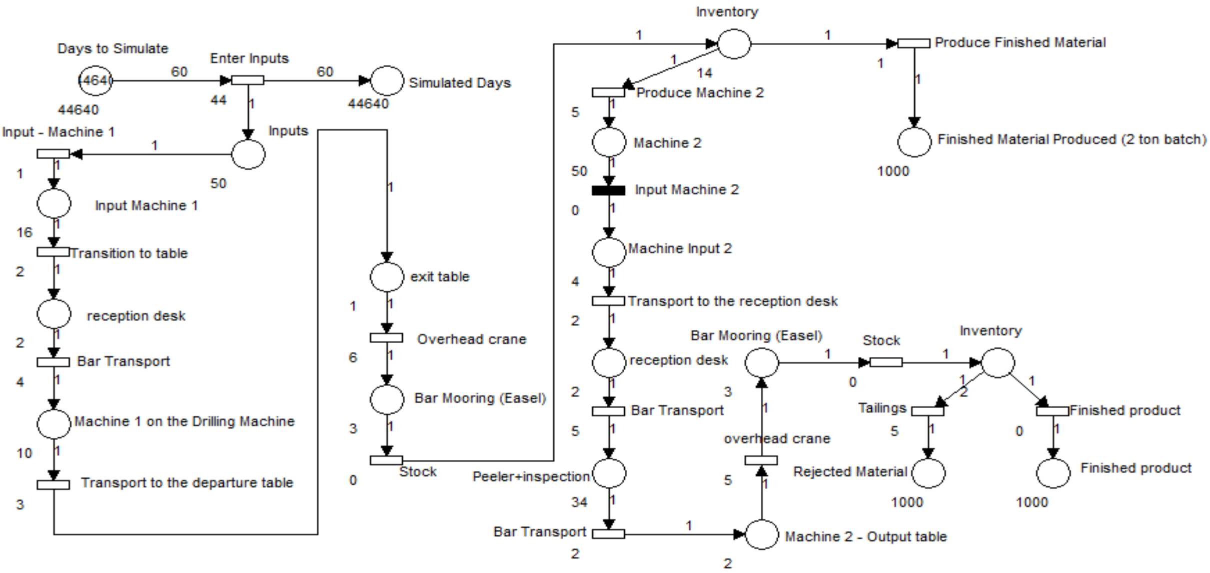
Table 7: Process times

Process times 1		Process times 2	
Bar Straightening		**Bar Peeling**	
Tone per day	50	Tone per day	50
monthly production	1550	monthly production	1550

Production time (min)	4379			Production time (min)	5420		
	Fraction approx.	**Min./day**	**Qty./day**		**Fraction approx.**	**Min./day**	**Qty./day**
Supply (Input)	31,00%	44	16	Supply	7,00%	4	12
Transition	1,00%	1	1	Transition	1,00%	1	2
reception table	4,00%	6	2	reception table	4,00%	2	7
Bar Transport	3,00%	4	2	bar transport	3,00%	2	5
straightener	20,00%	28	10	Peeler + inspection	68,00%	34	119
Transition	1,00%	1	1	bar transport	1,00%	1	2
exit table	2,00%	3	1	exit table	3,00%	2	5
overhead crane	4,00%	6	2	overhead crane	3,00%	2	5
Mooring Bars (easel)	6,00%	8	3	Mooring Bars (easel)	6,00%	3	10
overhead crane	1,00%	1	1	overhead crane	1,00%	1	2
Storage (toothpick holder)	27,00%	38	14	Storage (toothpick holder)	3,00%	2	5
% of product that results in input	83,00%			20% reject			
% of finished product	17,00%			80% finished product			

Source: (Authors, 2022)

Figure 3:Petri net simulation



Source: (Authors, 2022)

With the limit of up to 1,550 tons per month and the limitations of capacity and time, the simulation resulted in the following data: 675 batches of 2 tons of finished material were produced for the bar straightening machine; 63 batches of 2 tons of finished material were produced for the bar peeling machine and 6 batches of 2 tons of tailings, totaling 1,488 tons processed. Due to the limitations of the program, when inserting the probabilistic mathematical terms, the machines were forced to work with the adjustments imposed on the processes. However, the results were very satisfactory because the simulation had only a 4% margin of error compared to the real processes. In the simulation, it was also possible to observe the points that represent bottlenecks, and consequently the places that need optimization to improve the process, so that there is a better use of inputs and to achieve greater productivity. The simulation generated an analysis, described in tables 8 and 9, which shows the percentage of how long the material was in the process stages, making it possible to draw line graphs that show the points of greatest material occupation. The program ran the Petri net for 40,584 events.

Table 8: Percentage of Bar Straightening Machine Occupancy

Input	**straightening input**	**reception desk**	**Bar Straightener**	**exit table**	**Mooring Bars (easel)**	**Stock**	**Bar Peeling (2)**
1,83%	2,70%	8,64%	3,67%	12,83%	2,79%	2,69%	0,17%

Source: (Authors, 2022)

Table 9: Percentage of Bar Peeling Machine Occupancy

input	**reception desk**	**Peeler + Inspection**	**exit table**	**Stock**	**Mooring Bars (easel)**
0,44%	0,98%	0,51%	0,96%	0,24%	0,26%

Source: (Authors, 2022)

The processes on machine 1 take longer, and thus create more bottlenecks in production. The reception table and output table sets are the most critical ones in the process. In machine 2, the reception table, bar peeler, inspection and output table sets are the ones that most delay the process. This is just a first optimization step, as the company has many materials to be produced going through the same process. A more detailed analysis needs to be performed in the future, with the collection of a greater number of data aiming at more reliable results in the research.

4. Conclusion

The present research aimed to optimize the process flow of a steel company. The interactions between machines and the interaction of stocks were not addressed in the mathematical modeling because they mischaracterize the linearity of the system. Revisions were carried out addressing several concepts related to the case study, which provided understanding and enabled the analyzes to carry out the development of the study.

The results of the analyzes carried out in terms of mathematical modeling using the operational research and simulation of the Petri net were satisfactory and managed to achieve the expected results. The mathematical modeling was able to sequence the production, eliminating the delays of the analyzed month, delivering the materials ahead of schedule and still providing a view of availability, in a system where many products could not be delivered on schedule. The Petri net simulation was able to identify improvement points for greater production fluidity and efficiency, which would benefit the OEE of the equipment.

As a contribution to the other works, this research demonstrates itself as a case study where LP and modeling and simulation complement each other for the solution of a strategic problem.

For future work, more data will be incorporated into the study, following the production of all products present in the two machines from the raw material to the final product.

References

Bernardo, C. H. C., Chaves, V. H. C., SantAna, R. C. G. and Martínez, M. P. (2018). Operational research historical perspectives. Bolema-Mathematics Education Bulletin, 354-374.

Bridi, M. A. (2020) A pandemia Covid-19: crise e deterioração do mercado de trabalho no Brasil [The Covid-19 pandemic: crisis and deterioration of the labor market in Brazil] *Estudos avançados*, v. 34, p. 141-165, 2020.

Callegari, V. A (2021) Importância Do Mercado Siderúrgico Para A Economia Brasileira [Importance of the Steel Market for the Brazilian Economy]. Centro de Informação Metal Mecânica [Metal Mechanics Information Center] Available in: https://www.cimm.com.br/portal/artigos/21073-a-importancia-mercado-siderurgico-economia-brasileira .

Corradini, F., Fornari, F., Polini, A., Re, B., Tiezzi, F. and Vandin, A. (2021) A formal approach for the analysis of BPMN collaboration models. *Journal of Systems and Software*, v. 180, p. 111007.

Djordjevic, I., Petrovic, D., and Stojic, G. (2019). A fuzzy linear programming model for aggregated production planning (APP) in the automotive industry. Computers in Industry, 110, 48-63.

Instituto Aço Brasil (2020) *Mercado Brasileiro do Aço* [Brazilian Steel Market] Análise Setorial e Regional [Brazil Steel Institute] Available in:https://acobrasil.org.br/site/publicacoes/ .

Ribeiro, F.F (2020) O impacto da pandemia e das medidas do governo nas relações de trabalho no setor siderúrgico [The impact of the pandemic and government measures on labor relations in the steel sector] Centro de Estudos Sindicais e de Economia do Trabalho [Center for Union Studies and Labor Economics] Available in: https://www.cesit.net.br/wp-content/uploads/2020/06/Fl%c3%a1via_Ribeiro_IMPACTOS-DA-PANDEMIA-no-setor-siderurgico.pdf .

Souza, F. H. B.; Teixeira, G. A. and Simão, D.R.A (2019) Estudo de Caso da Aplicação de Métodos Lineares para Auxílio no Processo de Tomada de Decisão em uma Indústria de Laticínios [Case Study of the Application of Linear Methods to Aid in the Decision Making Process in a Dairy Industry] *IX Congresso Brasileiro de Engenharia de Produção*. Ponta Grossa.

Vianna, F.L.E. (2018) Indústria siderúrgica: investir em eficiência [Steel industry: investing in efficiency] Caderno Setorial ETENE [ETENE Sectorial Notebook.] Ano 3, Nº39.

Zhao, Z., Liu, S., Zhou, M., and Abusorrah, A. (2020). Dual-objective mixed integer linear program and memetic algorithm for an industrial group scheduling problem. IEEE/CAA Journal of Automatica Sinica, 8(6), 1199-1209.