Developing students' creative skills during the process of teaching geometry

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Abstract

This article discusses possible approaches for developing the creative abilities of secondary school students in the process of teaching geometry. The main disadvantages that hinder creative development discovered based on a comparative analysis of geometry textbooks by Russian scientists. These disadvantages are axiomatic methods, excessive theorization, non-systematic set of tasks, complex definitions etc. The author proposes the study of planimetry using the properties of simple geometric shapes without invoking axioms. As a concrete example of that, proof of triangle equality that is based on the property of "corresponding chords of angles" is demonstrated. Furthermore, the equality outcome is used to study the properties and features of other figures. Formulation of parallel postulates is simplified using formulas for the areas of a rectangle and a circle, and the circumference of a circle as the starting axioms. The author suggests a three-level system of tasks for each topic. The first level is designed to test the comprehensibility of the theoretical material. The second level applies the acquired knowledge in practice. The third level of tasks is intended to develop an independent study and critical thinking skills of a student. Additionally, at the end of each chapter, a wide range of extra tasks is given. Preparation for Science Olympiads and science fairs should be systematic. We succeeded at formulating general provisions for the management of scientific projects of schoolchildren in mathematics. The article provides an example of such implementation related to learning a special property of trapezoids.

Keywords: axiom, a chord of an angle, levelled tasks, auxiliary figures.

1. Introduction

Geometry and arithmetic have historically progressed to the scientific level of course construction. Geometry has evolved as an adjunct to philosophy. Precision in reasoning and the use of correct inferences were required in philosophical science. This has been the foundation for the scientific construction of geometry. Euclid is associated with geometry's rigorous construction. In his "Elements," Euclid formulated the axiomatic construction of geometry. However, Euclid's axioms were incomplete. The study of geometry's axiomatic construction has been going on for centuries, but it wasn't until the end of the nineteenth century that David Hilbert built a complete axiomatic course in geometry.

The school geometry course was designed using A. P. Kiselev's textbook "Elementary Geometry." A. N. Kolmogorov, A. V. Pogorelov, L. S. Atanasyan, A. D. Aleksandrov, and other prominent geometric scientists contributed significantly to the axiomatic construction of school geometry textbooks. Kazakh scientists are currently presenting a geometry course in the spirit of Russian authors that has been translated into Kazakh. In recent years, a new geometry textbook (by V. A. Smirnov and I. M. Smirnov) has been developed in Russia that is best suited to modern educational requirements.

Unfortunately, some flaws have been discovered in the use of an axiomatic approach in school geometry. The rigor of the presentation has hampered the creativity of the students. The axioms were memorized, but there was no practical advancement in their application. This has resulted in the formalization of students' knowledge. Furthermore, the large number of theoretical bases (theorems and definitions) in geometry in a comprehensive school does not provide adequate opportunities to solve a wide variety of specific geometric problems. Some textbooks contain definitions that are difficult for students to grasp. For example, A.N. Kolmogorov's geometry is scientifically written at a high level, but practice has shown that students and teachers are unprepared for the course's perception in the broad sense. The axioms in A.V. Pogorelov's geometry are closer to schoolchildren's perception, and the material problem is not well thought out in methodological terms. As one progresses through the relevant subjects, L. S. Atanasyan's geometry and axiomatics are gradually introduced. The imposition of axioms, on the other hand, is accepted in two ways: physically and mathematically. The range of tasks available is extensive. A.D. Aleksandrov's geometry is designed for classes that require a thorough understanding of mathematics. The main methodological ideas contained in this material, however, correspond to the development of students' creative abilities (practical problems, problems with construction, problems with research, problems with theorems and problems with the identification of new property figures and others). Today's pressing issue is the development of a geometry textbook for a comprehensive school that can also be used successfully for specialized classes (Dybyspayev, 2009).

Discussion

1.1 A new perspective on the construction of a school planimetry course.

It is essential to start a geometry course from not axioms, but from consideration of geometric facts that that are closer to schoolchildren in psychological terms and to teachers in terms of methodology. The concepts of line, ray, segment, and angle are well known to elementary school students. These concepts can be taken as primary and with the help of them try to build a systematic course in geometry. It is known that a segment is characterized by its length and the angle by a degree measure. Therefore, at first, it is useful to solve action problems with segments and angles both in arithmetic and geometric terms.

The segment with ends on the sides of the angel is called a **chord.** For two angles, we introduce the concept of corresponding chords. If for the angles O and O_1,

$\mathrm{OA} = \mathrm{O}_1 A_1$, $\mathrm{OB} = \mathrm{O}_1\mathrm{B}_1$ then the chords AB and $\mathrm{A}_1 B_1$ are called respectively. Equal corresponding chords contract equal angles. If $AB = \mathrm{A}_1 B_1$, then $\angle O = \angle O_1$.

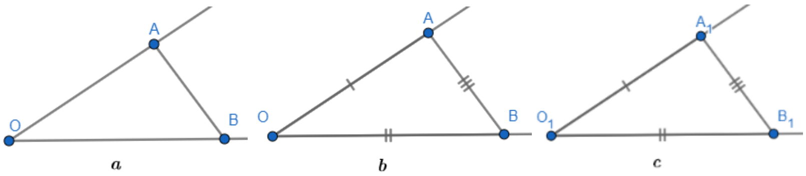


Figure 1.

Using the properties of the corresponding chords, it is to prove triangles equality criteria. First, we prove the criterion for the equality of triangles on three sides.

If triangles ABC and $A_1B_1C_1$ has the following properties: $AB = \mathrm{A}_1 B_1$, $AC = \mathrm{A}_1 C_1, \mathrm{BC} = \mathrm{B}_1 C_1$, then $\triangle ABC = \triangle A_1B_1C_1$.

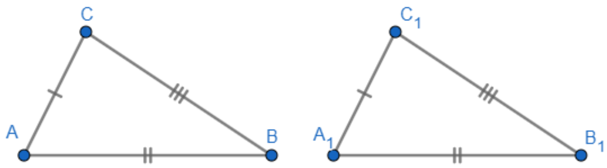


Figure 2.

Prove. Let us consider angels A и A_1 with $\mathrm{BC} = \mathrm{B}_1 C_1$, that are corresponding chords. So $\angle A = \angle A_1$. Likewise, for angels B and B_1(C and C_1), $\angle B = \angle B_1$ and $\angle C = \angle C_1$. So, $\triangle ABC = \triangle A_1B_1C_1$.

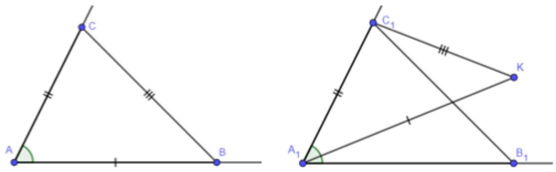


Figure 3.

If for triangles ABC and $A_1B_1C_1$ $\angle A = \angle A_1$, $AB = \mathrm{A}_1 B_1$, $AC = \mathrm{A}_1 C_1$, so $\triangle ABC = \triangle A_1B_1C_1$.

Prove. Let us consider, BC and $\mathrm{B}_1 C_1$ are not equal. For example, $\mathrm{BC} > \mathrm{B}_1 C_1$, then we can draw a new triangle A_1C_1K ($\mathrm{A}_1\mathrm{K} = \mathrm{AB}, \mathrm{C}_1 K = \mathrm{CB}$) that is equal to triangle ABC (by three sides). As $\angle A = \angle A_1 = \angle CA_1K$, $\mathrm{K} \in \mathrm{A}_1 B_1$. And if $AB = \mathrm{A}_1 K = \mathrm{A}_1 B_1$, so $K \equiv B_1$. That is $\mathrm{BC} = \mathrm{B}_1 C_1$. This contradiction proves that $\mathrm{BC} = \mathrm{B}_1 C_1$ (case when $\mathrm{BC} < \mathrm{B}_1 C_1$ has the same prove). In the result, $\triangle ABC = \triangle A_1B_1C_1$.

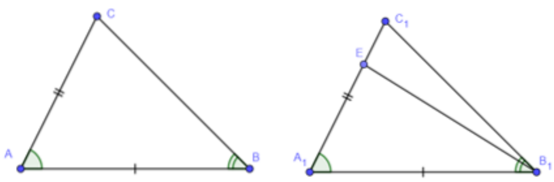


Figure 4.

If the triangles ABC and $A_1B_1C_1$ have $\angle A = \angle A_1, \angle B = \angle B_1$, $AB = \mathrm{A}_1 B_1$, so $\triangle ABC = \triangle A_1B_1C_1$.

Prove. It is enough to prove that $AC = \mathrm{A}_1 C_1$. Let's consider that $AC \neq \mathrm{A}_1 C_1$ ($AC > \mathrm{A}_1 C_1$ or $AC < \mathrm{A}_1 C_1$). We draw a new triangle A_1C_1E ($\mathrm{A}_1\mathrm{E} = \mathrm{AC}, \mathrm{B}_1 E = \mathrm{BC}, \mathrm{E} \in \mathrm{A}_1 C_1$) equal to triangle ABC. $E \equiv C_1$, since $\angle A_1B_1E = \angle ABC = \angle A_1B_1C_1$. So $AC = \mathrm{A}_1 C_1$, this contradiction means $\triangle ABC = \triangle A_1B_1C_1$(by two sides and angel) (Dybyspayev, 2001).

A.V. Pogorelov, with the help of axiom, proves the equality properties of triangles. In the future, the axioms will fade into the background, and the signs of triangle equality will serve as the basis for evidence. Our approach enables the continuation of a geometry study based on the criteria for the equality of triangles without the introduction of axiom systems.

Historically, Euclid's fifth postulate, the axiom of parallel lines, plays a crucial role in the construction of geometry. It is impossible to circumvent this axiom in a school geometry course. On the other hand, the strict formulation of the parallel axiom is difficult for schoolchildren. The way out is simple. For example, the axiom of parallelism of straight lines can be formulated as follows: "Through a point outside a straight line, you can draw only one straight line parallel to the given one." Along with this axiom, there are more subtle questions of mathematics related to the study of the area of figures and the length of a circle. Even in elementary school, students get acquainted with the area of a rectangle $S = a * b$, where a и b are natural numbers. Further in grades 5-6, this formula remains, but the numerical values can be both rational and irrational. Therefore, it is appropriate to consider the area of the rectangle, S = a * b, instead of the axiom. Similarly, the formulas for the circumference $\mathrm{C} = 2\pi r$ and the area $S = \pi r^2$ are known to schoolchildren in grades 5-6. In a systematic geometry course, these concepts are rigorously presented using the theory of limits, and this complicates the understanding of geometry by schoolchildren. It seems that these formulas should be accepted as a natural fact as a substitute for axioms.

It is ineligible to provide proof and consequences of theorems. If necessary similar proves uncovered, and some theorems may be combined into one. For example, signs of equal triangles, signs of parallel lines, signs of similarity of triangles and others. It is advisable to single out theorems according to their purpose, as property or feature, or by special names instead of numbering them. The above facts make it possible to free up time that can be used to solve higher quality problems aimed to develop creative abilities.

This paper suggests a three-level system of tasks for each topic. The first level is designed to test the comprehensibility of the theoretical material. The second level applies the acquired knowledge in practice. The third level of tasks is intended to develop an independent study and critical thinking skills of a student. For instance, **first-level problems** for the topic "simple geometric figures":

1. Draw a line a. Choose points A and B on the line a and point C and D out of the line a. Use symbols to write math notations.
2. Construct three-point A, B and C that are not collinear. Then draw lines AB, BC and AC.
3. Draw rays h and k with a common starting point O. Write down the resulting angles.

Second level problems:

1. Given four points. No three of them lie on one straight line. How many lines can you draw through these points?
2. How many undeveloped corners are there in three straight lines passing through one point?
3. Line a has three points A, B and C. Determine all line segments and all semi-lines.

Third level problems:

1. You are given four points A, B, C and D. Points $A, B,$ and C belong to the line a and points B, C and D belong to line m. Prove that the given four points belong to the same straight line.

2. You are given four points A, B, C and D that do not lie on one straight line. Line AB intersects CD, and line CD intersects AB. Prove that the segments AB and CD intersect.

3. Each two of the given four lines intersect. How many intersections points these lines define?

Additionally, at the end of each chapter, a wide range of extra tasks is given. The purpose of these tasks differs from tasks after each topic. While the tasks by topic are intended mainly to test knowledge related to a specific topic, the tasks for repetition involve the combination of knowledge on the covered topics. Therefore, with the help of these tasks, you can test the creative potential of the student.

There are many additional sources with high-quality problems. A creatively working teacher can choose a system of additional tasks that will be offered individually to students, taking into account their level of knowledge. The selection of tasks must meet the following requirements:

1. The wording of the objectives should be concise and understandable.
2. The problem should have a certain flavour that increases the student's interest.
3. It is advisable to select tasks that are close to practice.
4. Research tasks.
5. Tasks to determine new properties of figures.

Let us look at some examples of such tasks:

1. Construct an angle of 1 degree using the specified angle of 19 degrees.
2. Use a double-sided ruler to divide the line segment in half.
3. Investigate the conditions for the location of the centre of the circle described around the trapezoid.
4. Figure 5 shows a part of a broken disk. Rebuild the disk.
5. Establish a formula expressing the length of the bisector of a triangle in terms of the lengths of the sides and the lengths of the line segments on which the base of the bisector divides the third side.

Figure 5.

In the process of solving such problems, we offer students the initiative. They propose their ideas and refine the necessary theorems and formulas for the implementation of their theories. Often, various approaches to solving this problem can be proposed. If students cannot find the key to solving the problem, then it is advisable to offer them an easier problem of the proposed type or reformulate it. In climactic situations, the teacher, with the help of suitable landmarks, should direct the students' thought in the right direction. For example, consider Problem Five.

Of course, students propose to use bisector property, but in this proportion bisector itself is not included. The bisector AD is a side of triangles ABD and ACD. By applying the cosine theorem to one of the indicated triangles and expressing BD and CD through triangle sides, we can obtain a required formula. In this case, there are certain challenges, so the teacher suggests finding the easier way.

Participants may also propose the approach of the area, $S_{ABC} = S_{ABD} + S_{ACD}$. If $\angle CAD = \alpha, AD(AB + AC) = 2AB * AC * \cos\alpha$. To express the required formula, it is necessary to make proper calculations and transformations. This method is not easy as well. The teacher suggests using auxiliary figures. Searching for an auxiliary figure does not give immediate results. In such cases, the teacher must indicate the direction of the search. That is, this figure must be connected by ABC and the bisector or its extension.

Such a figure can be a circle circumscribed about ΔABC. AD extension intersects a circle at point E. So $\Delta ABD \sim \Delta AEC$ (by two angles). $\frac{AB}{AE} = \frac{AD}{AC}$. $AB * AC = AD * AE = AD^2 + AD * DE$. But $AD * AE = BD * CD$ (by the property of intersecting circle chords). Finally, $AD^2 = AB * AC - BD * CD$. As you can see using an auxiliary circle simplify algebraic calculations. This approach is relatively simple compared to those previously presented.

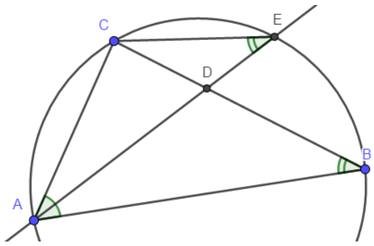


Figure 6.

1.2 Preparing students for geometry Olympiads

At present, the mathematics Olympiad among schoolchildren is widespread throughout the world. There are various types of math Olympiads, from intraschool to international. Systems for selecting schoolchildren for mathematics Olympiads have been established. A natural question arises. How to bring up an Olympiad in mathematics among children from secondary schools? First, it is necessary to instil in the student a love for the subject. Here the role of the teacher is essential. The teacher needs to start with the simplest problems that increase the student's interest in mathematics. Then gradually, increasing the degree of complexity, teach the student to independently find various ways of solving. As an example, the teacher should show various methods of solution using examples of several problems. Having previously familiarized schoolchildren with existing methods, both general and specific. Preparing students for the Olympiads involves a systematic approach.

In grades 5-6, geometric material is used as an application to arithmetic. By broadening the horizons of students in the study of the properties of geometric shapes, you can propose creative research problems. We offer the following directions for research activities to solve problems of an Olympiad nature (Dybyspayev, 2015):

1. Problems on the checkered plane.
2. Expansion of properties of some figures.
3. Tasks for the formation of spatial thinking.
4. Establishing patterns as general formulas.

When preparing students in grades 7-9 for mathematical Olympiads, we practice the use of the following methodological techniques (Dybyspayev 2009):

1. Working with students on problem acceptance.
2. Creation of reference notes to help find solutions to problems.
3. The use of guideline rules to find the main key for solving the problem.
4. Using problematic questions to help students find solutions to problems on their own.

5. Using elements of advanced learning.

When solving Olympiad geometric problems, students' attention should be drowned to the search for various ways to solve the same problem to identify the simplest way. This approach disciplines the direction of students' mental activity when solving non-standard geometric problems. In the process of searching for various ways of solution to one problem, an important role is assigned to the use of auxiliary figures. Using construction shapes reduces algebraic transformations.

For example, we will give the solution to one problem proposed at the regional Olympiad in the city of Nur-Sultan.

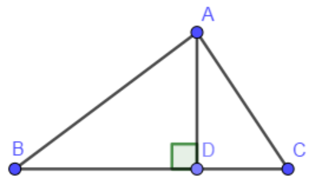


Figure 7.

Problem. In the triangle ABC, height AD is half of the side BC. Can the BAC angle be obtuse?

Solution:

Method 1. Suggest students use the cosine theorem.

Let us consider the case when the base of the height – point D – belongs to the side BC. If $BC = 2AD = 2h$, $BD = x$, $CD = 2h - x$. Applying twice the Pythagorean theorem, by the cosine theorem $\cos A = \frac{AB^2+AC^2-BC^2}{2AB*AC} = \frac{(h-x)^2}{AB*AC} \geq 0$. In this case $0° < \angle BAC \leq 90°$.

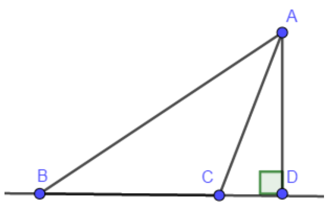


Figure 8.

In the case when point D does not belong to the BC. It is easy to find out that $\angle BAC > 90°$, consequently $\angle BAC$ can only be sharp and cannot be obtuse.

We separately note the case when $\cos A = 0$, that is $\angle BAC -$ a straight line. This is possible if and only if $x = h$, so triangle ABC is right-angled and isosceles.

Method 2. Let triangle ABC be rectangular and isosceles with height $AD = h$ dropped on the hypotenuse $BC = 2h$ (Figure 9).

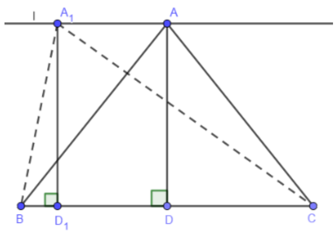


Figure 9.

Let us carry out the following additional constructions: through point A, construct a line l parallel to the base BC, choose an arbitrary A_1 on the line l and connect it with points B and C. Then, the set of triangles A_1BC contains all possible triangles with base BC and height equal to AD.

From the point A_1 draw the perpendicular to the line BC that intersects at the point D_1. Let $DD_1 = \varepsilon$, when $0 < \varepsilon < h$, so $BD_1 = h - \varepsilon$, $CD_1 = h + \varepsilon$.

$$\cos \angle BA_1C = \frac{\varepsilon^2}{A_1B*A_1C} > 0.$$

Consequently, in this case, $\angle BA_1C$ is acute. If point D_1 coincides with point B or C, respectively, the angle BA_1C - acute. Finally, if the point D_1 lies on the line BC outside the segment BC, for example, behind the point B, then the triangleBA_1C is obtuse, with an obtuse angle at the vertex B, which means that the angle BA_1C is acute.

When $\varepsilon = 0$, point A_1 coincides with point A, $\angle BA_1C = 90$ So, in any location of a point A_1 on the line l works the inequality $0° < \angle BA_1C \leq 90°$.

Method 3. Let us consider vectors $\overline{AB}$ and $\overline{AC}$. Angel BAC is an angle between vectors $\overline{AB}$ and $\overline{AC}$. We have $\overline{AC}-\overline{AB}=\overline{BC}$, so $(\overline{AC}-\overline{AB})^2=\overline{BC}^2$, it means $\overline{AC}^2-2\,\overline{AC}*\overline{AB}+\overline{AB}^2=\overline{BC}^2$. $\cos A=\frac{AB^2+AC^2-BC^2}{2AB*AC}$ since $\overline{AC}-\overline{AB}=\overline{AC}*\overline{AB}*\cos\angle ABC$. Further reasoning repeats the first method.

Method 4. Additional constructions are required. Construct a circle with diameter $BC=2h$ and draw a tangent line l parallel to BC (figure 10).

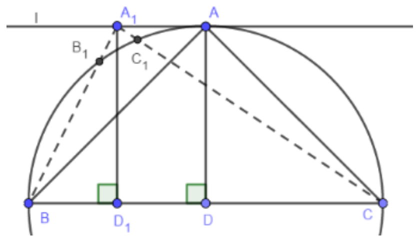


Figure 10.

Point A is the tangency point of line l with the circle, $AD\ =\ h$ is the radius of the circle and the height of the right-angled triangle ABC. Just as in the second method, we select an arbitrary point A on the line l and connect it with points B and C the intersection points of the segments BA_1 and CA_1, with the upper semicircle, we denote by B_1 and C_1, respectively. Up to similarity (defined by a specific value of h), the set of triangles A_1BC contains all possible triangles that satisfy the condition of the problem.

Note that if $A_1\neq A$, so we can apply the theorem on the angle between the secants, according to which the angle between two secants drawn from one point is equal to the half-difference of the degree measures larger and smaller than the arcs cut on the circle (sometimes it is called the ex-angle lemma). The large arc $\widetilde{BC}$ is a semicircle and has a degree measure of 180 °. So $\angle BA_1C=\frac{180°-\widetilde{BC}}{2}<90°$.

Method 5. The teacher suggests using trigonometric transformations.

Consider the case in figure 7. Let $\angle BAD=\alpha,\angle CAD=\beta$. So $\angle BAC=\alpha+\beta$. Consequently $\tan\alpha=\frac{x}{h},\tan\beta=\frac{2h-x}{h}$, $\tan\alpha+\tan\beta=2$.

$$\tan(\alpha+\ \beta)\ =\frac{2}{1-\tan\alpha\tan\beta}.$$

$\tan\alpha>0,\ \tan\beta>0$ since angles α and β are acute. According to the Cauchy inequality, $2=\tan\alpha+\tan\beta\geq2\sqrt{\tan\alpha\tan\beta}$, so $\tan\alpha\tan\beta\leq1$. $0°<\alpha+\beta<90°$ since $\tan(\alpha+\beta)>0$.

In the case when $\tan\alpha\tan\beta=1$, or when fraction on the expression $\tan(\alpha+\beta)$ does not make sense. In the Cauchy inequality, the sign of equality is possible only when $\tan\alpha=\tan\beta$, or $\alpha=\beta=45°,\alpha+\beta=90°$.

Of the five proposed methods for solving this problem, according to our students, the most original was the method for constructing an auxiliary circle. Indeed, the use of an auxiliary figure simplified algebraic transformations.

2. Notes from the experience of leading the scientific projects of schoolchildren in geometry.

One of the important government programs aims is to foster critical thinking, creativity, communication, and teamwork among students. Writing scientific projects in schools is a vivid example of how to properly educate students in the ability to work independently and develop these skills in themselves. This is one of the forms of development of creative abilities necessary for solving vital issues in the future.

Our many years of experience in leadership of scientific projects of schoolchildren allows us to formulate the most general provisions of the process of joint activities of a leader and a researcher (Dybyspayev 2019).

1. Selection of students
2. Preparing students to choose a research topic.
3. Work on the selected research topic.
4. Study of additional literature related to the topic under study.
5. Statement of the research problem.
6. Clarification of the research task.
7. Determination of research methods.
8. Design of the study.
9. Checking the quality of the study design by the head.
10. Defense of the project by the researcher in front of the head.
11. Clarification of questions submitted for defence.
12. Preparation of material for the presentation of the research.
13. Pre-presentation of the project for the school staff and reviewer.
14. Basic project presentation (City (Regional), Republican and International).

As an example, consider a scientific project on geometry related to the study of one remarkable property of the trapezoid (Dybyspayev, 2013; Dybyspayev & Dybyspayeva, 2014). In the beginning, we invite the student to complete a 5 - 10-minute message related to the interesting properties of the trapezoid. Usually, the student is limited to the materials of the school geometry textbook. We suggest using the Internet or additional literature to supplement your horizons with new facts related to the trapezoid. For example, segments of a trapezoid of length are expressed by the harmonic mean, geometric, arithmetic, quadratic values. The leader pays special attention to the root-mean-square segment. This segment divides the trapezoid into two equal trapezoids. Is there another straight line dividing the trapezoid into equal parts? It turns out that this is a straight line passing through four points: the midpoints of the bases, the point of intersection of the diagonals, and the point of intersection of the extension of the sides. Is there a point of a trapezoid through which infinitely many straight lines pass, dividing the trapezoid into equal parts? Oddly enough, there is such a point, and this is the middle of the midline of the trapezoid.

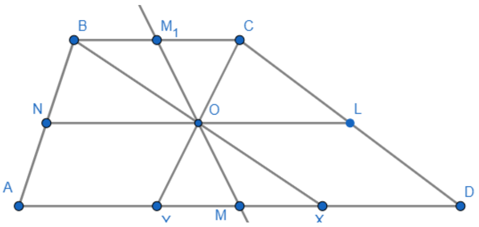


Figure 11.

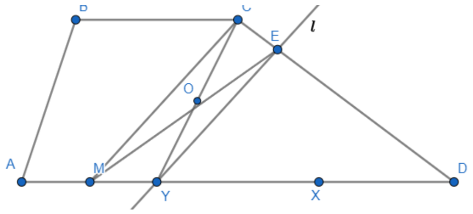


Figure 12.

$ABCD$ ($AD \parallel BC$) is trapezoid and point O is the middle of its midline NL, where $N \in AB, L \in CD$. Extension of BO and CO intersect base AD at the points X and Y respectively. If the point M belong to the segment XY or coincides with point X or Y, so the line MO splits the trapezoid into equal parts.

If point M belongs to the segments AY or DX, then how to construct a straight line dividing the trapezoid into equal parts? The answer is related to the remarkable property of the trapezoid (the diagonals of the trapezoid divide it into four triangles, two of which are with vertices at the ends of the lateral sides and the point of intersection of the diagonals of the same size). We obtain south line ME by drawing the YE ($E \in CD$) parallel to CM.

How to find a solution to the problem if the given point belongs to one of the lateral sides of the trapezoid? For points, S and T belong to AB and CD respectively, where ST is the mean square segment, satisfies the solution of the problem. Lines AP and DQ, where $Q \in AB$, $P \in CD$ is also a solution to this problem. The construction of these straight lines is based on the use of the above trapezoid property.

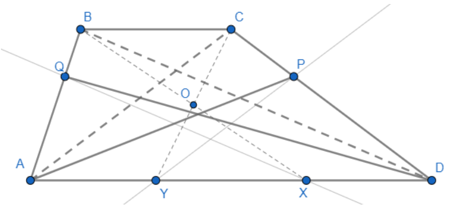


Figure 13.

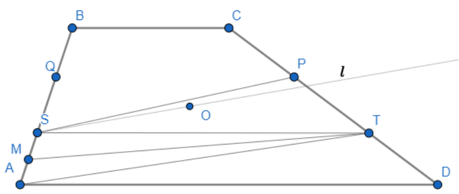


Figure 14.

Points S and Q (T and P) divide the sides of the trapezoid into three segments. How to solve the main problem if the given point belongs to one of the specified segments? The solution to this question is also related to the above trapezoid property. If point M belongs to AS, then the segment MM_1, where M_1belongs to TP, the trapezoidal diagonal MSM_1T serves as a solution to the problem. If M belongs to the segment SQ, the solution is similar. In the case when M belongs to the segment BQ, then the solution agrees with the previously considered position of the point M on the segments AY and DX.

The problem posed is solved for all points M running along the perimeter of the trapezoid. What if point M is located outside the perimeter of the trapezoid? Intuitively, the idea of using an auxiliary trapezoid arises. The sides of the auxiliary trapezoid must correspondingly

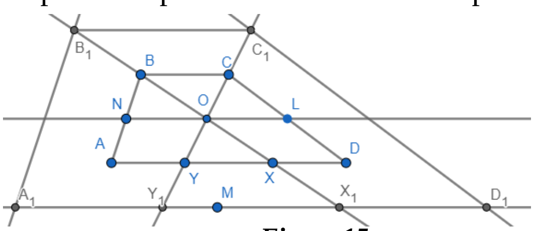
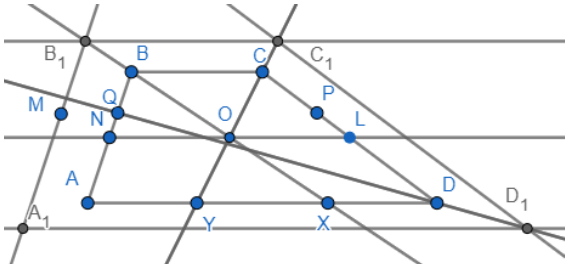


Figure 15.

parallel to the sides of this trapezoid, and point O must serve as the common midpoint of the midlines of the two trapezoids. Then the solution of the problem is reduced to the cases considered earlier, where the selected point will run along the perimeter of the auxiliary trapezoid.

Figure 16.



Let the point M_1 lie outside the trapezoid. Draw the line m $(m \parallel AD)$ through M_1. extensions BO and CO intersect line m at points X_1 and Y_1. Set aside points C_1 and B_1 for the continuation of OC and OB $(OC_1 = OY_1 = OB_1 = OX_1)$. Draw lines parallel to sides AB and CD through points B_1 and C_1, respectively. Their intersection with the line m is denoted by A_1 and D_1. As a result, we obtain the required auxiliary trapezoid $A_1B_1C_1D_1$, for which the given point M_1 belongs to the base A_1D_1. The problem has been reduced to the case considered earlier. The case when through a given point outside the trapezoid a straight line is drawn parallel to the lateral side the solution is similar (figure 16)

3. Conclusion:

A textbook intended for students plays an important role in promoting the creative abilities of schoolchildren in geometry. The textbook should serve as a reference book for the student, where the minimum necessary information, both theoretical and practical, is concentrated. The thinkers can use a variety of aids, taken from different sources, to nurture the broad interest of children in geometry. The article shows that this provision has been implemented. School children are prepared for the Mathematics Olympiads through extracurricular activities. To date, the content of the Olympiad problems has been quite extensive. Skilful selection of tasks for the preparation of the Olympics is an important challenge in the formation of the student's independent application of the knowledge gained in practice. In the article, we present one of the possible methodological approaches to the solution of this important problem.

The management of schoolchildren's science projects in mathematics is purely individual. The joint work between the head and the performer is a long one (sometimes 2-3 years). The leader's task is to develop the research skills needed to solve a particular problem. The performer, in turn, must offer independent solutions, which are incorrect, and the leader directs the researcher along the right path. In the end, this constant dialogue should lead to a victorious outcome. With this kind of work, the researcher will feel, in the smallest detail, the independent project he has completed. Such a project should receive an appropriate assessment during the defence.

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