Lexicographic Multi-Objective Optimization Approach for Welding Flux System Design

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Abstract

Multiple response optimization of welding flux performance has been found to be cost effective and useful for the achievement of the best balance among conflicting welding flux quality attributes. Many multi-criteria optimization methods (MCOM) have been applied in flux formulation situations where flux quality attributes are of comparable importance. However, information on applications of MCOM to flux design situations where quality attributes are in hierarchical order of importance is scarce in the open literature. In this study, a Lexicographic Multi-objective Optimization (LMO) model was proposed for handling flux design situations in which the attributes are in hierarchical order of importance. The model was applied using data from literature. Two priority levels were used: acicular ferrite (AF) maximization was assigned first priority while the maximization of polygonal ferrite (PF) content and weld-metal impact toughness (WIT) were assigned second priority subject to oxygen content constraint of 250 – 350ppm. The respective solutions for AF, PF, WIT and oxygen content were 51.19%, 21.80%, 23.70J at -20^{o}C and 315ppm. The corresponding flux formulation was CaO (25.90), MgO (15.00), $\mathrm{CaF_2}$ (31.10) and $\mathrm{Al_2O_3}(8.00\%)$. Various priority structures were used to explore trade-off options and to generate three more pareto efficient solutions from which the flux formulator can select the most preferred one. The proposed model has filled the existing gap in the literature being a pioneering work in the application of lexicographic multi-objective optimization method in welding flux design.

Keywords: hierarchical order of importance; multiple response optimisation; pareto efficient solution; priority levels; welding flux ingredients

1. Introduction

Welding flux design involves the consideration of many quality issues and characteristics that cover the entire lifecycle of the flux. Adeyeye & Oyawale, (2010 A) presented quality issues that affect each lifecycle stage of welding flux. For instance, matters that affect manufacturability of the flux such as extrudability and bonding are of concern to the manufacturer. To the welding and fabricating firm, storage requirements such as durability of flux coating and minimum moisture pick-up are important. Also, of importance are the operational requirements when the flux is put to use. Operational requirements such as arc stability, penetration control, spatter and slag detachability are among what determine the productivity of welding process. The health of welders and other environmental concerns require that the welding flux generates minimum fumes, toxic materials and noxious odors during welding. It is also required that the flux should be able to produce a weld deposit that possesses the required chemical, mechanical and metallurgical features for optimal performance of the welded structure when put in service. All quality requirements at each lifecycle stage should be taken into consideration during welding flux design.

The selection of the appropriate welding flux ingredients in their right proportions presents a big challenge in welding flux design because of the numerous quality specifications that often conflict with each other. There could also be conflict among stakeholders such as the manufacturer, welder/welding firm, user of welded structures and regulatory agencies. Due to conflict among the quality attributes and stakeholders, it is not possible to get a flux that will achieve the optimal values of all quality specifications simultaneously (Adeyeye et al., 2020). The target of the flux design process is to get a welding flux that achieves the best balance/compromise among the numerous quality attributes. In the last one decade, many multi-criteria optimization methods (MCOM) have been proposed for the design of flux to achieve the best compromise among the quality specifications (Adeyeye & Oyawale, 2009, 2010 B, Kumar, 2019, Singh et al., 2015, Jindal et al., 2013, Jindal et al, 2014 A, 2014 B and Singh & Singh, 2016). Most of the MCOM used so far in welding flux design are based on the assumption that all the flux design objectives/quality attributes are of comparable importance (Sui et al., 2006, Adeyeye & Oyawale, 2010 A, Bhandari et al., 2016, Sharma & Chhibber, 2019 A, 2019 B and Kumar, 2019). In the real-world engineering design environments, situations arise where some design objectives are of overriding importance when compared to the rest objectives. In such situations, the objectives are arranged and treated in hierarchical order of importance and Lexicographic Multi-objective Optimization (LMO) methods are used to determine the best compromise solution (Adeyeye & Oyawale, 2010 A). Although, there is a plethora of applications of LMO methods in engineering design, the applications of such methods are scanty in the welding flux design literature. The focus of this study, is to develop LMO model and solve the model to prescribe welding flux ingredient proportions that give the best balance among quality attributes that are in hierarchical order of importance. Literature review is presented in section two followed by a brief description of the pre-emptive optimization approach. Next, a numerical example is presented followed by the discussion of results and finally, the conclusion.

2. Literature

Interest of the welding flux research community has been on the increase since the early 1900s when research papers on welding flux design started appearing in the open literature (Ogden, 1924, Spraragen, 1924, Dallam et al., 1985, Datta & Parekh, 1989). The approach of designing new welding flux was based on lengthy trial-and-test experiments involving many iterations (Adeyeye & Oyawale, 2008, Fleming et al., 1996, De Rissone et al., 2001. de Rissone et al., 2002, Farias et al., 2004, Du Plessis et al., 2006, 2007 and Du Plessis & Du Toit, 2007). For instance, Fleming et al, (1996) developed welding flux for SMAW of HSLA-100 grade steel that would exhibit the excellent welding behavior found typically in a rutile electrode and balanced with the superior weld-metal properties deposited by a basic electrode using a sequential flux formulation methodology. Drawing upon the principles of physics, chemistry and metallurgy tempered with accumulated experience, the flux compositions were systematically varied starting with an initial flux that can be classified as rutile-based and ending up with a more basic flux. The flux development process involved nine iterations each with one substitution for a specific ingredient in the flux. The try-and-test approach is technically and economically inefficient, because of long lead-time and consumption of considerable amount of resources during lengthy experimental flux formulation and weld production (Adeyeye & Oyawale, 2008). The drawbacks were due to the paucity of prediction and optimization tools.

Kanjilal et al., (2004, 2005, 2006, 2007 A & 2007 B) used mixture experiments, a type of statistical design of experiment (DoE) to design experiments and the experimental data were used to develop regression equations for quality attributes such as weld-metal chemical composition, mechanical properties, microstructural features and process parameters. Apart from the usefulness of the regression equations for the prediction of quality attributes as a function of flux ingredients, their approach reduced the number of experiments and also provided insight into the direct and interaction effects of flux ingredients. The success of the application of DoE spurred other researchers to use DoE. Ren et al, (2006) used Uniform Design, Achebo and Ibhadode, (2008) used Hadamard Multivariate Design, Kumar, (2019) used Taguchi method while Singh et al., (2015) and Somal et al., (2015) used the Response surface method.

Adeyeye and Oyawale (2009) extended the work of Kanjilal et al., (2004, 2005, 2006, 2007 A, & 2007 B) by coupling it with mathematical programming method for the determination of optimal flux formulation for a single quality attribute case. This approach was novel but its usefulness was limited because the multiple attributes flux design cases are more common compared to the single attribute cases. Later, weighted-sum scalarization, non-pre-emptive GP, compromise programming and desirability function were suggested for multiple attributes flux design situations where the attributes are of comparable importance (Adeyeye & Oyawale, 2010 A). Pre-emptive GP was suggested for cases where some of the quality characteristics are of overriding importance when compared to the remaining quality characteristics (Adeyeye & Oyawale, 2010 A). In such instances, the quality characteristics are arranged in hierarchical or lexicographic order and solved sequentially. The optimization methods suggested for the cases where quality attributes are of comparable importance have been implemented and popularized by other researchers, while the cases where attributes are in lexicographic order have not received much attention (Somal et al., 2015, Rehal, 2015, Adeyeye & Allu, 2017, Mahajan & Chhibber, 2020, Mahajan et al., 2020 and Khan et al., 2020).

Flux situations where one or more attributes are of overwhelming importance when compared to the remaining attributes is yet to receive attention probably because the existence of such situation is yet to be identified. However, a careful look at the work of Kanjilal et al., (2007 A) showed that situations where LMO method should be used exist in welding flux design. Kanjilal et al., (2007 A) observed that samples with higher volume fractions of acicular ferrite (AF) and lower grain boundary ferrite (GBF), side plate ferrite (SPF) and ferrite with aligned second phase (FAS) possessed higher strength and higher weld-metal impact toughness (WIT). This was in agreement with previous studies that optimum strength and low temperature WIT is achieved in HSLA weld-metals containing high volume fraction of AF in the columnar region (Farrar, 1987, Mcgrath et al., 1988, Jung-Soo et al., 2001, Rishi et al., 2016, Mainak et al., 2018 and Beidokhti & Pouriamanesh, 2015). It was also observed that a sample with higher PF but identical AF, GBF, SPF and FAS volume fractions exhibited a higher WIT values than samples with lower PF volume fraction content in the microstructure. Generally, welding flux and welding process parameters are selected to obtain high proportion of AF in the weld-metal (Farrar, 1987, Mcgrath et al., 1988, Jung-Soo et al., 2001, Rishi et al., 2016, Mainak et al., 2018 and Beidokhti & Pouriamanesh, 2015). While AF is the primary contributor to improved mechanical properties, PF makes a secondary contribution as could be deduced from Kanjilal *et al.*, (2007 A). The implication of this is that welding flux designer should give first priority attention to AF while PF receives second priority attention and LMO approaches are the most appropriate in such welding flux design circumstances.

3. Method

3.1 Pre-emptive optimisation

Multi-criteria optimization methods have been used in many areas of engineering and management decisions such as design, production, construction, operations and project managements among others (Niyazi et al, 2014, Biswas, et al., 2018, Movafaghpour, 2019, Mirzaei, 2019 and Olabanji, 2020). The LMO method is among the MCOM. It is suitable for decision situations in which a priori articulation of stakeholder(s) preferences is possible and objectives are in hierarchy of importance or priority levels. The first priority objective(s) is/are of overwhelming importance or significance when compared to the second priority objective(s) and the second priority objective(s) is/are of overwhelming importance compared to third priority objective(s) and so forth. The first priority objective(s) receives first priority attention while second priority objectives are given second priority attention and so forth. Objectives that are of comparable importance may be at the same priority level but weights are assigned to them to reflect their relative importance within the level (Adeyeye & Oyawale, 2010 A, Ojha and Biswal, 2009 and Arora, 2017). The procedure suggested below may be useful for the application of lexicographic/pre-emptive optimization method to welding flux design.

Step 1: Identify the welding flux design criteria. Usually, the criteria are the quality attributes/specifications. Also, identify the nature of each objective if desirable or not. A desirable objective is a beneficial welding flux attribute for which higher values imply better performance. For instance, the ability of the flux to deposit weld-metal with higher volume fraction of AF is desirable. The AF content in weld-metal microstructure is beneficial and it should be maximized (Farrar, 1987, Mcgrath et al., 1988, Jung-Soo et al., 2001, Beidokhti & Pouriamanesh, 2015, Rishi et al., 2016 and Mainak et al., 2018). Non-desirable attributes are non-beneficial ones and common examples include diffusible hydrogen content, GBF, SPF and FAS where higher percentages or volume fractions in the weld-metal indicate poor performance and should therefore be minimized (Adeyeye et al., 2015).

Step 2: Arrange the quality attributes in hierarchical order of importance based on the technology of the flux design problem. First priority attribute(s) p_1 >>> second priority attribute(s), p_2 >>> third priority attribute(s) p_3>>> …. >>> p_I, the last priority level. Use DoE method to design experiment, conduct the experiments and use the experimental data to develop regression models for each of the attributes. The details of the DoE methods applicable to welding flux design have been described in the literature (Adeyeye & Oyawale, 2008, Kanjilal et al., 2004, 2005, 2006, 2007 A, 2007 B, Achebo & Ibhadode, 2008, Kumar, 2019, Singh et al., 2015 and Somal et al., 2015).

Step 3: Use the regression equations (i.e., the response functions) of the quality attributes to build the LMO model by arranging the objective functions in hierarchical order of priority levels. The problem may be expressed as; find the values of $x_1, x_2, \ldots x_n$ that maximize $f_{i,j}(x)$ in lexicographic order subject to the constraints of the problem. Note that x_n is the value/proportion of the n^{th} welding flux ingredient while $f_{i,j}(x)$ represents the response equation of j^{th} quality attribute with i^{th} priority level. For $j > 1$, we have 2 or more attributes at the same priority level. Such attributes are of comparable importance and are combined into single objective using standard weighting and normalization methods (Adeyeye & Oyawale 2010 A, 2010 B, Adeyeye & Allu 2017 and Movafaghpour, 2019). Mathematically, the LMO model is defined as;

$$\left.\begin{array}{ll} Maximise: & f_{i,j}(x) \\ Subject\ to; & \\ & x \in S \\ & f_{i-1,j}(x) = f^*_{i-1,j} \end{array}\right\} \quad (1)$$

The first constraint ($x \in S$) is the original technological/structural constraint while the second constraint represents the maximum value(s) of the immediate higher priority level.

Step 4: Solve the lexicographic optimization model as follows

(i): First priority level objective(s) is/are solved first as a single-objective problem subject to all the original constraints. Let the maximum value of first priority attribute be $f^*_{1,j}$. Then we move to (ii).

(ii) Second priority objective(s) is/are then solved again as a single-objective problem with an added constraint(s). The constraints are generally defined as $f_{1,j}(x) = f_{1,j}^{*}$. Let the maximum value of the second priority attribute be $f_{2,j}^{*}$. Then we move to step (iii).

(iii) Third priority attribute(s) are solved with 2 additional constraints added to the original constraints, $f_{1,j}(x) = f_{1,j}^{*}$ *and* $f_{2,j}(x) = f_{2,j}^{*}$. Generally, $f_{i-1,j}(x) = f_{i-1,j}^{*}$ is added for each $j \in J_{i-1}$ and $i \in I$. Observe that index i, represent priority level while $f_{i-1,j}^{*}$ is the maximum value of the j^{th} objective function of the immediate higher-level priority and J_i is the total number of objectives at priority level i. Note that $f_{i-1,j}^{*}$ is not necessarily the same as the independent optimum/maximum of $f_{i-1,j}(x)$. Their addition guarantee that improving the value(s) of lower quality attribute(s) does not diminish the performance/value(s) of the higher priority level attribute(s). The process is repeated, in which optimal solution obtained in the previous step is added as a new constraint, and the sequence of single-objective optimisation is solved, one problem at a time. The sequential solution process terminates once a unique optimum is determined. The unique optimum is identified when two consecutive optimization problems yield the same solution.

4. Illustrative Example

4.1 *Development of the LMO Model*

The proposed LMO approach for prescribing flux ingredients proportions that give the best balance among various flux quality characteristics is demonstrated by integrating it with Kanjilal et al, (2004, 2005, 2006, 2007 A and 2007 B) approach. Some of the confirmed response equations developed by Kanjilal et al, (2004, 2005, 2006, 2007 A and 2007 B) were selected for illustrative purposes. The composition of the welding wire, base metal, welding parameters and the various flux compositions as per the statistical design of mixture experiment with their corresponding response values are presented in Tab. 1 and 2, respectively. Bead-on-plate weld deposits were made at constant voltage (26 V), current (400 amp), speed (4.65 mm/sec) and electrode extension (25mm) on 100mm × 250mm × 18mm low-carbon steel plate with each of the fluxes using 3.15 mm-diameter low-carbon steel filler wire in the SAW process. The chemical, mechanical and metallurgical features of weld-metal are determined by many factors and complex interactions between welding wire, base metal, welding thermal cycles, cooling rate, flux composition and process parameters. Kanjilal et al, (2004, 2005, 2006, 2007 A and 2007 B) used the same welding wire, base metal, thermal cycle, cooling rate and welding process parameters. Welding flux composition was the only thing that vary from experiment to experiment (see Tab. 2). Therefore, the observed variations in the values of flux quality attributes were determined by the flux composition. The proportions of flux ingredients ($CaO, MgO, CaF_2,$ *and* Al_2O_3) were varied from experiment to experiment. The problem is how to find the proportion of the various flux ingredients that will give the best balance or compromise among the various quality characteristics while taking the hierarchical order of importance or priority levels of the attributes into account.

Table 1: Base metal and filler wire composition

Element	Carbon (wt.%)	Manganese (wt.%)	Silicon (wt.%)	Sulphur (wt.%)	Phosphorus (wt.%)	Nickel (wt.%)	Oxygen (ppm)	Nitrogen (ppm)
Base metal	0.22	0.77	0.25	0.03	0.02	-	350	50
Filler wire	0.10	0.56	0.05	0.02	0.01	-	380	60

Source: Kanjilal et al., (2004, 2005, 2006, 2007 A, and 2007 B)

Table 2: Experimental runs determined by mixture design and results

Experiment No	Flux composition (in wt. % of ingredient)				Fixed proportion flux ingredients (wt. %)					Flux quality attributes/response values			
	CaO	MgO	CaF_2	Al_2O_3	SiO_2	Fe-Mn	Fe-Si	Ni	Bentonite	AF (wt.%)	PF (wt.%)	CIT at-200C (J)	O_2 (ppm)
1	15.0	15.00	10.00	40.00	10.0	4.0	3.0	1.0	2.0	13	27	8.8	560
2	15.0	15.00	40.00	10.00	10.0	4.0	3.0	1.0	2.0	12	27	9.8	570
3	15.0	32.40	10.00	22.60	10.0	4.0	3.0	1.0	2.0	15	30	10.5	520
4	15.0	17.00	40.00	8.00	10.0	4.0	3.0	1.0	2.0	14	30	9.8	500
5	15.0	32.40	24.60	8.00	10.0	4.0	3.0	1.0	2.0	13	27	7.8	530
6	35.0	15.00	10.00	20.00	10.0	4.0	3.0	1.0	2.0	24	24	22.2	580
7	17.00	15.00	40.00	8.00	10.0	4.0	3.0	1.0	2.0	16	25	13.7	490
8	35.00	15.00	22.00	8.00	10.0	4.0	3.0	1.0	2.0	19	29	14.4	480
9	29.60	32.40	10.00	8.00	10.0	4.0	3.0	1.0	2.0	28	20	16.7	330
10	35.00	27.00	10.00	8.00	10.0	4.0	3.0	1.0	2.0	16	29	14.7	480
11	24.43	23.14	24.43	8.00	10.0	4.0	3.0	1.0	2.0	35	20	26.0	300
12	15.67	15.67	40.00	8.66	10.0	4.0	3.0	1.0	2.0	26	24	15.8	350
13	25.92	24.36	10.00	19.72	10.0	4.0	3.0	1.0	2.0	28	27	23.5	320
14	23.40	15.00	24.40	17.20	10.0	4.0	3.0	1.0	2.0	36	25	25.5	300
15	19.87	32.40	14.86	12.87	10.0	4.0	3.0	1.0	2.0	35	18	24.1	320
16	15.00	22.36	24.92	17.72	10.0	4.0	3.0	1.0	2.0	10	31	9.1	600
17	35.00	19.00	14.00	12.00	10.0	4.0	3.0	1.0	2.0	20	28	14.2	470
18	22.67	21.63	21.63	14.07	10.0	4.0	3.0	1.0	2.0	16	28	11.6	540

Source: Kanjilal et al., (2004, 2005, 2006, 2007 A, and 2007 B)

Now consider a situation where the quality attributes of interest to the flux designer are AF, WIT, PF and Oxygen content of weld-metal. His/her preferences are determined a priori as follows: Acicular ferrite is at the first priority level while WIT and PF are at the second priority level. It is also required that the oxygen content in the weld-metal be in the range 250-350ppm. The PF and WIT are presumed to be of equal importance, hence the weight assigned are w_{PF} and w_{WIT} respectively and $w_{WIT} = w_{PF} = 0.5$. The problem is to find the proportions of $CaO, MgO, CaF_2,\ and\ Al_2O_3$ to use in the flux formulation such that AF is maximized at first priority level while WIT and PF receive second priority attention subject to O_2 content constraint and the other technological constraints of the problem. The priority structure, response equations from Kanjilal et al, (2004, 2005, 2006, 2007 A and 2007 B) and direction of optimization are presented in Tab. 3. The lexicographic multi-objective optimization model is presented in Eq. 2. Note that the lower and upper limits of the flux ingredients, $CaO, MgO, CaF_2,$ and Al_2O_3 as per the technology of the flux formulation problem are the constraints of the model. Experiments were performed using different priority structures (i.e., cases 2, 3 and 4). The priority structure for cases 2, 3, and 4 respectively are presented in Tab. 4. In all cases, attributes that are at the same priority level are presumed to be of equal importance. In real world flux formulation, quality attributes at the same priority level may have different weights depending on the situation.

Although, this is an illustrative example to demonstrate the feasibility of the application of LMO to welding flux design, we have tried to be realistic in setting the priority structure. For instance, as discussed in Section 2 and in agreement with previous studies, higher AF volume fraction in weld-metal microstructure improves weld-metal mechanical properties (Farrar, 1987, Mcgrath et al., 1988, Jung-Soo et al., 2001, Beidokhti & Pouriamanesh, 2015, Rishi et al., 2016 and Mainak et al., 2018). The contribution of PF to the improvement of mechanical properties is marginal and far below that of AF (Kanjilal et al., 2007 A). Hence, AF was assigned first priority while PF receive second priority attention. The WIT is an important weld-metal property and may be given first or second priority attention depending on the goal of optimization and the technology of the flux design problem. Oxygen content requirement was also set realistically between 250 and 350ppm because previous studies have proposed that the optimum oxygen content in high strength low alloy steel is in the range 200-350ppm (Potapov, 1993 and Seo, 2013). The oxygen content constraint was put at 250-350ppm which is within the optimum range specified in the literature.

The LMO model is given by;

$$\left.\begin{array}{ll}
Maximise, f_{AF} & (\,first\ priority) \\
Maximise, f_{PF} & (\,second\ priority) \\
Maximise, f_{WIT} & (\,second\ priority) \\
Subject\ to; & \\
\quad x_{CaO} + x_{MgO} + x_{CaF_2} + x_{Al_2O_3} = 80 & \text{(sum of the proportions} \\
 & \text{of } CaO, MgO, CaF_2, and\ Al_2O_3) \\
\quad 15 \le x_{CaO} \le 35 & (lower\ and\ upper\ bound\ of\ CaO\ in\ flux) \\
\quad 15 \le x_{MgO} \le 32.40 & (lower\ and\ upper\ bound\ of\ MgO\ in\ flux) \\
\quad 10 \le x_{CaF_2} \le 40 & (lower\ and\ upper\ bound\ of\ CaF_2\ in\ flux) \\
\quad 8 \le x_{Al_2O_3} \le 40 & (lower\ and\ upper\ bound\ of\ Al_2O_3\ in\ flux) \\
\quad 250 \le f_{O_2} \le 350 & \text{(Oxygen content in weld} - \text{metal constraint)}
\end{array}\right\} \quad (2)$$

where $x_{CaO}, x_{MgO}, x_{CaF_2}$ and $x_{Al_2O_3}$ are the respective proportions of $CaO, MgO, CaF_2,$ and Al_2O_3 while f_{AF}, f_{PF}, f_{WIT} and f_{O_2} are as defined in Tab. 3.

Table 3: Response equations with priority structure of quality attributes

S/N	Quality Attribute	Response equation Kanjilal, et al, (2004, 2005, 2006, 2007 A and 2007 B)	Flux designer's desires
1	Acicular ferrite	$f_{AF} = -4.8335x_{CaO} + 2.0808x_{MgO} - 0.3680x_{CaF_2} - 0.6867x_{Al_2O_3} + 0.0756x_{CaO}x_{MgO} + 0.1551x_{CaO}x_{CaF_2} + 0.1701x_{CaO}x_{Al_2O_3} - 0.0731x_{MgO}x_{CaF_2} - 0.0721x_{MgO}x_{Al_2O_3} - 0.0068x_{CaO}x_{Al_2O_3}$	Maximise at first priority level
2	Polygonal ferrite	$f_{PF} = 2.2848x_{CaO} - 1.2764x_{MgO} + 0.3102x_{CaF_2} + 0.1682x_{Al_2O_3} - 0.0135x_{CaO}x_{MgO} - 0.0540x_{CaO}x_{CaF_2} - 0.0646x_{CaO}x_{Al_2O_3} + 0.0461x_{MgO}x_{CaF_2} + 0.0656x_{MgO}x_{Al_2O_3} + 0.0145x_{CaO}x_{Al_2O_3}$	Maximise at second priority level
3	Weld-metal impact toughness	$f_{WIT} = -3.31038x_{CaO} + 0.62389x_{MgO} - 0.26209x_{CaF_2} - 0.84441x_{Al_2O_3} + 0.06680x_{CaO}x_{MgO} + 0.10098x_{CaO}x_{CaF_2} + 0.12913x_{CaO}x_{Al_2O_3} - 0.03063x_{MgO}x_{CaF_2} - 0.02394x_{MgO}x_{Al_2O_3} - 0.00737x_{CaO}x_{Al_2O_3}$	Maximise at second priority level
4	Oxygen	$f_{O_2} = 5186x_{CaO} - 1064x_{MgO} + 533x_{CaF_2} + 1359x_{Al_2O_3} - 6171x_{CaO}x_{MgO} - 10314x_{CaO}x_{CaF_2} - 13419x_{CaO}x_{Al_2O_3} + 5602x_{MgO}x_{CaF_2} + 5055x_{MgO}x_{Al_2O_3} + 2397x_{CaO}x_{Al_2O_3}$	Range 250-350ppm

Table 4: Priority structure for the various cases

Cases	First Priority	Second Priority
1	AF	WIT, PF
2	PF	AF, WIT
3	WIT	AF, PF
4	AF, PF	WIT

4.2 *Solution and Experiment*

The lexicographic multi-objective model was solved using the procedure described in step 4 of section 3.

(i) The first priority objective (maximize f_{AF}) was solved subject to the original constraints of the problem to obtain the maximum or the ideal volume fraction of acicular ferrite (f_{AF}^{*}) in the weld-metal under the existing conditions. With all the lower priority objectives removed, the model is expressed as follows:

$$\left.\begin{array}{ll} Maximise, f_{AF} & (first\ priority) \\ Subject\ to; & \\ & x_{CaO} + x_{MgO} + x_{CaF_2} + x_{Al_2O_3} = 80 \\ & 15 \leq x_{CaO} \leq 35 \\ & 15 \leq x_{MgO} \leq 32.40 \\ & 10 \leq x_{CaF_2} \leq 40 \\ & 8 \leq x_{Al_2O_3} \leq 40 \\ & 250 \leq f_{O_2} \leq 350 \end{array}\right\} \quad (3)$$

All the models were solved using Lingo 18 software. The maximum/ideal value of the volume fraction of acicular ferrite in the weld-metal microstructure is f_{AF}^{*} =51.19% under the existing conditions. Observe that the value(s) attribute(s) obtained from the solution of a higher priority level becomes a constraint in the lower priority level. Hence, f_{AF}^{*} =51.19% becomes a constraint in the next step.

(ii) Next, the second priority objectives (f_{PF} *and* f_{WIT}) were normalised and then combined into one function. The combine objective was then solved as a single-objective problem with all the constraint in Eq. 3 and additional constraint (i.e., $f_{AF} = f_{AF}^{*} = 51.19$) from the solution of Eq. 3. The model to be solved is given by Eq. 4:

$$\left.\begin{array}{l} Maximise,\ F = f_{PF}^{N} + f_{WIT}^{N} \qquad\qquad\qquad\qquad (second\ priority) \\ \quad Subject\ to; \\ \qquad\qquad x_{CaO} + x_{MgO} + x_{CaF_2} + x_{Al_2O_3} = 80 \\ \qquad\qquad 15 \le x_{CaO} \le 35 \\ \qquad\qquad 15 \le x_{MgO} \le 32.40 \\ \qquad\qquad 10 \le x_{CaF_2} \le 40 \\ \qquad\qquad 8 \le x_{Al_2O_3} \le 40 \\ \qquad\qquad 250 \le f_{O_2} \le 350 \\ \qquad\qquad f_{AF} = 51.19 \end{array}\right\} \quad (4)$$

Observe that f_{PF}^{N} and f_{WIT}^{N} are the normal forms of $f_{PF}\ and\ f_{WIT}$ respectively and F is the combined function. Normalization was done using the weights ($w_{WIT} = w_{PF} = 0.5$) and following the approach described in (Adeyeye & Oyawale, 2010, A and Adeyeye, et al., 2020). The final solution of the LMO gives $x_{CaO}^{*},\ x_{MgO}^{*},\ x_{CaF_2}^{*} and\ x_{Al_2O_3}^{*}$ which are the proportions of flux ingredients that give the best balance among the corresponding values of flux quality attributes (f_{AF}^{*}, f_{PF}^{*} and f_{WIT}^{*}).

4.3 Results and Discussion

The results of the LMO for the flux design problems are presented in Tab. 5. The proportions of the flux ingredients, oxygen content in the weld-metal and the values of acicular ferrite, impact toughness and polygonal ferrite were obtained for each case. In all the cases, the desire of the flux formulator to achieve oxygen content within the range 250 and 350ppm was realized. For all the cases, the LMO was able to achieve the best balance among the attributes without deteriorating the value(s) obtained for higher level priority. For instance, in case 1, the second priority attributes (polygonal ferrite and weld-metal impact toughness) were maximized (see Eq. 4) without deteriorating the ideal value (51.19%) obtained for AF by solving Eq. 3. Although the 51.19% for acicular ferrite is the ideal/optimum value, it does not imply that the LMO algorithm and the single objective model will give the same result. The LMO algorithm searches among the alternative solutions to Eq. 3 to identify the one that maximizes the lower priority level attributes (i.e., PF and WIT) without compromising the value obtained for AF in case 1 (see Tab. 5.). The attribute value (f_{AF}^{*}) obtain for AF may not be the same as the individual optimum of AF. For example, in case 4 where AF and PF were assigned first priority, the value obtained for f_{AF}^{*}, was 50.80% as against 51.19% when AF was the sole attribute in priority 1. This is so because the solution is a compromise between the volume fraction of AF and PF.

Table 5: Results of the Lexicographic Multi-objective Optimization Problem

Case	Flux Quality Attributes				Flux composition (in wt. % of ingredient)			
	Acicular ferrite, f_{AF}^{*} (%)	Weld-metal impact toughness, f_{WIT}^{*} [at -20⁰C (J)]	Polygonal ferrite, f_{PF}^{*} (%)	Oxygen, f_{O_2}(ppm)	CaO (x_{CaO}^{*})	MgO (x_{MgO}^{*})	CaF_2 ($x_{CaF_2}^{*}$)	Al_2O_3 ($x_{Al_2O_3}^{*}$)
1	P_1 (51.19)	P_2 (23.70)	P_2 (21.80)	315	25.90	15.00	31.10	8.00
2	P_2 (35.05)	P_2 (22.92)	P_1 (25.50)	350	30.77	21.06	10.00	18.17
3	P_2 (42.40)	P_1 (28.90)	P_2 (20.20)	250	27.20	15.00	31.10	27.80
4	P_1 (50.80)	P_2 (23.70)	P_1 (22.20)	318	26.55	15.00	30.45	8.00

The LMO approach provides some flexibilities to the welding flux formulator. For instance, the flux formulator may want to explore trade-off options available to him or to generate the pareto efficient solutions for evaluation so that he could be properly guided in selecting the most preferred solution. The trade-off options enable the flux formulator to know how much he/she needs to give up in one or more attributes in order to gain improvement in one or more of the other attributes. For example, the value of AF content dropped from 51.19 to 50.80% while the value of PF increased from 21.80 to 22.20% for cases 1 and 4 respectively. Different weight structures may also be used either alone or in combination with different priority structures for trade-off exploration. As a result of conflict among the attributes, it is impossible to find a point within the attribute space at which all the attributes would assume their optimum/ideal values simultaneously. The LMO approach always give nondominated solutions as indicated in Tab. 5. The flux formulator need not use trial-and-error again when handling attributes that are in hierarchy of importance.

5. Conclusion

A multi-criteria optimization method known as lexicographic multi-criteria optimization model was proposed for the achievement of best balance among conflicting flux quality attributes in flux design situations where attributes are in various priority levels. The major conclusions are as follows:

(i) Situations where welding flux quality attributes are in hierarchical order of importance exist in welding flux formulation as illustrated by the contribution of acicular ferrite and polygonal ferrite to weld-metal mechanical properties.

(ii) Some quality attributes may be introduced into the optimization model as constraints as the case of oxygen content in the weld-metal instead of limiting the constraints to the lower and upper bounds of flux ingredients.

(iii) The lexicographic multi-criteria model was able to prescribe the proportion of various flux ingredients that give the best realization of the objectives of the flux formulator.

(iv) The model provides increased flexibility to the flux formulator to use different priority structures and/or weights to explore trade-off options and to also generate more pareto efficient solutions for him/her to choose the most preferred welding flux formulation.

(v) The proposed model has filled the existing gap in the literature being a pioneering work in the application of lexicographic multi-objective optimization method in welding flux design.

Acknowledgement

The first author appreciates the financial assistance received from the Department of Industrial and Production Engineering, University of Ibadan and the Innovators of Tomorrow (IOT) grant under the Federal Government of Nigeria and World Bank STEP-B programmed. He is also grateful to Professors Alexandre Queiroz Bracarense of Universidade Federal de Minas Gerais, Pampulha, Belo Horizonte, Brazil; Tapan K. Pal of Welding Technology Centre (WTC) Jadavpur University India and Gregory F. Piepel of Statistical Sciences of Pacific Northwest Laboratory, Richland USA who made their research works available and the useful communications in the course of this work. The authors are indebted to Lindo Systems Inc. Chicago, USA for the free research license granted for the use of the extended version of Lingo software.

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