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Coupled Stochastic Geomechanical Reservoir Black-Oil Flow Modeling

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Abstract

The problem of poroelasticity effect of oil reservoir has been investigated by many authors, having concrete results which often coincided with reality. However, in some cases, many such poroelasticity problems have not agreed with the practical results. Moreover, the reason has been precisely the difficulty of dealing with the problem theoretically and practically. However, with the rapid evolution of computer calculations, there has been a development in this direction in recent years. The stochastic nature, for the sake of truth, has remained a little bit untreated. Even in those few cases treated from a stochastic point of view, it is time-consuming on the computer. The Uncertainty Quantification method, based on these algorithms, is not yet a well-defined, consolidated method. We will address this problem in this paper in a stochastic manner, but without the Uncertainty Quantification technic. The discretization of the values resulting from continuous functions is that what we propose. The results are elaborated in software such as MRST-SINTEF and are calibrated with other commercial software, like Eclipse.

Keywords: Coupled modeling; Stochastic geomechanics, poroelasticity, continuous discretization functions, black-oil flow,

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Introduction

It is well known now that deformations of the matrix rock can be caused from fluid flow in porous media. The Biot's linear theory is the most vastly used to describe this phenomenon. According to this theory even smaller changes in fluid density and compression are associated with the small deformations of the porous rock. There are only a few works to deal with the problem of stochasticity in black-oil reservoir. Almost all the works are based to the approaches to the coupled fluid flow and geomechanical reservoir simulation based on the Biot theory [9]. These approaches that are based on the linear Biot theory, do not take into account the stochastic nature of the problem. One of the early works [12] treat the problem of poroelasticity in a similar manner of ours work and dealing with the stochasticity. In [12] the system of equation derived for solving the problem has a similar form with our work. This model, but even other models on literature relies on physical parameters which are not well known or inaccurate which allow a stochastic description of them.

Methodology

We begin by taking in consideration the work model from [1]. The standard time-dependent model in this work is derived by applying a discretization scheme over implicit time. In this formulation, given a volumetric source/sink term g and a body force f and, the goal is to find the associated fluid pressure p_F and the displacement u of the saturated poroelastic medium that satisfy the following system of equations (1) and (2)

$$-\nabla \cdot \sigma = f \quad \text{in } D, \quad (1)$$

$$-s_0 p_F - \alpha \nabla \cdot u + \tau \nabla \cdot (\tilde{k} \nabla p_F) = g \quad \text{in } D, \quad (2)$$

with homogeneous boundary conditions (for simplicity)

$$\sigma n = 0, \quad p_F = 0 \quad \text{on } \partial D_p \quad (3)$$

$$u = 0 \quad (\tilde{k} \nabla p_F) \cdot n = 0 \quad \text{on } \partial D_u. \quad (4)$$

Where, τ ($0 < \tau < 1$) is the time-step we have chosen for this case. The strain and stress quantities are given as:

$$\sigma := 2\mu \epsilon(u) + \lambda \nabla \cdot u \mathbf{I} - \alpha p_F \mathbf{I}, \quad \epsilon(u) := \frac{(\nabla u + (\nabla u)^T)}{2}, \quad (5)$$

It is evident in the boundary-value problem (1) - (4). the number of important physical parameters. The (spatially varying) permeability stands for the Biot-Willis coefficient $\alpha \in (0,1]$, $\tilde{k} > 0$. The

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coefficients $E > 0$ and $\nu \in (0, 0.5)$ are linked to the usual Lamé coefficients $\lambda > 0, \mu > 0$ as follows;

$$\mu = \frac{E}{2(1+\nu)}, \quad \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}. \quad (6)$$

The so-called parameter s_0 (that is the storage coefficient) for an incompressible fluid can be written as follows;

$$s_0 = \frac{\alpha - \phi}{2\mu d^{-1} + \lambda}, \quad (7)$$

Reader can refer to [3,4,5,6] for more detail on parameter s_0 . In [3], for the problem of linear poroelasticity with Biot random inputs, there has been done a vast, fully compressive and well posedness analyses

With the same technic as in [7] and [8], we obtain the 'overall pressure' $p_T := -\lambda \nabla \cdot u + \alpha p_F$ and the following system of equations is obtained:

$$-\nabla \cdot \sigma = f \quad (8)$$

$$-\nabla \cdot u - \lambda^{-1}(p_T - \alpha p_F) = 0 \quad (9)$$

$$\lambda^{-1}(\alpha p_T - \alpha^2 p_F) - s_0 p_F + \nabla \cdot (k \nabla p_F) = g \quad (10)$$

Here $\sigma := 2\mu \epsilon(u) - p_T I$.

In real-world cases, the values of E, k, ν, α and s_0 are not well known, the precise values are often *uncertain* and may have very different order of magnitude.

The New Model.

Biot's Consolidation model obtained from Stochastic Galerkin Mixed FEM

Here we introduce vectors of parameters $y = (y_1, \dots, y_{M_1})$ and $z = (z_1, \dots, z_{M_2})$, for defining the new model. We take in consideration that E and k are expressed as follow:

$$E(x, y) = e_0(x) + \sum_{k=1}^{M_1} e_k(x) y_k, \quad x \in D, y \in \Gamma_y := \Gamma_1 \times \dots \times \Gamma_{M_1}, \quad (11)$$

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$$(12) \quad k(x, z) = k_0(x) + \sum_{k=1}^{M_2} k_k(x) z_k, \quad x \in D, z \in \Gamma_z := \Gamma_1 \times \dots \times \Gamma_{M_2}.$$

In (11) and (12) can be noted that they have the same form as Karhunen-Loève expansion. The Lamé coefficients are functions of the parameter y . So we have :

$$\mu(x, y) = \frac{E(x, y)}{2(1 + \nu)}, \quad \lambda(x, y) = \frac{E(x, y)\nu}{(1 + \nu)(1 - 2\nu)}, \quad (13)$$

And lastly, we give the two other variables $p_1 := (p_T - \alpha p_F)/E$, $p_2 = p_F/E$ and the rescaled Lamé coefficients become

$$\tilde{\mu} := \frac{2\mu}{E} = \frac{1}{1 + \nu}, \quad \tilde{\lambda} := \frac{\lambda}{E} = \frac{\nu}{(1 + \nu)(1 - 2\nu)}, \quad \tilde{s}_0 := Es_0. \quad (14)$$

For further development of the algorithm the reader can refer to [1].

Here briefly we describe the procedure for finding the velocity u and the pressure p . The solution of (8), (9) and (10) after some transformation is given by:

$$\begin{pmatrix} \mathcal{A} & \mathcal{B}^T \\ \mathcal{B} & -\mathcal{C} \end{pmatrix} \begin{pmatrix} \mathbf{v} \\ \mathbf{p} \end{pmatrix} = \begin{pmatrix} \mathbf{b} \\ \mathbf{c} \end{pmatrix} \quad (15)$$

The block structure of the solution vector has the form;

$$\mathbf{v} = \begin{pmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} \mathbf{p}_F \\ \mathbf{p}_T \end{pmatrix} \quad (16)$$

Here $\mathbf{u}_{1,2} \in \mathbb{R}^{n_u \times n_y}$, $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_T \in \mathbb{R}^{n_p \times n_y}$ and $\mathbf{p}_F \in \mathbb{R}^{n_u \times n_y}$ and the other quantities of (15) are given as follow;

$$\mathbf{b} = \begin{pmatrix} \mathbf{g}_0 \otimes \mathbf{f}_1 \\ \mathbf{g}_0 \otimes \mathbf{f}_2 \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \in \mathbb{R}^{2(n_u + n_p)n_y}, \quad (17)$$

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$$\mathbf{c} = \begin{pmatrix} \mathbf{g}_0 \otimes \mathbf{g} \\ \mathbf{0} \end{pmatrix} \in \mathbb{R}^{(n_0+n_p)n_u} \quad (18)$$

Here $\mathbf{p}_F^\top = (\mathbf{p}_{F,1}^\top, \mathbf{p}_{F,2}^\top, \dots, \mathbf{p}_{F,n_y}^\top)$ is the vector associated with the fluid pressure and $\mathbf{p}_{F,j} \in \mathbb{R}^{n_0}$ for $j = 1, \dots, n_y$. The blocks of the coefficient matrix, assuming ordering of degrees of freedom in (15) are given by

$$\mathcal{A} := \begin{pmatrix} \tilde{\mu} \sum_{k=0}^{M_1} G_k \otimes A_{11}^k & \tilde{\mu} \sum_{k=0}^{M_1} G_k \otimes A_{21}^k & 0 & 0 \\ \tilde{\mu} \sum_{k=0}^{M_1} G_k \otimes A_{12}^k & \tilde{\mu} \sum_{k=0}^{M_1} G_k \otimes A_{22}^k & 0 & 0 \\ 0 & 0 & \tilde{\lambda}^{-1} \sum_{k=0}^{M_1} G_k \otimes \tilde{C}_k & 0 \\ 0 & 0 & 0 & \tilde{s}_0 \sum_{k=0}^{M_1} G_k \otimes \tilde{C}_k \end{pmatrix} \quad (19)$$

$$\mathcal{B} := \begin{pmatrix} 0 & 0 & \alpha \tilde{\lambda}^{-1} I \otimes C_b & \tilde{s}_0 I \otimes C_b \\ I \otimes B_1 & I \otimes B_1 & \tilde{\lambda}^{-1} I \otimes C & 0 \end{pmatrix}, \mathcal{C} := \begin{pmatrix} \sum_{k=0}^{M_2} \tilde{G}_k \otimes D_k & 0 \\ 0 & 0 \end{pmatrix} \quad (20)$$

The discrete system can be written in various ways. The linear system in so-called Kronecker form can be written in the following form:

$$\left(G_0 \otimes \mathcal{K}_0 + \sum_{k=1}^{M_1} G_k \otimes G_k + \sum_{k=1}^{M_2} \tilde{G}_k \otimes \tilde{\mathcal{K}}_k \right) \mathbf{x} = \mathbf{z} \quad (21)$$

This the final stochastic form of our solution of stochastic Biot linear problem.

In the above (21) system of equations we have

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$$\mathcal{K}_0 := \begin{pmatrix} \tilde{\mu}A_{11}^0 & \tilde{\mu}A_{21}^0 & 0 & 0 & 0 & B_1^\top \\ \tilde{\mu}A_{12}^0 & \tilde{\mu}A_{22}^0 & 0 & 0 & 0 & B_2^\top \\ 0 & 0 & \tilde{\lambda}^{-1}\tilde{C}_0 & 0 & \alpha\tilde{\lambda}^{-1}C_b^\top & -\tilde{\lambda}^{-1}C \\ 0 & 0 & 0 & \tilde{s}_0\tilde{C}_0 & \tilde{s}_0C_b^\top & 0 \\ 0 & 0 & \alpha\tilde{\lambda}^{-1}\tilde{C}_b & \tilde{s}_0C_b & D_0 & 0 \\ B_1 & B_2 & -\tilde{\lambda}^{-1}C & 0 & 0 & 0 \end{pmatrix}, z = g_0 \otimes \begin{pmatrix} f_1 \\ f_2 \\ 0 \\ 0 \\ g \\ 0 \end{pmatrix} \quad (22)$$

and for $k = 1, \dots, M_1$ and $l = 1, \dots, M_2$ we have

$$\mathcal{K}_k := \begin{pmatrix} \tilde{\mu}A_{11}^k & \tilde{\mu}A_{21}^k & 0 & 0 & 0 & 0 \\ \tilde{\mu}A_{12}^k & \tilde{\mu}A_{22}^k & 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{\lambda}^{-1}\tilde{C}_k & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{s}_0\tilde{C}_k & 0 & 0 \\ 0 & 0 & 0 & 0 & D_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \tilde{K}_l := \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & D_l & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (23)$$

Our solution methodology now is complete, so let's illustrate with a simple example the Biot linear problem with uncertain inputs and later implementing the same procedure for real application cases making use of MATLAB software MRST and modification to some of their code (MRST)

Example.

This example is similar with the implementation that we will follow ours work for the real cases. This example represents a general framework for the real cases that will see below. Next, we take in consideration that $D = (-1,1)^2$ with $\partial D_u = [-1,1] \times \{-1\} \cup \{-1\} \cup [-1,1]$ and $\partial D_p = (-1,1] \times \{1\} \cup \{1\} \cup (-1,1]$. We choose $f = (1,1)^\top$ and $g = 0$. Here the permeability k and the module of Young, E and are modeled as

$$E(x, y) = e_0 + \sigma_E \sum_{m=1}^{M_1} \sqrt{\lambda_m} \varphi_m(x) y_m, \quad k(x, y) = k_0 + \sigma_k \sum_{m=1}^{M_2} \sqrt{\lambda_m} \varphi_m(x) z_m, \quad (22)$$

where y_m, z_m are random variables of images of independent $U(-\sqrt{3}, \sqrt{3})$, $\{(\lambda_m, \varphi_m)\}$

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$$C(x, x') = \exp\left(-\frac{1}{2} \|x - x'\|_1\right), \quad x, x' \in D \quad (23)$$

Table 1

k_0	α	level	$\nu=.4$	$\nu=.499$	$\nu=.49999$	$\nu=.4$	$\nu=.499$	$\nu=.49999$
		$l = 5$ $l = 6$	79(4.23) 81(22.7)	98(5.33) 98(27.3)	96(5.31) 98(27.5)	81(16.0) 82(90.2)	99(18.9) 101(113.0)	99(18.5) 101(113.8)
$k_0 = 1$	$\alpha = 10^{-2}$	$l = 5$ $l = 6$	79(4.33) 79(23.1)	96(5.29) 98(27.4)	95(5.18) 97(27.1)	80(14.6) 80(86.9)	99(18.4) 99(110.6)	97(17.9) 99(107.5)
	$\alpha = 10^{-4}$	$l = 5$ $l = 6$	77(4.12) 78(22.1)	95(5.15) 97(27.1)	95(5.12) 96(27.0)	78(15.1) 79(87.3)	97(17.7) 99(108.3)	96(17.4) 98(106.7)
$k_0 = 10^{-5}$	$\alpha = 1$	$l = 5$ $l = 6$	85(4.88) 85(24.3)	99(5.42) 100(29.0)	98(5.35) 100(29.1)	86(15.7) 87(94.0)	100(18.3) 102(112.3)	100(18.3) 101(109.0)
	$\alpha = 10^{-2}$	$l = 5$ $l = 6$	80(4.38) 81(22.7)	98(5.28) 100(27.9)	96(5.37) 99(27.7)	81(14.9) 83(89.6)	100(18.8) 101(109.8)	99(18.8) 101(109.5)
$k_0 = 10^{-10}$	$\alpha = 10^{-4}$	$l = 5$ $l = 6$	79(4.34) 79(22.5)	96(5.4) 98(27.3)	95(5.11) 97(27.3)	80(15.2) 81(87.4)	99(18.1) 99(107.1)	97(18.0) 99(107.1)
	$\alpha = 1$	$l = 5$ $l = 6$	96(5.58) 97(28.7)	91(5.22) 96(27.3)	98(5.71) 100(29.3)	97(19.7) 98(114.4)	93(17.9) 97(113.8)	100(20.6) 102(119.3)
$k_0 = 10^{-10}$	$\alpha = 10^{-2}$	$l = 5$ $l = 6$	93(5.20) 94(26.9)	99(5.34) 100(28.0)	98(5.32) 100(28.2)	95(17.4) 95(103.4)	100(18.6) 102(111.5)	100(18.3) 102(111.6)
	$\alpha = 10^{-4}$	$l = 5$ $l = 6$	81(4.40) 81(22.8)	98(5.28) 100(28.1)	96(5.19) 99(27.7)	81(14.8) 83(90.3)	100(18.4) 101(109.6)	99(18.0) 101(109.6)

MINRES iteration counts and time in seconds (in parathesis) for varying $\nu, \alpha, k_0, \sigma_k = 0.1 \times k_0$

In the above data in table 1 our new method is facilitated forward from Uncertainty Quantification.

In the figures below, we will simulate a real case, that of the Norne oil field. Several simulations will be done with different geomechanical coefficients. This is done to show the influence of these parameters in situ stress and strain and even the influence when are taken into account the different geomechanical parameters; all nodes belonging to outer faces have displacement equal to zero for the cases with no displacement. The bottom nodes have zero displacements for the 'bottom fixed' option, while a given pressure is imposed on the external faces that are not bottom. The stochastic values for E Young's module, for Poisson's ratio ν and alpha's Biot's coefficient, are derived from a table similar to Table 1. In other words, we have taken the value of the module of Young uncertain from an interval: $\mathbf{E} = [0; 3]$ and found a representative value $\mathbf{E}=1$. The same discussion for the other two parameters where we have found $\nu=0.3$ and $\alpha=1$.

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Figure 2. Pressure distribution in the initial stage of simulation in the Reservoir of Norne - real case. Young module E , Poisson's ratio ν and Biot's coefficient α are all set to zero. Simulation of black-oil flow with coupled with poroelasticity of the Norne Field, with one injection (water) well in the left corner and one producer (black-oil) well in the right corner. The pressure is given in Pascal. Axes are given in meter. (D.Zeqiraj & MRST, 2021)

Figure 3. Pressure distribution after 101 day of water injection for the simulation of the Norne oil Reservoir - real case.. Simulation of black-oil flow with coupled with poroelasticity of the Norne Field, with one injection (water) well in the left corner and one producer (black-oil) well in the right corner. The pressure is given in Pascal. Axes are given in meter. (D.Zeqiraj & MRST, 2021)

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Figure 4. Pressure distribution in the initial stage of simulation of the Norne oil field with. In this case-figure the pressure distribution is of 1 order of magnitude smaller as we see when comparing figure (2) and (3), (see the column bar in the left). This is because of the pore pressure (D.Zeqiraj & MRST, 2021)

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Figure 5. Pressure distribution after 101 days of injection of water of in the Norne oil field with Young module $E = 1$ giga Pascal, $\nu = 0.3$ and $\alpha = 1$. Comparing with figure (4) in this case-figure the pressure is approximately 2 times bigger at nearly every point of the reservoir. This is because of water injected with a pressure of 270 bar (D.Zeqiraj & MRST, 2021)

Figure 6. Vertical displacement in meter after 101 days of injection of water of in the Norne oil field with Young module $E = 1$ giga, $\nu = 0.3$, $\alpha = 1$. The displacement takes values from negative to positive (see the color bar to the left) and this dependent from the structure of reservoir and from reference depth of injector well (D.Zeqiraj & MRST, 2021)

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Figure 7. Total displacement ($u = u_1 + u_2 + u_3$) in meter after 101 days of injection of water of in the Norne oil field with Young module $E = 1$ giga Pascal, $\nu = 0.3$, $\alpha = 1$. We see that the displacement takes values from negative to positive and this due to the structure of reservoir and from reference depth of injector well (D.Zeqiraj & MRST, 2021)

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Figure 8. The rate of oil at producer for the case $E = 1$ giga Pascal, $\nu = 0.3$, $a = 1$ (D.Zeqiraj & MRST, 2021)

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Figure 9. The rate of oil at producer for the case $E = 0.000001$ giga Pascal, $\nu = 0.000003$, $\alpha = 0.0000001$ (D.Zeqiraj & MRST, 2021)

Conclusions.

Many reservoir simulation problems do not consider the geomechanical effects, poroelasticity, and especially when it comes to their stochastic nature. In this work we have shown that geomechanical aspects are important in terms of stress and strain of the porous structure with strict stochastic approximations; in many real case problems, the values of geomechanical parameters are not known precisely but are given as components of a specific interval where they are part. We have chosen a robust methodology for our problem with uncertain data; the problem we are discussing is brought in a deterministic approach form. This allows calculation of other quantities, like the amount of oil that we can obtain with the effect of poroelasticity without it or changing the values of a different order of magnitude. This conclusion is demonstrated with figure (8) and (9)

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