A Bi-Objective Model for Determining an Optimal Warehouse Capacity, And Product Allocation in a Green Multi-Product, Multi-Period Distribution Network

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Abstract

Nowadays, the growing trend of environmental concerns is significantly higher than in the past while the business also focuses on the supply chain more to serve the customer demand using the lowest resources possible which leads to two main decisions which are capacity, and product allocation. Therefore, this research aims at designing a multi-product, multi-period, and multi-echelon supply chain network with factories, internal warehouses, external warehouses, and customers while also trying to lower the environmental effect of the supply chain through gas emission. For this problem, large data of products and 20 periods of planning horizon of a real-world polymer industry are considered. The product deliveries use direct shipment from both internal and external warehouses. We develop bi-objective mixed-integer linear programming to find the most optimal product allocation and capacity while minimizing the total logistic costs which include warehouse and transportation cost and total CO_2 emission through transportation. The problem is solved by the min-max approach through a Mixed integer linear programming model using CPLEX software. After we get the result, we compare it with the single-objective model's result to determine the trade-off between the total logistic cost and emission gas. Our base case result shows a better overall satisfaction level among all the models.

Keywords: Bi-objective programming, four-echelons supply chain, green supply chain, min-max approach, and Mixed-integer linear programming

1. Introduction

In recent years, environmental concerns have grown significantly due to global warming and environmental regulation by governments around the world. In Thailand, TISI 2315-2551 is established to regulate pollutants such as carbon dioxide (CO_2). These regulations and the climate changes result in making the companies adjust their supply chain to be greener. The total carbon dioxide emission in Thailand was estimated at 279.31 mega-tons in 2018 while in 2019, it is slightly reduced to 275.06 mega-tons.

As for the distribution network, this research aims to improve the performance of the supply chain by optimizing the capacity and product allocation in one of Thailand's polymer distribution networks. Their supply chain network is a 4-echelon supply chain consisting of factories, internal warehouses, external warehouses, and customers from both overseas and domestic as shown in Figure 1. The factory produces multiple products which will be shipped to the internal warehouse for packaging. Due to the limited capacity of the internal warehouses, some of the products will be sent to the external warehouse. Therefore, the customer can receive the products from both internal and external warehouses. Since the objective is to minimize the total cost which includes transportation and warehousing cost as well as the CO2 emission, a bi-objective mixed-integer linear programming (MILP) model is formulated. The model is solved by using a commercial solver together with the min-max method (Sun et al., 2019).

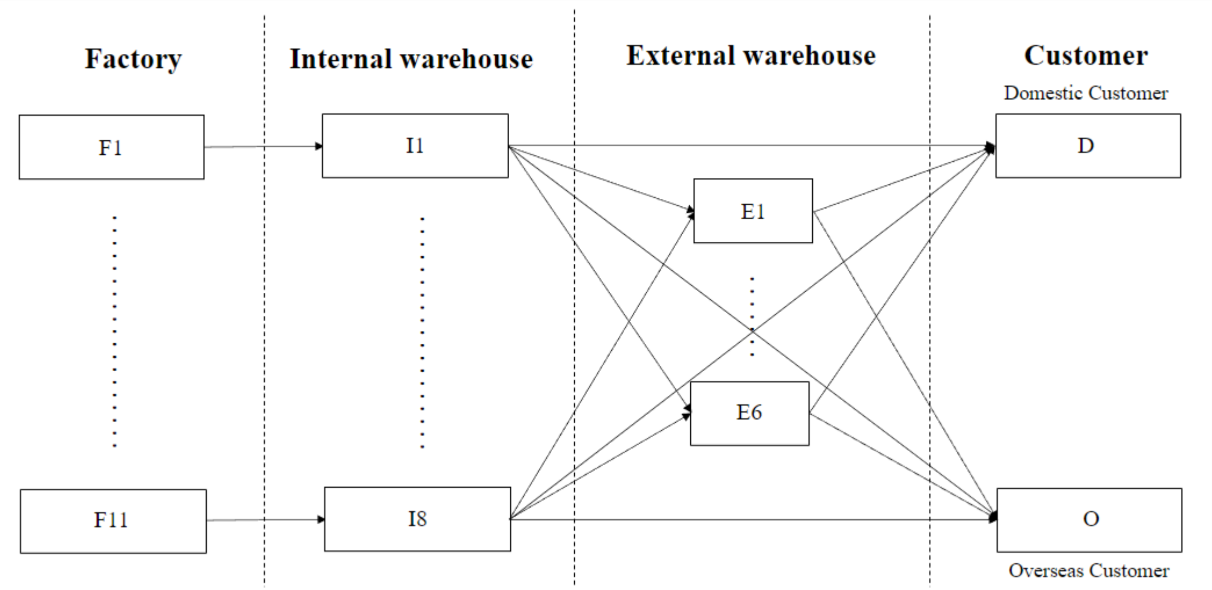

Figure 1. Model diagram

1.1 Literature Review

The supply chain network performance always involves capacity and product allocation. Therefore, the relevant studies in the past are reviewed to make new contributions. Table 1 summarizes the decision variable, characteristics, and method of approach. For easier understanding, we refer to the studies as the number given in table 1.

According to table 1, the number of echelons is ranged from two (1,2,4,8), three (3,10,11), four (5,9) to five (6,7). The two echelons supply chain (1,2,4,8) is the supply chain that involves direct shipment, while three echelons (3,10,11) consider warehouse. For Jirachai et al. (2019), the study includes the external warehouse in which the customers can receive the product from both internal and external warehouses. For Shankar et al. (2013), the supply chain includes suppliers, plants, distribution centers, and customers.

For the decision variable, the optimal location (3-6,8-11) is considered by listing the possible location and using the binary variables to select the location to open the facility. The study with facility capacity (1,3,5-11) also considers the number of products that can be kept in the facility and the study that considers multiple products (2-8,10) will have to allocate the product to match with the time frame. Studies (6,7,10,11) considered the environmental aspect in the supply chain. Jamshidi et al. (2012) minimize the amount of nitrogen oxide, carbon monoxide, and volatile organic produced in the supply chain, while Paksoy et al. (2010) and Wang et al. (2011) minimize the amount of carbon dioxide emission. Lastly, Sun et al. (2019) minimize the impact of transport and facilities on the population's health.

Mixed-integer linear programming is used to find the optimal solution (3-9,11). Askin et al (2013). also, use genetic algorithm and heuristic approach to solve the problem. Santosa & Kresna (2015) applied a simulated annealing method in addition. Memetic using Taguchi method is applied in Jamshidi et al. (2012). Lagrangian relaxation is used in Sadjady & Davoudpour (2012). Hybrid particle swarm optimization is used in Shankar et al. (2013). The normalized constraint method is used in Wang et al. (2011) and the min-max approach is applied to Sun et al. (2019). For the non-linear programming model (1,10), the heuristic method is used in Lee and Elsayed (2005), while the normalized constraint method is used in Wang et al. (2011.) Lastly, Zhu et al. (2020) used the heuristic method to solve the problem.

Table 1. Research gap

No	Study	Echelons	Decision variable			Characteristic		Method
			Location	Capacity	Product Allocation	Multiple products	Environment aspect	
1	Lee & Elsayed (2005)	2		✓				NLP, Heuristic
2	Zhu et al. (2020)	2			✓	✓		Heuristic
3	Askin et al. (2013)	3	✓	✓	✓	✓		MIP, GA, Heuristic
4	Santosa & Kresna (2015)	2	✓		✓	✓		MIP, SA
5	Jirachai et al. (2019)	4	✓	✓	✓	✓		MIP
6	Jamshidi et al. (2012)	5	✓	✓	✓	✓	✓	MIP, Memetic using Taguchi method
7	Paksoy et al. (2010)	5-Forward, 5-Reversed		✓	✓	✓	✓	MIP

8	Sadjady & Davoudpour (2012)	2	✓	✓	✓	✓		MIP, Heuristic, Lagrangian relaxation
9	Shankar et al. (2013)	4	✓	✓				MIP, Hybrid particle swarm optimization
10	Wang et al. (2011)	3	✓	✓	✓	✓	✓	NLP, Normalized constraint method
11	Sun et al. (2019)	3	✓	✓			✓	MIP, min-max approach
This study		4		✓	✓	✓	✓	MIP, min-max approach

1.2 Problem Statement

The major details of this case study including the distribution network, transportation mode, product properties, warehouse operations, and gas emission will be described in this section

1.2.1 Distribution network

In this case-study supply chain, there are eleven factories, eight internal warehouses, six external warehouses, one overseas customer, and domestic customers scattered in 44 provinces of Thailand. All products are produced at the factory and then sent to the internal warehouse for packaging and store. Due to the limited capacity of the internal warehouses, some of the products will be transferred to the external warehouse, which means the products will be sent either from internal or external warehouses. For the transportation modes, there is a total of 12 modes, e.g., 10-wheels truck, 18-wheels truck, truck, etc. In which different modes are used for transporting products from each node.

1.2.2 Products

Each factory produces one type of polymer product which in total has over 100 different grades. Later they will be packed in different package sizes and types, e.g., 20 kg bag, 500kg bag, slab, etc. Different modes of transportation are required for these packages. Therefore, the products combinations from factories, grade, and package type are around 711 product combinations.

1.2.3 Gas Emissions

Carbon dioxide emissions data are obtained from the Life cycle assessment laboratory of The National Metal and Materials Technology Center. In the data, the amount of emission is measured by mode and per kilometer.

1.2.4 Assumptions

- Production capacity from the factories is to satisfy all the demand.
- The warehouse's storage space is calculated using the summation of the required space overall products. The requirement is calculated by multiplying the inventory day of that product with the product's annual flow
- The planning horizon is 20 years long

2. Methodology

2.1 Mathematical Model

The problem is formulated as mixed-integer linear programming (MIP) models, where each model minimizes the different objective functions. In addition, we formulated another model that seeks a compromise solution among two objective functions. The models' indices, parameters, and decision variables are given below:

Set and index*:* products, $p \in P$; factory, $f \in F$; internal warehouses, $i \in I$; external warehouses, $e \in E$; domestic customers, $d \in D$; oversea customers $o \in O$; modes, $m \in M$; time period, $t \in T$; internal warehouse operation, $a \in A$; external warehouse operation, $b \in B$; transferred product warehouse operation, $h \in H$; factory and internal warehouse, $(f, i) \in FI$; factory and product, $(f, p) \in FP$; internal warehouse and product, $(i, p) \in IP$.

Parameter*:* $C_{f,i,m,t}$ unit transportation cost from factory f to internal warehouse i using mode m in period t. $M_{f,i}$ transportation mode set from factory f to internal warehouses i, $D_{f,i}$ distance from factory f to internal warehouses i, G_m carbon dioxide emission using mode m; C_i , C_e unit cost of performing all warehousing operations for products stored at internal warehouse i and external warehouse e, respectively. $C_{i,e}$ unit cost of performing all warehousing operations for products stored at internal warehouse i to external warehouse e; W_i, W_e warehouse capacity at internal warehouse i and external warehouse e, respectively; $W_{f,p,t}$ amounts of products p supplied from factory f in period t; $R_{d,p,t}$, $R_{o,p,t}$ domestic customer d and oversea customer o demand, respectively of product p in period t, I_p inventory days of product p

Decision variables*:* $X_{f,i,m,p,t}$ flow of product p from factory f to internal warehouse i using mode m in period t. $Y_{i,p,t}$, $Y_{e,p,t}$ amounts of products p stored at internal warehouse i and external warehouse e in period t, respectively. $Z_{i,p,t}$ amounts of products p transferred from internal warehouse i in period t. $S_{i,t}$, $S_{e,t}$ total storage space required at internal warehouse i and external warehouse e in period t, respectively.

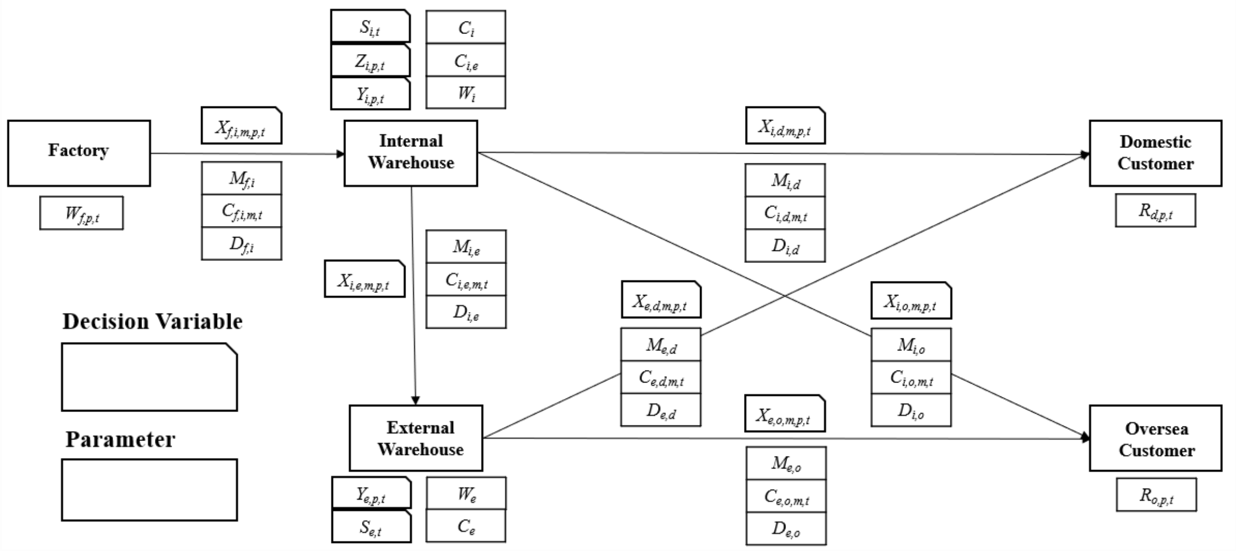

Figure 2. Model diagram

2.1.1 MIP model for minimizing total cost

Minimize:

$$O_c = \sum_{f\epsilon F}\sum_{i\epsilon I}\sum_{m\epsilon M_{FI}}\sum_{t\epsilon T}\sum_{p\epsilon P} C_{f,i,m,t}X_{f,i,m,p,t} + \sum_{i\epsilon I}\sum_{e\epsilon E}\sum_{m\epsilon M_{IE}}\sum_{t\epsilon T}\sum_{p\epsilon P} C_{i,e,m,t}X_{i,e,m,p,t} +$$

$$\sum_{i\epsilon I}\sum_{o\epsilon O}\sum_{m\epsilon M_{IO}}\sum_{t\epsilon T}\sum_{p\epsilon P} C_{i,o,m,t}X_{i,o,m,p,t} + \sum_{e\epsilon E}\sum_{o\epsilon O}\sum_{m\epsilon M_{EO}}\sum_{t\epsilon T}\sum_{p\epsilon P} C_{e,o,m,t}X_{e,o,m,p,t} +$$

$$\sum_{i\epsilon I}\sum_{d\epsilon D}\sum_{m\epsilon M_{ID}}\sum_{t\epsilon T}\sum_{p\epsilon P} C_{i,d,m,t}X_{i,d,m,p,t} + \sum_{e\epsilon E}\sum_{d\epsilon D}\sum_{m\epsilon M_{ED}}\sum_{t\epsilon T}\sum_{p\epsilon P} C_{e,d,m,t}X_{e,d,m,p,t} +$$

$$\sum_{a\epsilon A}\sum_{i\epsilon I}\sum_{t\epsilon T}\sum_{p\epsilon P} C_i Y_{i,p,t} + \sum_{b\epsilon B}\sum_{e\epsilon E}\sum_{t\epsilon T}\sum_{p\epsilon P} C_e Y_{e,p,t} + \sum_{h\epsilon H}\sum_{i\epsilon I}\sum_{t\epsilon T}\sum_{p\epsilon P} C_{i,e} Z_{i,p,t} \tag{1}$$

Subject to:

$$\sum_{m\ in\ M_{FI}} X_{f,i,m,p,t} = W_{f,p,t}\ ,\forall(f,p) \in FP\ ,\forall(f,i) \in FI\ ,\forall t \in T \tag{2}$$

$$\sum_{m\ in\ M_{FI}} X_{f,i,m,p,t} = Y_{i,p,t} + Z_{i,p,t}\ ,\forall(i,p) \in IP\ ,\forall t \in T \tag{3}$$

$$\sum_{d\ in\ D}\sum_{m\ in\ M_{ID}} X_{i,d,m,p,t} + \sum_{o\ in\ O}\sum_{m\ in\ M_{IO}} X_{i,o,m,p,t} = Y_{i,p,t}\ ,\forall(i,p) \in IP\ ,\forall t \in T \tag{4}$$

$$\sum_{e\ in\ E}\sum_{m\ in\ M_{IE}} X_{i,e,m,p,t} = Z_{i,p,t}\ ,\forall(i,p) \in IP\ ,\forall t \in T \tag{5}$$

$$\sum_{m\ in\ M_{IE}} X_{i,e,m,p,t} = Y_{e,p,t}\ , \forall e \in E, \forall (i,p) \in IP\ , \forall t \in T \quad (6)$$

$$\sum_{d\ in\ D}\sum_{m\ in\ M_{ED}} X_{e,d,m,p,t} + \sum_{o\ in\ O}\sum_{m\ in\ M_{EO}} X_{e,o,m,p,t} = Y_{e,p,t}\ , \forall (i,p) \in IP\ , \forall t \in T \quad (7)$$

$$\sum_{e\ in\ E} X_{e,o,m,p,t} + X_{i,o,m,p,t} = R_{o,p,t}\ , \forall o \in O, \forall (i,p) \in IP, \forall p \in P\ , \forall t \in T \quad (8)$$

$$\sum_{e\ in\ E} X_{e,d,m,p,t} + X_{i,d,m,p,t} = R_{d,p,t}\ \ , \forall d \in D\ , \forall (i,p) \in IP, \forall p \in P, \forall t \in T \quad (9)$$

$$Y_{i,p,t} + \sum_{e\ in\ E} Y_{e,p,t} = W_{f,p,t}\ \ , \forall (i,p) \in IP, \forall p \in P\ , \forall t \in T \quad (10)$$

$$S_{i,t} = \sum_{p\ in\ P} \left(\frac{I_p}{365}\right) Y_{i,p,t}\ \ , \forall i \in I, \forall t \in T \quad (11)$$

$$S_{e,t} = \sum_{p\ in\ P} \left(\frac{I_p}{365}\right) Y_{e,p,t}\ , \forall e \in E, \forall t \in T \quad (12)$$

$$S_{i,t} \le W_i\ \ , \forall i\ \in I, \forall t \in T \quad (13)$$

$$S_{e,t} \le W_e\ \ , \forall e\ \in E, \forall t \in T \quad (14)$$

$$\sum_{i\ in\ I}\sum_{m\ in\ M_{IE}} (\frac{I_p}{365}\ X_{i,e,m,p,t})\ \le\ W_e\ , \forall e \in E\ , \forall t \in T \quad (15)$$

The objective function (1) is to minimize the total transportation costs from each node combined with the total warehousing costs in each warehouse. Constraints (2) make sure that the factory production capacity must equal to the amount of product supplied from the factory to the internal warehouse. Constraints (3) state that the number of products flowing into the internal warehouse must either be stored there or transferred to the external warehouse. Constraints (4) force the stored products in the internal warehouse to be sent to either domestic or overseas customers. Constraints (5) force the transferred product from the internal warehouse must be shipped to the external warehouse. Constraints (6) state that the number of products flowing into the external warehouse must be stored there. Constraints (7) force the stored products in the external warehouse to be sent to either domestic or overseas customers. Constraints (8,9) make sure that domestic and overseas customer's demand must equal the number of products being sent from the internal and external warehouse to domestic and overseas customers respectively. Constraint (10) forces the stored product in both internal and external warehouses to be equal to the production capacity. Constraints (11,12) describe the storage area at the internal and external warehouse to be adjusted by the products' inventory day. Constraint (13,14) make sure that the storage areas at the internal and external warehouse must not surpass its capacity. Constraint (15) make sure that the flow forms the internal to external warehouse that is already adjusted by the inventory day to not exceed the external warehouse’s capacity.

2.1.2 MIP model for minimizing total carbon dioxide emission

Minimize:

$$O_{co} = \sum_{f\ in\ F}\sum_{i\ in\ I}\sum_{m\ im\ M_{FI}}\sum_{p\ in\ P}\sum_{t\ in\ T} X_{f,i,m,p,t}D_{f,i}G_m + \sum_{i\ in\ I}\sum_{e\ in\ E}\sum_{m\ im\ M_{IE}}\sum_{p\ in\ P}\sum_{t\ in\ T} X_{i,e,m,p,t}D_{i,e}G_m + \sum_{i\ in\ I}\sum_{d\ in\ D}\sum_{m\ im\ M_{ID}}\sum_{p\ in\ P}\sum_{t\ in\ T} X_{i,d,m,p,t}D_{i,d}G_m + \sum_{e\ in\ E}\sum_{o\ in\ O}\sum_{m\ im\ M_{EO}}\sum_{p\ in\ P}\sum_{t\ in\ T} X_{e,o,m,p,t}D_{e,o}G_m + \sum_{i\ in\ I}\sum_{o\ in\ O}\sum_{m\ im\ M_{IO}}\sum_{p\ in\ P}\sum_{t\ in\ T} X_{i,o,m,p,t}D_{i,o}G_m + \sum_{e\ in\ E}\sum_{d\ in\ D}\sum_{m\ im\ M_{ED}}\sum_{p\ in\ P}\sum_{t\ in\ T} X_{e,d,m,p,t}D_{e,d}G_m \tag{16}$$

Subject to constraints (2) - (15)

The objective function (16) is to minimize the total carbon dioxide emission from each node.

2.1.3 MIP model for determining compromise solutions

We formulated a compromise solution using the min-max approach which is used to find the solution that minimizes the deviations from the ideal result.

Let O_c^*, O_{co}^* be the optimal solution obtained from models 2.11 and 2.12. Furthermore, let O_c^{worst}, O_{co}^{worst} be the worst result that can be obtained from models 2.11 and 2.12. We can now find the deviation between the ideal result and the current result by normalizing the functions below.

$$\sigma_c = \frac{O_c - O_c^*}{O_c^{worst} - O_c^*} \tag{17}$$

$$\sigma_{co} = \frac{O_{co} - O_{co}^*}{O_{co}^{worst} - O_{co}^*} \tag{18}$$

We can now develop a min-max algorithm to the deviations of our objectives by adding a new decision variable Y as follows.

Minimize:

$$Y \tag{19}$$

Subject to:

$$\sigma_c \leq Y \tag{20}$$

$$\sigma_{co} \leq Y \tag{21}$$

$$(1) - (16) \tag{22}$$

The goal of the min-max algorithm is to minimize the largest deviation from the optimal result among the two objectives.

3. Results

3.1 Case study data

There are four products $P = \{1, 2, 3, 4\}$, two factories $F = \{1, 2\}$, two internal warehouses $I = \{1, 2\}$ which has two operations within the warehouse $A = \{1, 2\}$ and two operations to transfer products to external warehouses $H = \{1, 2\}$. There are two external warehouses $E = \{1, 2\}$ which has two operations $B = \{1, 2\}$, four domestic customers $D = \{1, 2, 3, 4\}$ and one oversea customer since all shipments go through the same seaport $O = \{1\}$ and five time periods $T = \{1, 2, 3, 4, 5\}$. There are total of three transportation modes $M = \{1, 2, 3\}$ where different modes are used depend on the origin and destination. The amount of generated Carbon dioxide is dependent on transportation mode and distance between the nodes.

3.2 Base case results

The summary of the total cost and gas emission is shown in Table 1. The results from the MIP model for minimizing total cost are shown in column "Minimizing Total Cost", the total overall cost is equal to 240,434,539 Baht which is composed of transportation cost (95,836,968 Baht) and warehousing cost (144,597,571 Baht). The total gas emission is equal to 1,880,372,221 grams. The satisfaction level of total cost and gas emission is at 100% and 0% respectively, indicating that this model gives the best optimal result for the total cost while giving the worst result for the gas emission. The results from the MIP model for minimizing total gas emission are shown in column "Minimizing Total Gas Emission", the optimal overall cost is equal to 319,645,599 Baht which is composed of transportation cost (175,048,028 Baht) and warehousing cost (144,597,571 Baht). The total gas emission is equal to 1,867,521,491 grams. The satisfaction level of total cost and gas emission is at 0% and 100% respectively, indicating that this model gives the best optimal result for the total gas emission while giving the worst result for the total cost.

Table 2. Result of single and multiple objective optimizations

		Single Objective		**Multiple Objective**
		Minimizing Total Cost	Minimizing CO2	Minimizing Total Cost and CO2
Total Cost	Total Transportation Cost	95,836,968	175,048,028	118,757,452
	Total Warehousing Cost	144,597,571	144,597,571	144,597,571
	Total Overall Cost	240,434,539	319,645,599	263,355,023
	Satisfaction Level	100%	0%	71%
Total CO2 Emission	Total CO2	1,880,372,221	1,867,521,491	1,871,239,974
	Satisfaction Level	0%	100%	71%
Average Satisfaction Level		50%	50%	71%

For the compromise solutions, the results are shown in column "Minimizing Total Cost and Gas Emission", the total overall cost is equal to 263,355,023 Baht which is composed of transportation cost (118,757,452 Baht) and warehousing cost (144,597,571 Baht). The total gas emission is equal to 1,871,239,974 grams. The satisfaction level of total cost and gas emission is both at 71% respectively. For the average satisfaction level, the compromised solution provides the best average satisfaction level (71%).

4. Conclusion and Future work

In this paper, we formulate a MIP model for determining the product allocation and capacity in a multi-product, multi-period distribution network to minimize total cost and total gas emission. Later, we developed a model to find a compromised solution using a min-max approach to minimize the deviation from the ideal result. The limitation of this model is that it can only solve a small problem instance for now. The future work is to add more types of emission gas, develop a model that can solve a large-scale problem, and use the stochastic data type instead.

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References

M.-K. Lee and E. A. Elsayed (2005). Optimization of warehouse storage capacity under a dedicated storage policy. *International Journal of Production Research*, vol. 43, issue 9, pp. 1785-1805.

Zhu, S., Hu, X., Huang, K., and Yuan, Y. (2020). Optimization of product category allocation in multiple warehouses to minimize splitting of online supermarket customer orders. *European Journal of Operational Research*, vol. 290, issue 2, pp. 556-571.

Askin, R. G., Baffo, I., and Xia, M. (2013). Multi-commodity warehouse location and distribution planning with inventory consideration. *International Journal of Production Research*, vol. 52, issue 7, pp. 1897-1910.

You, M., Xiao, Y., Zhang, S., Yang, P., and Zhou, S. (2019). Simulated Annealing to Solve Single Stage Capacitated Warehouse Location Problem. *Industrial Engineering and Service Science*, vol. 4, pp. 62-70.

Buddhakulsomsiri, J., Jeenanunta, C., Dumrongsiri, A., and Chau, T. (2019). Optimization of warehouse storage capacity under a dedicated storage policy. *International Journal of Logistics Systems and Management*, Vol. 34, No. 4, pp. 510-532.

Jamshidi, R., Fatemi Ghomi, S. M. T., and Karimi, B. (2012). Multi-objective green supply chain optimization with a new hybrid memetic algorithm using the Taguchi method. *Scientia Iranica*, vol. 19, issue 6, pp. 84-96.

Paksoy, T., Ozceylan, E. and Weber, G.W. (2010). Optimization of warehouse storage capacity under a dedicated storage policy. *Global Journal of Technology and Optimization*, vol. 2, pp. 1785-1805.

Sadjady, H. and Davoudpour, H. (2012). Two-echelon, multi-commodity supply chain network design with mode selection, lead times, and inventory costs. *Computers & Operations Research*, vol. 39, pp. 1345-1354.

Shankar, B.L., Basavarajappa, S., Chen, J.C.H. and Kadadevaramath, R.S. (2013). Location and allocation decisions for multi-echelon supply chain network – a multi-objective evolutionary approach. *Expert Systems with Applications*, vol. 40, issue 2, pp. 551-562.

Wang, F., Lai, X. and Shi, N. (2011). A multi-objective optimization for green supply chain network design. *Decision Support Systems*, vol. 51, issue 2, pp. 262-269.

Olapiriyakul S., Pannakkong W., Kachapanya W., and Starita S. (2019). Multiobjective Optimization Model for Sustainable Waste Management Network Design. *Journal of Advanced Transportation*, vol. 2019.