

Enhancing Pre-Service Teachers' Understanding of A Mathematical Concept Through Practises: A Constructivist Perspective

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Abstract

This paper addresses the ways that Tanzanian university pre-service mathematics teachers (PSTs) practised teaching the concept of pi in a daylong research meeting. The research meeting focused on enhancing the PSTs' understanding of a mathematical concept through practises. In particular, the activities were designed and implemented based on practise as one of the five components of the concept-rich instruction (CRI). The intention in using the activities based on practise was to enhance PSTs' pedagogical knowledge and skills through participating in designing and performing micro-teaching for teaching the concept while using local circular objects as teaching and learning resources. Nine PSTs volunteered to participate in this study. The PSTs worked in small groups while designing the lesson plans and performing micro-teaching. Audio-video recordings and group learning notes were used to collect qualitative data in the research meeting. The collected data were analysed using thematic analysis while focused on the ways that PSTs practised related to the concept. NVIVO software was used in the coding process (QSR international Pty Ltd. Version 11.4, 2017). It was found that the activities helped PSTs to enhance their understanding on how to teach pi as a concept, including designing the lesson plans for teaching the concept using local circular objects, performing micro-teaching and demonstrating measuring the diameter and circumference of a circular object using a set of vernier callipers, a string and a ruler. These findings have implications in the teaching and learning of mathematics in university mathematics classes, including PSTs can gain pedagogical knowledge and skills on how to engage learners in a participatory way of learning as well as learning by doing.

Keywords: A mathematical concept; pi, practises, pre-service mathematics teachers, instructional activities

1. Introduction

Practise related to a concept is unavoidable for enhanced delivery of a lesson in university mathematics classes. However, university pre-service teachers' (PSTs') practises a mathematical concept are usually done after class. They practise a concept through working mainly on assignments. Because of that the pre-service teachers (PSTs) have limited time for practising a concept in university mathematics classes including designing a lesson plan, performing micro-teaching and demonstrating a concept (Deogratias, 2020). But we expect these PSTs to be future teachers engaging their learners in a participatory way of learning in either primary or secondary schools. In doing so, PSTs will have limited pedagogical knowledge and skills for teaching the concept after completion of their degree programs. This study gave PSTs an opportunity to practise the concept in the research meeting including designing a lesson plan, performing micro-teaching, and demonstrating teaching a mathematical concept.

Practise is the process of learning a mathematical concept by doing (Ben-Hur, 2006; Deogratias, 2020). Practises involve all homework and classwork activities about a concept including group work assignment, demonstration, and exercise (Ben-Hur, 2006). The homework activities are not easily assessed by the teacher because a teacher cannot know how the activities proceeded. Learners work on the activities while at home and submit the work to the teacher at school for marking. Classwork activities are done by the learners in the class and a teacher makes follow-up on how the activities proceed among learners in the class. In doing so, teachers get the opportunity to assess individual learners' understanding of a mathematical concept.

PSTs' learning of a mathematical concept requires appropriate and sufficient practise because we expect them to be teachers after completion of their degree programs. The 'practise' component of Ben-Hur's (2006) CRI points to the idea that practise involves new activities that are all based on the corresponding concept, which includes homework and class work. Furthermore, Ben-Hur suggests to the teachers that learners should avoid repeating similar work while practising the concept. Also, he suggests that learners should be given an opportunity to reflect on their practises the concept. Moreover, Ben-Hur proposes that "to achieve a high level of [learners'] success, teachers must avoid practise tasks that are too easy as well as practise tasks that are too difficult" (2006, p. 16). The PSTs worked on non-repeated tasks to practise the concept, including designing and presenting the lesson plan for teaching π as a concept while exploring the research question: How do university pre-service teachers practise a mathematical concept in a research meeting?

2. Social constructivist theory

Knowledge development is a result of social interactions (Cobb, 1996; Cole, 1996; Ernest, 1998; Gee 2008; Jones & Brader-Araje, 2002), and language plays a major role for knowledge development among learners in a social context (Ernest, 1996; Gee, 2008). Hence,

knowledge is socially constructed and shared among people rather than individual experience (Cole, 1996; Ernest, 1998; Watson, 2001).

In the classroom context, the role of a teacher is to facilitate learning (Lave & Wenger, 1991). For example, the teacher can develop learners' knowledge through questioning. In the research meeting, I was a facilitator. I played a major role in asking questions to the PSTs on how they can develop a lesson and teach learners so that they understand pi as a concept.

Learning process is a social event (Watson, 2001). During the teaching and learning process, PSTs were actively participating in creative activities and self-organisation in small groups by working together in designing a lesson plan for teaching pi as a concept using local circular objects. Then, they presented their designed lesson in the research meeting. After that, there were class discussions to elaborate the lesson and delivery.

Learners need to work in a social and collaborative environment to improve delivery of the lessons (Watson, 2001). In a daylong research meeting, the PSTs worked in small groups while developing the lesson plan for teaching pi as the concept. Then, individual members represented their group lessons through performing micro-teaching. Other group members were moving around to facilitate teaching and learning processes to the individual PSTs as well as to the small groups.

3. Methodology

This qualitative case study focused on enhancing pre-service mathematics teachers' pedagogical knowledge and skills through practises. This study was qualitative in nature because it was not meant to generalize the findings but explorative in nature (Merriam, 1988; 1998).

Nine PSTs volunteered, participating in a daylong research meeting which started from 9:00am to 4:30 pm. They worked in small groups while designing their lesson plans and performing micro-teaching. There were three groups in the meeting and each group consisted of three PSTs.

Four tools were used to collect data. The first tool was the designed activities that encouraged PSTs to practise the concept, through asking them to work in small groups on how they can design a lesson for teaching pi as a concept and perform micro-teaching to the concept while using local circular objects. The second tool was audio and video recordings which focused on collecting data when PSTs were working in small groups to design the lesson plan for teaching pi as a concept. The same tool was used to record data while PSTs were performing micro-teaching to the concept. The third tool was collective notes which focused on collecting written notes about what PSTs discussed in small groups while designing the lesson plans for teaching pi as the concept. The fourth tool was reflective journal which was used to collect data focused on PSTs' practises on what they were not unfamiliar about before practising teaching pi as the concept.

The collected data were analysed using thematic analysis (Clarke & Braun, 2006). NVIVO software was used in the coding process (QSR international Pty Ltd. Version 11.4, 2017) and

Ben-Hur (2006) was used in naming categories focused on the ways PSTs practised teaching π as the concept.

4. Findings

4.1. Designing lesson plans for teaching the concept

A lesson plan is one of the practises performed by PSTs in the research meeting by developing the lesson on how to teach π as a concept. As evidenced in the past studies (for instance, Meador, 2017; Olusanjo, 2010), the lesson plan helps a teacher to deliver the lesson well and remain focused in the class, because the teacher organises the what, how, when, whom and why to teach before going in the class.

PSTs demonstrated their understanding of a mathematical concept by being able to practise developing their lesson plan. Before participants performed the micro-teaching, they started developing the lesson plan in small groups (G1, G2, and G3). G1, G2, and G3 worked in their small groups to develop the individual group lesson plans for teaching the concept using local materials as teaching and learning aids so that students come to understand π as a concept.

The duration for developing the lesson plan was 120 minutes. The groups developed the lesson plans by adapting the lesson plan template for Tanzanian secondary school. This adoption was not a challenge to the participants, because all participants had already participated twice in teaching practises in secondary schools at the time when they participated in the research meeting focused on π using a concept-rich instructional approach. The Tanzanian mathematics syllabus for secondary school (TIE, 2005) was used by the participants to develop the lesson plans. Also, Tanzanian mathematics textbooks or reference books for secondary schools (TIE, 2009) were used in the process of preparing the lesson plans to develop lesson notes.

Three lesson plans for teaching π in 30 minutes were developed by the participants in their respective groups. The participants filled the evaluation and remarks which were the required parts of the lesson plan after presenting their developed lesson plans.

The PSTs developed their lesson plans on how to teach π as a radian measure using participatory approaches. For instance, G3 demonstrated their understanding of how to develop the lesson plan for teaching the concept as a radian measure. Below I present the group discussions (G3D) while developing the lesson plan.

G3D: $C = 2\pi r \equiv 360^\circ$ $C = 2\pi r \equiv 360^\circ$ where C is the circumference of a circle.

Sub-concept 1: The relationship between arc length, s and central angle, θ .

Sub-concept 2: conversion of angles from angle measure to radian measure.

G3D1: I prepare the teaching aids. I will show the central angle and arc length-relationship. [As shown in Figure 1 below.]

G3D2: Prepare lesson notes!

G3D3: Once we know to derive up to here, then we will give two examples how to apply it.

G3D1: We can use the central angle to get the concept of π .

G3D2: Here we need to show on how $\pi \text{ rad} \equiv 180^\circ$

G3D3: We need to show $\frac{l}{r} = \frac{l}{r} = \theta$ where l is arc length, r is radius of a circle, and θ is radian measure.

G3D2: We need to remind students about this formula, $l = \frac{\pi r \theta}{180}$. In the introductory part, we need first to derive, this formula before showing that $\pi \text{ rad} \equiv 180^\circ$. Also, in the introduction, we will show a relationship between l and θ .

G3D1: Introduction will take 5 minutes, presentation will take 15 minutes

(demonstration about $\pi \text{ rad} \equiv 180^\circ$), in the reflection, we will give a question to the students, it will take 3 minutes (questions and answers), reinforcement will take 5 minutes; conclusion will take 2 minutes (we will give an assignment). During teaching, we will not write the lesson notes.

G3D3: What are we going to write about competence?

G3D1: Introduction; we will introduce how to derive the relationship between l

and θ from the formula $l = \frac{\pi r \theta}{180}$. We will use 5 minutes.

Competence: We look at the skills for this topic.

Main topic: Circle.

Sub-topic: Central angle.

Specific objectives are obtained from the sub-topic.

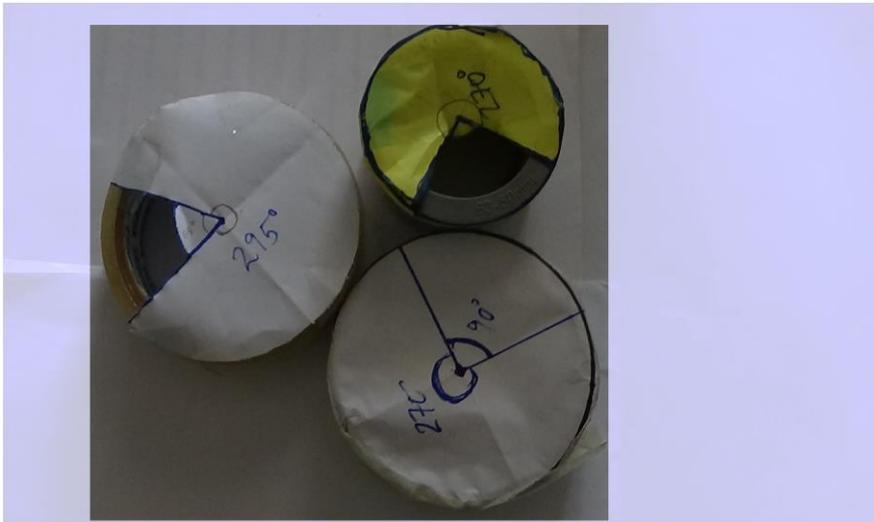
General objectives are obtained from the topic. We look at the abilities.

G3D: Teaching strategies are demonstration, discussion, and questions and answers. Teaching aids/resources are sole tape, plain paper, ruler, and protractor.¹ (Transcribed data).

From the above discussions, G3 developed a lesson plan for teaching π as a concept focusing on students' understanding of its meaning as a radian measure, and how to convert angle measure from degree to radian measure and vice versa.

¹ Nodes from the CRI Practise Designing the lesson plan (2)

Figure 1. Teaching and learning aids developed by the group.²



From the developed lesson plan, the participants have demonstrated their understanding how to develop the lesson plan for teaching π as a radian measure using local materials. They developed the lesson plan for teaching the meaning of the concept as a radian measure, and the way to obtain its value in radian, which is equivalent to 180 degrees from the conversion of angle measure in degrees into radians. The lesson plan focused on using participatory approaches of learning, such as group discussion and question and answers. G3 developed the lesson plan for teaching π as a concept in Form III class. G3 noticed that the concept of π was not represented in the syllabus as either a topic or sub-topic. As a result, it was a challenging time for the group members to identify the main topic and the subtopic for teaching π as a concept from the syllabus while developing the lesson plan. However, they prepared the lesson so that the students can understand the meaning of the concept as a radian measure, and the conversion of angles from degree measure to radian measure and vice versa. The protractor was used as one of the teaching and learning aids to develop the lesson plan to help students measure the central angle(s) of the drawn circular figure on a plain paper in their small groups. The intention of including the idea of converting angles from degree to radian measure and vice versa in their lesson plan was to help the students recognize that the value of π in radian is equivalent to 180 degrees.

4.2. Presenting the designed lesson plans for teaching the concept

Lesson presentation is one of the practises performed by PSTs in the research meeting by presenting a lesson how to teach π as a concept. As evidenced in the past studies (for instance, Allen & Eve, 1968; Olusanjo, 2010), the lesson presentation is important to help

² Nodes\\Nodes from the Four-day CRICKET\\Practise\DSC00301-CRI

students understand the concept in the class. This presentation is fostered by an affectively designed lesson plan (Bruce *et al.*, 2009; Meador, 2017; Olusanjo, 2010).

The PSTs demonstrated their understanding of π through practising teaching the concept. After developing the lesson plan, each group (G1, G2, and G3) was allowed to present their lessons for 30 minutes. This micro-teaching was performed after designing the lesson for teaching π as a concept. The lesson presentations performed by G1P, G2P, and G3P were part of the session two activities. During lesson presentations G1P, G2P, and G3P acted as teachers in the class, and other participants were students. G1P was the first to present the lesson, followed by G2P and G3P. For instance, below I present the lesson delivered by G1P.

G1P: Good morning! [Greetings from the teacher.]

Participants: Good morning! [Replied by students.]

G1P: Today's session, we are going to learn about the concept of π [while writing on a board]. About learning the concept of π , we are learning about circular objects. Now, in a group of 3 students, you can give us what you know as examples of the circular objects. Work in a group and discuss. 2 minutes group work discussion; give at least two circular objects. [Two groups were formed, namely, Group 1 and Group 2 for this activity.]

Group 1: Examples of circular objects are the top cover of a water bottle.

Group 2: Sugarcane stem.

G1P: Another example of the circular objects are coins and cylindrical figures

[teacher adding other examples]. Now let us come to our concept, we are learning about the concept of π . So, we can define the word π .

Now, what is the meaning of the word π ? [Teacher asking the question orally to the students.]

Participant 1: π refers to the ratio of the circumference of a circle to its diameter.

G1P: Is there another one? [Teacher asking for more contributions.]

Participant 2: I think, π is a constant value, which has been used in different calculations in mathematics or other fields.

G1P: Okay. Thanks for your contributions. We can define π as the ratio of the circumference of a circle to its diameter. When you are dealing with any activity concerning π , you have to learn about the circumference and diameter of a circle. We can see that circumference is a parameter or distance around a circular body, and diameter is a straight line cutting into two equal halves of a circular body. Now, I am giving you these circular objects in your group. From these objects, you can learn how to obtain the value of π , and you use this vernier calliper and a string. The string will be used in measuring the circumference, and a ruler you have on your desk. Now, when you are finding the value of circumference, you have to measure. You measure the circumference by using the string you have

given. You will use string to round a circular object, and you have given a vernier calliper, you will use it to measure the diameter of circular objects. Okay! Now, take your string and round outside of a circular object. Please be free to answer your questions or ask any questions in your group. After rounding the string, then measure the distance you have given on the ruler. Also, take the vernier calliper, and measure the diameter outside of a circular body. Use four minutes to work on this [activity] (teacher said). [Group work activities continuing in small groups shown in Figure 2 below.] (Teacher is silent only observing the ongoing learning on the activity in the two groups, namely, Group 1 and Group 2). Okay, let us start with Group 2; what is the measure of circumference?

- Group 2: 6.5cm [replied by Group 2 members].
 G1P: And what diameter have you found?
 Group 2: 2.05cm [replied by Group 2 members].
 G1P: Excellent, this is the measurement you got for Group 2. Let us come for Group 1. What is the measurement have you recorded for the circumference?
 Group 1: Circumference is 6.4cm [Replied by Group 1 members].
 G1P: Good! How about the diameter?
 Group 1: We are still proceeding [replied by Group 1 members].
 G1P: Let us give you 20 seconds. [While the activity was taking place in Group 1, teacher and Group 2 members were silent waiting for the answer]. What is the measurement of diameter? [Teacher asked Group 1 members].
 Group 1: 16.35mm [replied by Group 1 members].
 G1P: We can convert it into centimetres. What do we get?
 Group 1: 1.635 cm
 G1P: Okay! Excellent in your discussion. Now, let us come and say our values. For Group 2, you have gotten that circumference is 6.4cm and diameter is 2.05cm, and Group 1 has gotten that circumference is 1.635 cm. Now, according to our definition, π is the ratio of circumference to diameter of a circular object. We can find the ratio.

The ratio as $\pi = \frac{c}{d}$. For Group 2, let us say circumference, c as c_2 , and diameter, d as d_2 , and for Group 1, c as c_1 and diameter, d as d_1 . Now, we are going to get the ratio of circumference to diameter. Now, take the ratio of circumference, you have got in your group divided with the diameter you have got in your group. For Group 1, $\pi = \frac{c_1}{d_1} = \dots$ For Group 2, $\pi = \frac{c_2}{d_2} = \dots$ (teacher said). After group

- work discussion, let us start with Group 2, what value of π have you got?
- Group 2: 3.17073....
- GIP: For Group 1, what is the value of π ?
- Group 1: 3.914....
- Group 2: 3.17073.....
- GIP: Okay. This is the value of π we have obtained from the groups. For Group 2, you have gotten that π is 3.17073.... From this, we can obtain that the value of π as an irrational number has no ending decimal places. And Group 1 has obtained the value of π as 3.914... The teacher has given you the same object, but the measurements are in different values. So, for Group 1, I think, there are some errors you have obtained when you were measuring the units. Is it? (Teacher asked). So, the correct value of the given object, its circumference was 6.5cm and diameter as 2.04cm. These were the correct values obtained from measuring the diameter and circumference of the object. So, when measuring the value of π as a ratio, we can get 3.14... which is approximately equivalent to 3.14. When you obtain the value of π by getting the ratio of the circumference to diameter, you have to obtain the value of π which can be approximated to about 3.14 (into 2 decimal places). Now, if you obtain about 3.17073.... which is approximated to 3.17 (into 2 decimal places), and it is near to about 3.14. So, it is the value of π . Okay! Thanks! Now, we can see that for the given value of π , we can say that π is the ratio of circumference over diameter. This ratio can give us another definition of π . We can see that $\pi = \frac{c}{d}$. So, $c = 3.14d$, $c = 3.14d$, because 3.14 is the value which is the number of diameters to complete the circumference of a circle. So, $diameter = \frac{c}{3.14}$ $diameter = \frac{c}{3.14}$. This implies that $d = \frac{c}{\pi}$ or $d = \frac{c}{\pi}$. So, from $c = 3.14d$, We can say that π is the number of diameters to complete one circle or one circumference. This is another definition of π .
- GIP: Have you understood?
- Participants: Yes!
- GIP: Let us come and see, what is the application(s) of π ? In your group. Discuss and give at least two applications of π in your daily life. Where is π in your daily life? How can it be applied? (Teacher asked). (3 minutes group work discussion).

- G1P: Let us see from Group 1, what is the application of π ? As you have discussed.
- Group 1: It is used in electronic devices, and it is used in the calculation of the circumference of a circle.
- G1P: Okay. Good! (Teacher rewarding Group 1). Let us see for Group 2 what are the applications of π ? (Teacher asked).
- Group 2: It is used in road construction, and it is used in mathematical computations.
- G1P: Okay. Good idea! (Teacher rewarding Group 2). So, we can see that these are the applications of π . π is used in different electronic devices, and in calculating the circumference of a circle. Any circular body, you can calculate circumference, areas, and volume. For example, a sphere is circular. Also, π is used in road constructions. Also, π is used in mathematical computations. Those are applications of π . Also, π is used in statistics. Those are some of the applications of what? (Teacher asking students).
- Group 1 and 2: π . (Students replied).
- G1P: Now let us see the summary of this lesson, we have talked about π , we have defined it. π is the ratio of circumference to diameter. So, we measure the circumference by using what? (Teacher asked).
- Group 1 and 2: String! (Students replied).
- G1P: And which instrument do you use to measure the diameter? (Teacher asked the students).
- Group 1 and 2: Vernier callipers (students replied).
- G1P: So, we define π as the ratio of circumference to diameter, and π gives as approximately equivalent to 3.14. So, 3.14 (approximate) is used as a constant value of π in all calculations. Also, we have said that π is the number of diameters to complete a circumference circle. These are some definitions of π . So, these are summaries of what we have learned. So, before calculating the value of π , we have seen that we can find the circumference, and then diameter. After finding the diameter and circumference, you can find the ratio of circumference to a diameter which gives the value of π . Is it clear? Have you understood? (Teacher asked students).
- Group 1 and 2: Yes! (Students replied).
- G1P: Okay. So, in our level (Form I), we can define π as the ratio of circumference to diameter of a circle. But, when you will be on another level, you can find that the concept has different meanings which are the relationship about π in a unit circle. So, thank you, and this is the end of our lesson. (Teacher said to the students).

Group 1 and 2: Thank you! (Students replied).³ (Transcribed data).

PSTs working in small groups measuring and recording the diameter and circumference of a stem of pawpaw, as shown below.

Figure 2. Group work activities taking place for finding the value of π of a stem of pawpaw.⁴



In the above transcript, the participants have demonstrated their understanding how to teach π as a concept using local materials by being able to perform micro-teaching for teaching the meaning of the concept and the way to obtain its value. They used participatory approaches of learning, such as group discussion. G1P used a stem of pawpaw as the teaching and learning resources to help students understand how to define the concept and find its value from measuring its circumference using a string and a ruler, and diameter using a set of vernier callipers. Also, students in small groups were able to experience the applications of π in their daily environment, including finding the circumference of a circular figure or circle.

4.3. Demonstrating the concept

Demonstration is one of the practises performed by the PSTs in the research meeting, including measuring the diameter and circumference of a circular object. As evidenced in the past studies (Bruce, *et al.*, 2009; Chikuni, 2003), demonstration helps learners to develop a better understanding of the mathematical concepts in the class, because the process is visually oriented.

Demonstration of measuring the diameter of a circular object is some of the practical work performed by PSTs in the research meeting. During micro-teaching, the group presenters (G1P, and G2P) used demonstration as one of teaching and learning strategies for teaching π as a concept. For instance, G1P demonstrates:

Look here; when you read on a set of vernier callipers starting at zero, look here, what is this space? This space from here to here, there is a need to look at these units

³ Nodes\\Nodes from the CRI\\Practise\\Presenting the lesson (1)

⁴ Nodes\\Nodes from the CRI\\Practise\\00004-CRI

[readings], and you add to this reading. Why do we forget these units? It is true when we add these units here; the readings become more accurate. Okay! So, you may take this circular object, and then you may put it this way. Okay. Then, you measure the diameter. Look here. In this case, here there are two measurements. I mean scale, the top one, and the bottom one. Okay! So, the top one [upper reading], this one which reads zero up to 150 mm. Okay! You read it first. After that, you record a certain numerical figure. From there you come to find another figure or measure to the bottom one [lower reading] which reads 0.05mm. After getting the numerical value here, how can you identify this one? This one, it will be identified by looking at the line or measure which will correlate with the top one and bottom one. Okay, where they will stand in a straight line or where they will be equal [lower and upper lines intersecting]. Okay! Then, if it is, let say 3, you will multiply it with 0.05, after that, you will add to the measurement you got at the top one. For example, here, let's say it is 80.5 (the top one). Okay! But when you look clearly, the line where there is an intersection, I mean correlation is 80 and 3, so, you take 3 as a reading here times 0.05 plus 80.5. Okay! I will demonstrate clearly in your group. So, if you get the concept of how to measure this diameter, then you record the figure, I mean the measurement.⁵ (Transcribed data).

In the above quote, it seems that the participants have described their understanding of how to obtain the readings of the diameter of a circular object on a set of vernier callipers by being able to demonstrate the proper way of measuring the diameter of a circular object using a set of vernier callipers. G1P demonstrated to the research meeting how to measure the diameter of a circular object by using a set of vernier callipers (as shown in Figure 3 above). G1P also demonstrated to the meeting a correct way of getting the readings (upper reading and lower reading) of the diameter of a circular object on a set of vernier callipers. Measuring the diameter of a circular object using a set of vernier callipers is important to obtain the value of π , because the number of diameters which is required to complete the circumference of the object gives the value of π .

For peer teaching, the entire group was reformulated into their three sub-groups. Despite the fact that PSTs demonstrated measuring the diameter of a circular object using a set of vernier callipers, they also demonstrated measuring the circumference of the object using a string and a ruler. For instance, G2P demonstrates:

This distance obtained from the starting point here to here, this is called the length of the circumference of a circle. Do you get the concept? So, after here you may take this distance you get and then you measure it to the ruler to know the exact numerical value of that distance surrounding the circumference or surrounding a circle which is called the circumference. So, let's say, we get 29 cm as a certain numerical value after

⁵ Nodes\\Nodes from the CRI\\Practise\\Demonstrating the concept (1)

placing it on a ruler. But this practise you will do on your own in a group.⁶
(Transcribed data).

In the above transcript, it seems that the participants understand how to measure the circumference of a pipe by being able to demonstrate the proper way of measuring the circumference of the object using a string and a ruler. G2P demonstrated to the small groups (G1, and G2) during performing the micro-teaching an approach for measuring the circumference of the circular object to obtain the value of π of the object. Still, G2P insisted on using a string and a ruler to measure the circumference of the circular object. The length of the string used to measure the circular object represents the circumference of the object. Measuring the circumference of a circular object is important to obtain the value of π in a variety of ways, including finding the ratio of the two measures (circumference and diameter) of the object.

5. Discussions of the findings

Designing a lesson helps teachers to develop their teaching skills (Shen, Zhen & Poppink, 2007). This process also helps individual teachers to deepen their understanding of pedagogical content knowledge, realise the ways that a subject matter is represented in the mathematics curricula, design the instructional activities and methods that actively engage individual learners on a concept, and realise how individual learners can understand the concept and achieve learning (Wang & Paine, 2003). In the research meeting, the PSTs gained skills on how to design a lesson for teaching a mathematical concept to foster a participatory way of learning. For instance, designing the lesson for teaching π as a concept in the research meeting was a first-hand experience to PSTs. The reason is that the concept is not taught in university mathematics class, only PSTs use it to solve mathematical problems involving π . In doing so, they did not participate and practised teaching π as a concept in their university mathematics classes.

Creativity is the quality value of a society for the 21st century because of innovative and rapid development in the society (Inweregbuh, Ugwuanyi, Nzeadibe, Egara & Emeji, 2020). Creativity is encouraged in the education system because fostering creativity provides an opportunity for generation of new ideas (Despina & Marianthi, 2016). Teachers play a major role in a class to practise creativity through using varieties of teaching and learning strategies, approaches, and resources (Bolden, Harnes & Newton, 2010; Inweregbuh, Ugwuanyi, Nzeadibe, Egara & Emeji, 2020; Lauren, 2018; Sriraman, 2009). Creativities existed among PSTs in the research meeting during designing the teaching and learning aids. The PSTs were asked to design a lesson for teaching π as a concept. PSTs worked in three formed small groups (Group 1, 2, and 3). Each group used circular objects available in their local environment. For instance, Group 1 used stems of pawpaw as one of their teaching and learning aids for teaching π as a concept. The objects were local and circular. Group 2 used a

⁶ Nodes\\Nodes from the CRI\\Practise\\Demonstrating the concept (3)

top cover of a water bucket. Group 3 used plane papers folded on a round sole tape. These designed teaching and learning aids show that PSTs were creative in using the resources available in their local environment to design the lesson for teaching π as a concept. As such, CRI empowered PSTs by designing the teaching and learning aids that they used to practise teaching the concept in the research meeting.

Teacher confidence is critical for positive effect among learners (Stipek, Givvin, Salmon & MacGyvers, 2001) and for quality design and delivering a lesson (Norton, 2017). This results in achieving learning among learners (Ben-Hur, 2006). In the research meeting, the PSTs gained confidence on how to teach π as a concept during group lesson presentations. Also, the confidence was seen during demonstration on how to obtain the value of π from a circular object using a set of vernier callipers, a string and a ruler. The PSTs demonstrated in the research meeting how to measure a diameter of a circular object using a set of vernier callipers. Also, they demonstrated in the meeting how to measure the circumference of the same object using a string and a ruler. These two measures (circumference and diameter) of the object were important to obtain the value of π , because the ratio of two measures (circumference and diameter) of the circular object gives the value which is approximately equivalent to 3.14.

6. Conclusion

Practice related to a concept in mathematics class is necessary to help PSTs gain pedagogical content knowledge and skills for teaching the concept. We see connections in this to the concept-rich instruction activities developed by Ben-Hur (2006) in the North American context. We also see connections in the instructional activities designed by Deogratias (2020) in the Tanzanian context while conducting research with university pre-service mathematics teachers. In both American and Tanzanian contexts, the authors have found that learners need sufficient practises related to the concept for in-depth understanding of the concept and to achieve learning. In this study, it is suggested that practises related to the concept could be done in mathematics classes in primary and secondary schools, colleges, and universities because of the benefits that individual learners gain through practises.

Fortunately, we have strong examples of teachers and teacher educators engaging learners in such ways. Deogratias's (2020) doctoral dissertation and Ben-Hur's (2006) book on concept-rich instructional activities can be used by teachers, teacher educators, and researchers in mathematics and serve for designing the instructional activities. Deogratias (2022) also advocates using real objects as teaching and learning resources that can contribute for the delivery of the instructional activities that foster learning by practising a mathematical concept.

As it is, the mathematics teaching and learning process suffers from being distanced from learning by doing—practises related to a concept (Ben-Hur, 2006; Deogratias, 2020). This is especially the case at Tanzanian university level, where practices related to a concept are

rarely done by PSTs in mathematics classes (Deogratias, 2020). Ignoring practices related to a concept limits PSTs to actively engage in learning mathematics.

Mathematics teachers and teacher educators can encourage a participatory way of learning through designing instructional activities that encourage learners to practise the concept in mathematics class. While doing so, teachers and mathematics educators need to ask themselves a question: What does it mean by practising a mathematical concept in each learning context? Responding to this question while designing and implementing instructional activities in mathematics class, can foster a valuable practice related to the concept among learners.

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