A Multiple Rolling Turning Point Detection Method

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Abstract.

Detecting time series turning points is crucial in the financial field where series are characterized by several changes in their trajectories. This paper proposes a multiple rolling test of hypothesis of a regression model slope change where the entry and exit windows contain more than one observation, thus contributing to a significant reduction of false signals and the corresponding probability of wrong decisions. To give evidence of the procedure's performance in predicting turning points, we consider – as a preliminary analysis – a set of twenty stocks selected from the EURO STOXX 50 Index, covering the historical period 2010 – 2021. The model is run with different values of the main parameters, providing additional information in investment decision making.

Keywords: Turning point detection, financial time series, time varying parameters, probability-based approach

1. Introduction

Timely detection of turning points is a task of great importance and a challenge in trading and investment activity for planning profitable investment decisions and control actions especially in financial time series characterized by high frequency data set. They are defined as the periods where a series changes its local slope (Zellner et al., 1991) and are usually referred both to the mean and volatility of the series.

Typically, financial series will contain several turning points. There is therefore a growing need to be able to search for such changes efficiently. Many techniques have been developed during the years for peak and trough estimation. For a general overview see e.g. (Aminikhanghahi & Cook, 2017) and (Chen et al., 2005).

In this paper we propose an extension of the probability-based approach for turning point identification developed by (Bramante et al., 2019). The procedure is based on hypothesis testing of a regression model slope change by adding/removing observations to a rolling fixed time window. Good financial performances are obtained considering only one entering/leaving observation but, with the aim to reduce further the effect of false signals, our proposal takes into account a greater number of observations entering/leaving the test window. Along with the description of the methodology, we apply as an example the proposed procedure to twenty stocks selected from the EURO STOXX 50 Index, considering data on a daily and weekly frequency covering the period 2010 – 2021. In order to evaluate the sensitivity of the methodology, the model is run with different values of the main parameters, providing additional information in investment decision making.

The rest of the paper is organized as follows. In Section 2 the methodological aspects of the model are discussed. In Section 3 we apply the proposed procedure to a selection of financial time series. Section 4 concludes.

2. Methodology

2.1 About trend lines in technical analysis

All technical analysis softwares provide a wide variety of indicators to identify turning points and corresponding buy and sell signals. Among them, the trendline is is simply a linear regression line drawn between two points using the least squares fit method. The trend line is a "momentum indicator" and can slope in different directions depending on the number of observations considered. Regression trendlines can be used to draw trend channels by plotting two parallel, equidistant lines above and below the trend line. Regression channels contain price movement, with the bottom channel line providing support and the top channel line providing resistance, thus giving evidence of a reversal in trend if prices remain outside the channel for a longer period of time. The main issue of this approach, widely used by financial analysts, is that lines are subjectively drawn inspecting the chart and then connecting the located two points.

2.2 Test on the difference between two consecutive slopes

Let Y_t, $t = 1, 2,\ldots, T$ be a time series and t_{cp} the time event of a turning point ($1 < t_{cp} < T$). Starting from trend change detection within linear models, we assume the trend approximates a straight line estimated using a rolling fixed time window of size K:

$$Y_{t,K} = \alpha_{t,K} + \beta_{t,K}t + \varepsilon_t, \qquad k = 1, 2, \ldots, K \tag{1}$$

Here $Y_{t,K}$ are the most recent values from time t-K+1 to t, $\alpha_{t,K}$ and $\beta_{t,K}$ are the corresponding rolling parameters and ε_t is the error component.

Denoting with $r = 1, 2,..., R$ the rolling counter increment historical windows ($t = K+R-1,\ldots, T$) are usually rolled through only one observation at a time ($R = 1$) so as to detect a change as soon as possible after the turning point has occurred. This choice can lead to the announce of too many false alarms (i.e. a signal of a turning point is generated provided the change has not occurred, type II error), therefore values of R greater than one can serve as a confirmation of the signal at $r = 1$.

To identify the turning point at time $t+R$, we use the following system of hypotheses:

$$\begin{cases} H_0: \beta_{t+R,K} = \beta_{t,K} \\ H_1: \beta_{t+R,K} \neq \beta_{t,K} \end{cases} \tag{2}$$

where $\beta_{t+R,K}$ is the rolling parameter at time $t+R$. More specifically, under the null hypothesis there is no signal of turning point at time $t_{cp} = t+R$, otherwise for the alternative one.

The difference between $\hat{\beta}_{t+R,K}$ and $\hat{\beta}_{t,K}$, that represent the ordinary least squares estimates of β at time $t+R$ and t is

$$\hat{\beta}_{t+R,K} - \hat{\beta}_{t,K} = \frac{1}{2K\sigma_t^2}\left\{\sum_{r=1}^{R}[K - 2(R-r) - 1]\left[y_{t+R} - \bar{y}_{(t-K+1)+R:t}\right] + \right.$$
$$\left. +[K - 2(r-1) - 1]\left[y_{t-K+R} - \bar{y}_{(t-K+1)+R:t}\right]\right\} \tag{3}$$

where $\sigma_t^2 = \frac{K^2-1}{12}$ $\forall R$, and $\bar{y}_{(t-K+1)+R:t}$ is the average of the K-R observations from $(t$-$K+1)+R$ to t. The special case with $r = R = 1$ is described in (Bramante et al., 2019).

Based on Eq. 3, the test on the difference between $\beta_{t+R,K}$ and $\beta_{t,K}$ switches to a series of tests on the difference between the $2R$ observations entering or leaving the rolling windows and the overlapping mean. In this case one of the two samples of the classical t-test contains only one observation, so it does not contribute to the degrees of freedom or to estimate the pooled variance within groups; see (Sokal & Rohlf, 1969) for details.

For observations entering (E) the rolling time window, we consider a one-tailed right test, and the test statistic is:

$$\tau_{K-R-1;E} = \frac{y_{t+R} - \bar{y}_{t-K+1+R:t}}{\sqrt{\frac{K}{K-1} s_{t-K+1+R:t}^2}} \sim t_{K-2} \tag{4}$$

where $s_{t-K+1+R:t}^2$ is the sample variance of the K-R observations from $(t$-$K+1)+R$ to t.

The decision rule is splitted according to whether a peak or a trought is considered. Specifically, a lower turning point has been detected at time $t_{cp} = t+R$, if

$$P\left(t_{K-2} \leq \tau_{t+R;E}\right) > 1 - P(t_{K-2} \leq \tau_{K-R-1}^*) \tag{5}$$

Otherwise, an upper turning point has been detected at time $t_{cp} = t+R$, if

$$P\left(t_{K-2} \leq \tau_{t+R;E}\right) < P(t_{K-2} \leq \tau_{K-R-1}^*) \tag{6}$$

The benchmark under the null hypothesis which allows the turning point detection rule to be set up is the value of the test statistic in case of a perfect straight line:

$$\tau^*_{K-R-1} = \sqrt{3}\frac{K+R-1}{\sqrt{(K-R+1)^2}} \tag{7}$$

In fact, when Eq. (1) corresponds to a perfect straight line, the test statistic both for the entering and leaving observations is equal to τ^*_{K-R-1}, that can be interpreted as the benchmark under the null hypothesis (ie, no turning points have been detected).

As the observation entering the rolling window is the most recent data point, usually it is enough to set up the turning point identification following the procedure described above. Anyway, it is possible to include in the decision process also the observations leaving (*L*) the rolling time window, considering a one-tailed left test with test statistic:

$$\tau_{K-R-1;L} = \frac{y_{t-K+R} - \bar{y}_{t-K+1+R:t}}{\sqrt{\frac{K}{K-1} s^2_{t-K+1+R:t}}} \sim t_{K-2} \tag{8}$$

and reversing the test probabilities in Eq. 5 and 6.

3. Results and Discussion

This section gives an example of the proposed model to give evidence of the performance in predicting turning points. Twenty stocks, identified for their market capitalization – liquidity and number of data points available, are selected from the EURO STOXX 50 Index. Turning points estimates are evaluated using data on a daily and weekly frequency covering the period 2010 – 2021; the model is run with different values of the two parameters *K* and *R*.

In Fig. 1 an example on a selected stock for two different values of *r* is depicted when the weekly time series is considered, and *K* is set to 50. Turning points selection clearly vary over the sample data; the number of generated signals reduces when *r* increases even though trend identification seems to be preserved.

Figure 1: Turning points for a sample stock (K=50; r=1 and 5)

Regarding the performance of the model, evaluated in terms of the evaluated strategies gross profitability, the Return on Average Capital Employed (ROACE) is employed by collecting information on all trade outcomes related to each value of *K* and *R* connected to a turning point signal.

In Tab. 1 ROACE estimates for the five tested values of *K* (three only in the weekly frequency analysis since a maximum value of *K*=50 is used) are reported, together with the evidence on the percentage of winning trades, both in terms of long and short strategies. An increase of profitability is achieved, in most of the reported cases, when both *K* and *R* increase and when the lowest (weekly) frequency time series is considered. Having a look to the percentage of winning trades, the overall win/loss ratio seems quite constant throughout the strategies whereas it is clear that buy strategies (a minimum turning point is detected) perform better than the opposite short ones (a peak is detected). This can be related to the capability of the model to respond quickly in trending down markets that might merit further investigation using a large number of securities.

Table 1: STOXX 50 ROACE and % winning trades by K and R

		Daily				**Weekly**			
			% Winning Trades				% Winning Trades		
K	*R*	ROACE	ALL	LONG	SHORT	ROACE	ALL	LONG	SHORT
	1	-10.57	33%	38%	28%	-2.48	35%	46%	23%
	2	-9.64	33%	39%	27%	0.84	36%	50%	22%
10	3	-7.30	35%	42%	28%	-1.26	34%	46%	21%

	4	-4.39	34%	42%	27%	-0.15	35%	48%	21%
	5	-2.01	35%	42%	29%	3.18	37%	51%	24%
	1	-4.83	33%	41%	24%	2.07	37%	51%	22%
	2	-3.40	34%	43%	26%	3.73	37%	56%	18%
20	3	-3.29	36%	45%	27%	1.99	37%	53%	20%
	4	-2.64	35%	45%	26%	1.42	36%	54%	18%
	5	-1.66	34%	42%	26%	4.56	39%	58%	19%
	1	0.33	34%	45%	23%	4.65	33%	50%	14%
	2	-0.83	34%	43%	24%	6.01	37%	55%	15%
50	3	-1.26	34%	44%	25%	9.24	37%	57%	13%
	4	-0.85	36%	49%	22%	8.28	40%	61%	15%
	5	-0.04	37%	49%	24%	11.42	41%	62%	15%
	1	1.44	31%	45%	16%				
	2	1.50	32%	46%	18%				
100	3	1.66	35%	51%	18%				
	4	1.72	35%	50%	20%				
	5	1.61	35%	53%	16%				
	1	2.75	32%	48%	13%				
	2	3.65	36%	53%	17%				
200	3	4.19	36%	53%	19%				
	4	4.44	35%	53%	16%				
	5	4.39	35%	54%	14%				

In Tab. 2 (daily frequency) and Tab. 3 (weekly frequency) only the strategies that belong to the highest ROACE for each stock are considered, leaving out in this way “sub-optimal” turning points detection schemas. Results, if compared with the ones reported in Tab. 1, are clearly better both in terms of profitability and win/loss ratios. The highest values of the couple *K*-*R* prevail in both the time series frequencies used, thus confirming that the use of *R* as a signal confirmation parameter avoids over identification of turning points candidates and allows major patterns to stand out while de-emphasizing minor price movements.

Table 2: STOXX 50 ROACE and % winning trades by K and R (Daily Data – Best Strategies)

	% Assets		% Assets	% Winning Trades			
K		*R*		ROACE	ALL	LONG	SHORT
10	10%	4	50%	2.41	37%	43%	32%
		5	50%	7.19	32%	38%	27%
20	10%	3	50%	-0.02	37%	44%	30%
		5	50%	3.87	39%	56%	23%
50	10%	1	25%	3.02	48%	45%	30%
		4	25%	6.52	37%	64%	8%
		5	50%	8.11	36%	48%	23%
100	20%	4	50%	6.38	53%	78%	25%
		5	50%	7.08	54%	86%	17%
200	50%	2	10%	4.95	43%	75%	
		3	20%	6.07	48%	46%	30%
		4	20%	8.72	44%	78%	
		5	50%	7.83	41%	58%	20%

Table 3: STOXX 50 ROACE and % winning trades by K and R (Weekly Data – Best Strategies)

	% Assets		% Assets	% Winning Trades			
K		*R*		ROACE	ALL	LONG	SHORT
	15%	1	33%	1.61	53%	56%	50%
10		2	33%	4.74	41%	53%	29%
		5	33%	8.16	41%	47%	36%
20	30%	1	17%	0.78	41%	56%	25%
		2	17%	2.81	50%	80%	20%
		3	17%	4.81	48%	61%	35%
		5	50%	3.86	38%	67%	14%
50	55%	1	18%	2.61	37%	40%	33%
		3	9%	4.84	67%	92%	
		4	9%	6.54	67%	93%	
		5	64%	11.72	58%	88%	10%

4. Conclusion

Timely and accurate identification of turning points is crucial in financial analysis. In this paper, an extension an algorithm based on regression trend lines with multiple observations in the entering/leaving window is proposed. The procedure's performance is illustrated, as a preliminary example, considering a case study focused on twenty stocks selected from the EURO STOXX 50 Index covering the period 2010 – 2021. The empirical analysis shows that the model allows an adequate detection of peaks and troughs signals, and the use of R as a signal confirmation parameter reduce the announce of false alarms allowing major patterns to stand out.

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