Mathematical technologies for modeling of cardiological data: Heart Rate Variability

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Abstract.

Cardiovascular diseases are widespread worldwide, causing a decline in the quality of life of the population and many deaths each year. For these reasons, the study and modeling of cardiac data is a challenge for the research community. Heart rate variability derived from electrocardiographic and Holter recordings is a time series formed by the intervals between successive heartbeats. This paper presents an overview of mathematical technologies for heart rate variability modeling. The most effective and the most used mathematical models in the scientific literature are presented of heart rate variability: the Zeeman model, Gaussian functions model, mathematical model based on the Integral Pulse Frequency Modulation and others. A mathematical algorithm using two Gaussian functions was analyzed. The use of Wavelet analysis technology in the processing of cardiac data and their mathematical modeling is presented. Various wavelet bases used in heart rate variability modeling have been investigated. The results presented show the impact of these bases on the proposed mathematical model. The effect of the size of the created time series on the program execution time of the algorithm was also investigated. The mathematical model presented can be used to model realistic cardiac intervals and be used in training future physicians, as well as to research new algorithms for cardiac data processing and analysis.

Keywords: Cardiological Data, Heart Rate Variability, Modeling, Wavelet Analysis, Gaussian Functions

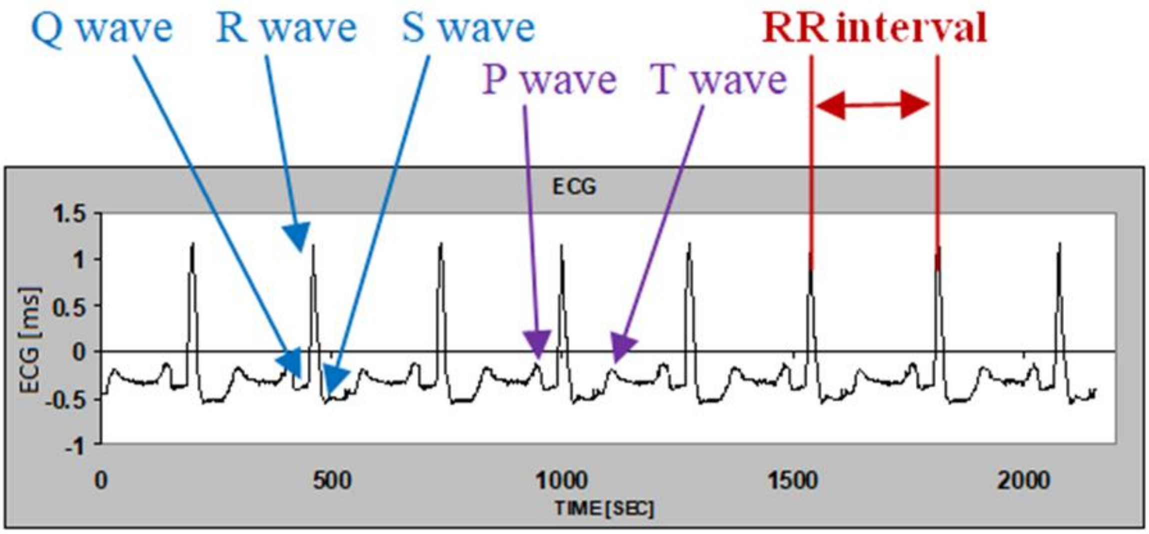
1. Introduction

An electrocardiogram is a graphical record of the electrical activity of the heart during cardiac cycles (during depolarization and repolarization of the atria and the chamber of the heart) from different points of the body and is an effective tool for analysis and diagnostics of the work of the heart. Electrical activity is detected by electrodes attached to the skin. Real electrocardiological (ECG) signals have a number of disturbances and it is important to separate them before performing the analysis.

The registration of cardiac activity is performed in several leads - in clinical practice it is usually done with 12 standard leads. Figure 1 shows a diagram of a typed ECG signal corresponding to a second standard lead, with its characteristic points. The following essential elements of a cardio cycle are noted:

- P wave - smooth low-amplitude wave;
- QRS complex - high amplitude part with steep slopes. It consists of a negative Q peak, a high positive R peak, and a negative S peak;
- T wave - medium amplitude smoothed wave;
- U wave - there may be a low amplitude wave after the T wave.

Figure 1: ECG signal



Source: (authors)

2. Methods for Heart Rate Variability modeling

Heart Rate Variability is a means of examining a person's cardiovascular activity and is considered an indicator of the heart condition and health of the individual. A number of areas for the study, processing and analysis of cardiac data have evolved in recent years and are continuing to create new ones (Georgieva-Tsaneva, 2019).

The phenomena that affect Heart Rate Variability are (Attarodi et al, 2011):

- physiological control processes: Respiratory Sinus Arrhythmia (RSA) and blood pressure (Mayer waves);

- physical and mental activity;

- rhythms of blood circulation;

- angiotensin harmonic system caused by extracellular volumes of fluid;

- stages of wakefulness, falling asleep, sleeping.

The mathematical modeling of heart rate variability is used in the analysis of physiological signals and the prediction of their development over time, for the evaluation of created algorithms for analysis, for educational purposes (presenting the signals in an appropriate form for training and working with them). The purpose of heart rate variability modeling is to create artificial Heart Rate Variability sequences that model real Heart Rate Variability series under different physiological conditions. In recent years, the world scientific community has worked hard in all scientific disciplines to use new technologies and thus serve humanity (Dimitrova et al, 2019; Zahariev, Valchova, 2019).

There are many mathematical models of Heart Rate Variability created, the most commonly used ones:

1. The Zeeman Mathematical model.
   Zeeman uses nonlinear dynamic differential equations to model the heart rhythm (Zeeman, 1972), given the fact that the source of the mechanism for generating cardiac impulses is nonlinear.
   Zeeman's mathematical model (based on the Van der Pol-Lienard equation) describes the dynamics of heart activity and the causes of muscle fiber contraction. The effects of normal or beta distribution can be added through the parameters of the mathematical system, thus introducing the element of unpredictable fluctuations in the activity of the heart.
   The model is based on the following important characteristics of the heart: stable equilibrium, thresholding for triggering an action potential, return to equilibrium. Zeeman's mathematical model has proven to be successful and many researchers have worked on it over the years (Abad et al 2009; Ayatollahi, 2005). The authors (Jafarnia-Dabanlooa et al, 2007) modify the Zeeman model by incorporating the effects of the sympathetic and parasympathetic nervous systems to be able to generate appropriate peaks in the HRV power spectrum.
   Other researchers such as (Suckley, Biktashev, 2003) argue against the assumptions underlying this model and suggest different models.

2. McSharry Mathematical model.
   A mathematical model using Gaussian functions (McSharry, 2003) is another effective method of HRV simulation. The use of Gaussian functions enables a good interpretation of the properties in the power spectrum. To incorporate the Low frequency (LF) and High frequency (HF) into the HRV spectrum, in (McSharry, 2003), two Gaussian functions are used.
   Gaussian Mixture Mathematical Model as a linear combination of Gaussian mixture decomposition of the signal is presented in (Costa et al, 2012). The model allows the modeling of arbitrary complex distributions. The presented model covers well-researched data, with dominant models in the HRV being captured by the distribution of the presented components.
   In (Georgieva-Tsaneva et al, 2012) a modification of the algorithm described in (McSharry, 2003) using the means of wavelet theory to create the time series (instead of Fourier transform) is presented.
   A work (Georgieva-Tsaneva, 2019) presents an algorithm for generating a realistic HRV based on three Gaussian distributive functions modeling the spectral components of the HRV in the three frequency ranges: Very Low frequency (VLF), Low frequency, and High frequency.

3. Mathematical model based on the Integral Pulse Frequency Modulation.
   HRV series can be modeled using mathematical relationships that are based on Integral Pulse Frequency Modulation. The existing models in the scientific literature based on IPFM are based on the use of arbitrary threshold or fixed threshold; evaluation of the functioning of the autonomic nervous system; based on vector space theory; based on neural networks and more.
   An HRV mathematical model based on integral impulse frequency modulation simulating the operation of the autonomic nervous system in the spectral domain is presented in (Attarodi, et al, 2013; Attarodi et al, 2011).
   Safdarian (2014) bases his method on vector space theory and time-varying applications of the IPMF model, special functions, and two different HRV methods. The work is based on an assessment of the effects of the sympathetic and parasympathetic nervous systems and the internal entrance to the Sinoatrial node. The results of this method are the interval series of cardiac intervals that are very close to the actual intervals of the human heart. The results obtained are compared with real data and the effectiveness of the proposed model is confirmed. The author points out, as a possible application of the proposed model, its use for the evaluation of diagnostic ECG processing devices; when signal compression and telemedicine application.

4. Stochastic models.
   Stochastic models are based on the ability of the cardiovascular system to self-regulate. In work (Amaral, 1999) a stochastic model with a feedback of the neuro-autonomous regulation of heart rate is presented. The model considers the main characteristics of cardiac variability, including energy spectrum, functional form, and scaling of the distribution of variations in intervals and correlations in the Fourier phases.

3. The Gaussian mathematical HRV model

The Gaussian mathematical Heart Rate Variability model is giving by the sum of two Gaussian frequency distributions (McSharry, 2003):

$$S_1(f) = \frac{\sigma_1^2}{\sqrt{2\pi c_1^2}}.exp\left(\frac{(f-f_1)^2}{2c_1^2}\right), \tag{1}$$

and

$$S_2(f) = \frac{\sigma_2^2}{\sqrt{2\pi c_2^2}}.exp\left(\frac{(f-f_2)^2}{2c_2^2}\right). \tag{2}$$

Where: f - frequency;
f_1 and f_2 - means of frequency;
c_1 and c_2 - standard deviations;
σ_1^2 and σ_2^2 - power in the LF and HF frequency bands.

The energy in the low frequency (LF) and high frequency (HF) regions is given by σ_1^2 and, respectively, with the total energy in the two regions being (McSharry, 2003):

$$\sigma^2 = \sigma_1^2 + \sigma_2^2 \tag{3}$$

The LF/HF ratio is determined by σ_1^2/σ_2^2.

The implemented algorithm for the generation of HRV series uses two Gaussian distribution functions, calculates complex frequency sequence and uses inverse wavelet transformation to represent the generated frequency sequence in the time domain. This algorithm can simulate HRV data with temporal and spectral characteristics corresponding to healthy subjects.

4. Results and Discussions

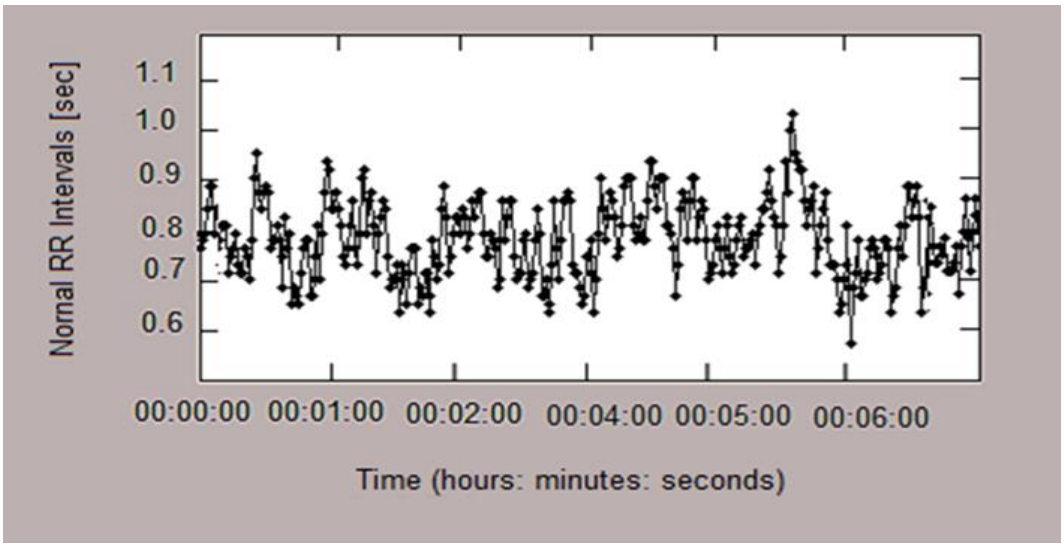
The above results are obtained through the software implementation of a mathematical model of Heart Rate Variability using two Gaussian frequency distributions.

Figures and tabular results are obtained through the software implementation of the algorithm.

Figure 2 presents a time series obtained by using the presented mathematical method to simulate an HRV with two Gaussian distribution functions and a wavelet transform to represent the resulting frequency series in the time domain.

The generated HRV series (6 minutes) fluctuates around an average of 0.8 seconds for the duration of the intervals received. The figure shows that the time series has a wide amplitude of variation along the ordinate axis. The interval time series obtained corresponds to an interval of good health with no cardiovascular problems.

Figure 2: Heart Rate Variability Model (6 minutes)



Source: (authors)

The presented algorithm is implemented in MATLAB environment. Table 1 shows the lead times for the HRV generation procedure (Daubechies wavelet basis Db4 used), depending on the length of the series generated. The results obtained show that time series are generated in a short time and the algorithm is effective as software implementation.

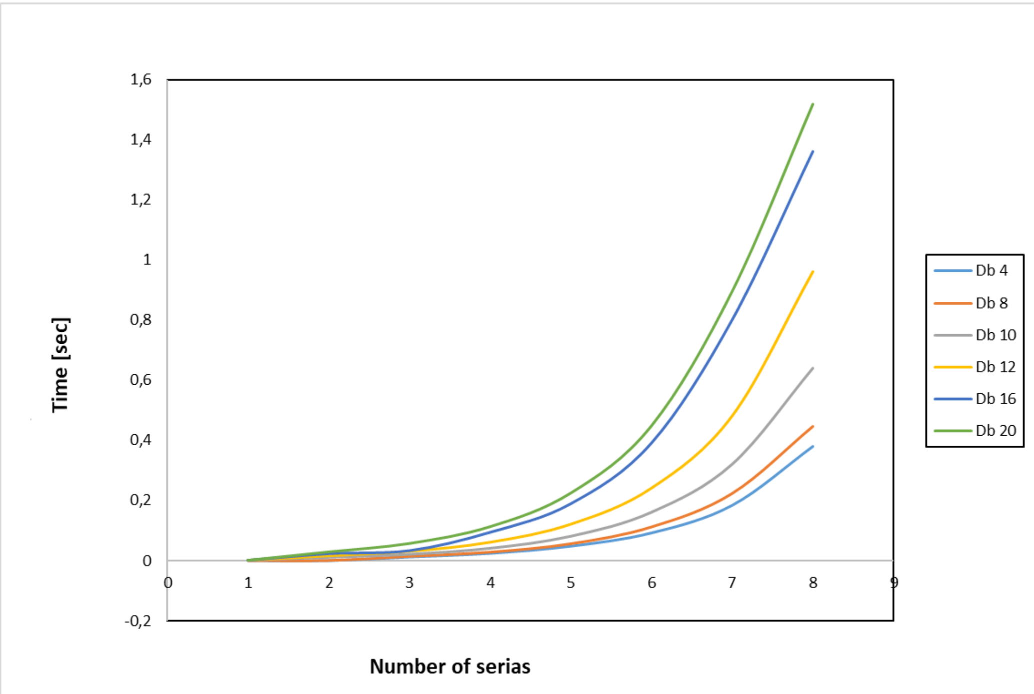
Table 1: Simulation time, an algorithm with Db4 basis

Number of intervals in the series	1024	2048	4096	8192	16384	32768	65536	131072
Time [sec.]	0.00	0.0	0.014	0.029	0.056	0.109	0.213	0.441

Source: (authors)

Figure 3 graphically represents the relationship between processor time and the selected base for the HRV series implementation. The studies were done with Daubechies wavelet bases with a different number of coefficients: 4, 8, 10, 12, 16 and 20. Times for the execution of the program procedure is lower when using Daubechies wavelet bases with fewer coefficients.

Figure 3: Relationship between CPU time and selected basis



Source: (authors)

Areas of application of the proposed algorithm:

- The mathematical model of the generated HRV time sequence can be used to evaluate the performance of various processing algorithms developed, to analyse and compress biomedical signals.
- The RR synthetic series can be used to understand the theoretical mechanisms of action of the cardiovascular system.
- Using a mathematical model of heart activity, a database of realistic, modelled HRV series can be created. The database can be used for research and educational purposes.
- For educational purposes.
- Created database of modelled HRV series can be used to test statistical hypotheses.

5. Conclusion

The paper presents mathematical methods for modeling the HRV series. An overview of commonly used methods in the scientific literature is made: The Zeeman Mathematical model, the mathematical model using Gaussian functions, Mathematical model based on the Integral Pulse Frequency Modulation, models based on stochastic analyzes. The paper presents the results of using a Gaussian mathematical HRV model. Shown is a graph of a generated 6-minute HRV series created with the presented model based on Gaussian functions. The dependencies between the length of the generated series and the execution time of the created procedure are shown. The relationship between the CPU time and the chosen basis for the HRV series implementation is presented graphically.

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