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Extension of commuter train timetabling problem, mathematical modeling and Simulated Annealing solution approach

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Abstract.

The Optimal utilization of railroad transport capacity is one of the main goals in train scheduling problem. The sequences of the trains dispatch as well as train stop schedule at stations are two main factors in the optimal use of railroad transportation capacity. Train timetabling has been modeled by researchers with varying constraints, most research aimed to minimize delays. In this paper we are looking for timetabling from both passenger and manager perspective regarding to the constraints and assumptions of the rail road transportation system. Two East-North and East-West single line routes are considered. Each route contains a certain number of stations and blocks. The model was solved using Simulated Annealing algorithm and GAMS software. The results show that the algorithm provides a near-optimal solution.

Keywords: Railroad transportation, Timetabling, Commuter train, Simulated Annealing.

1. Introduction

The train timetabling problem is one of the most difficult scheduling issues in transportation systems. With the increase in travel demand and development of railroads, the importance of timetabling and sequencing issues has increased. The purpose of train timetabling is to minimize the trains travel time from origin to destination point to reduce all operational costs, fuel, personnel and to satisfy the passengers and owners by reducing delays and maximizing the capacity utilization of the lines, stations and crew. If rail networks have an optimal schedule, travel costs and travel times and delays reduced and consequently passenger satisfaction and service quality will improve and affect their decisions. In busy networks, there are many stations and hundreds of trains a day, usually with complex routes

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and numerous obstacles such as (crossing, repairs, physical condition of the network) along the way. On the other hand, there are different types of trains, travel times for each train, different routes and preferred lines. Increasing the share of transportation in the economies of countries, as well as increasing the speed of computers and use of software has led researchers to provide new approaches to solve problems every day.

2. Literature of review

Train stop scheduling and timetabling play pivotal role in railway operations. To achieve quality timeline, an integrated optimization model including a stop plans and time-dependent passenger demand was proposed. The problem was presented as a mixed-integer nonlinear programming model to optimize passenger performance. This model includes the total waiting time at the stations, delay time in trains departure due to the train stop and minimize total train travel time (Dong et al., 2020). In the research of (Palmqvist et al., 2020) dwell time delay with the number of passengers for commuter trains in Stockholm and Tokyo are considered. The study of (warg et al., 2019) evaluates the effects of commuter trains adjustments on both the passenger and the train operator. Costs are estimated based on train travel and network operations. In this study, the impact of changes at departure times was investigated. There have been various researches on the optimal train timetabling since the 1960s. What is obvious in the literature review is that there are numerous problems (such as complexity and large dimensions of the problem) in finding the optimal solution, solving methods tend to find an acceptable solution rather than an optimal solution. The Line Planning Problem (LPP) and the Train Schedule Problem (TSP) are two pivotal elements at the strategic and tactical level that lay the foundation for a high level of service quality for railway operation. In the investigation of (Yan & Goverde, 2019) a combination of line planning problem and train timetabling problem is designed with timetable robustness, discipline, travel time of passengers, then multi-objective mixed integer linear programming (MOMILP) model is proposed for set of assumption. (Ait Ali et al., 2017) studied the socio-economic advantages of using the commuter train services in Stockholm. In the present research a simulation-based optimization approach is used to solve the problem of train timetable in single and two line rail networks. In the proposed approach, a simulation model is used to generate scheduling programs in ED software environment. The results show that the implementation of the proposed model in comparison with the current Iranian railway schedule has led to a significant improvement in non- scheduled train stops. Also, the proposed algorithm is able to create a schedule of commuter trains within a reasonable period of time according to the time allowed for prayer (Nayebi et al., 2016). (Yaghini, 2011) modeled the train timetabling regarding to stop time consideration for prayers. In the research, the model has been solved for on-way path. (Javanshir & Mossadeghi , 2010) presented a multi-objective mathematical model for trains movement scheduling in the single-rail lines, taking into account the intersection.

This mathematical model has two objective functions. The first goal minimized the total delay time of trains in a specific time interval and the second goal minimized costs. (Kianfar & Jamilli, 2009) presented the article "train movements timetabling using hyper heuristic simulated Heat Treatment Algorithm", where the object minimized delays by the way that the problem constraints are satisfied. The problem defined as integer programming and solved using conventional methods such as branch and bound. However, because the optimal solution is not achieved through exact methods due to the increase of variables and constraints over an acceptable period of time, the simulated Heat Treatment algorithm has been used. (Bayhan & Yalcınkaya, 2012) simulated the freight train scheduling problem to obtain timetable for all trains in the system. (Narayanaswami & Rangaraj , 2013) rescheduling optimized train timetabling with disruption that minimizing delays was aimed. (Heydar et al., 2013) developed a mathematical model using mixed integer programming to minimize train timetabling period and maximize routes capacity on single-path by considering two types of trains. In research of (Jamili, 2011) the problem of periodic train scheduling in rail networks and infrastructure capacity determination has been examined. Robust approach for practical capacity scheduling was presented. It has also proposed several algorithms for calculating theoretical and practical capacity using robust scheduling methods. In the studies of (Liu & Kozan, 2010) the problem of train scheduling by consideration of priority for each train is modeled in the form of flexible job-shop scheduling with no delay and limit capacity assumption between the machines. In the proposed model, high speed passenger trains have higher priority than others and must arrive to destination point without unscheduled stops.

3. Definition of problem

The problem of trains scheduling is one of the most important issues in the planning of trains which ensures maximum passenger capacity utilization in addition to ensuring passenger safety. Because of many variables and constraints, it is one of the most complex problems that cannot be solved in exact methods. The objective functions are also different in these problems but in most cases the purpose is to reduce travel time and increase line capacity. The train scheduling is an NP hard problem (CAI & GOH, 1998). The rail networks in most of the countries in the world are single-path for economic reasons. Hence only one train can travel in a block at a time (distance between two stations). In other words, only trains at the stations are allowed to cross. In such a situation, if the trains timing is not properly scheduled, it has caused the trains cross the border and irreparable life and financial risks created. This study needs to find near optimal train schedule using the meta-heuristic algorithm. In this case, the route is single-path and has one origin and two destination points. Also, different conditions for railway transportation systems, including safety and emergency stops are considered to minimize travel time and related costs that passengers and managers expect. With respect to the size of the problem, it was solved using meta-heuristic Simulated Annealing algorithm.

3.1. Model assumption

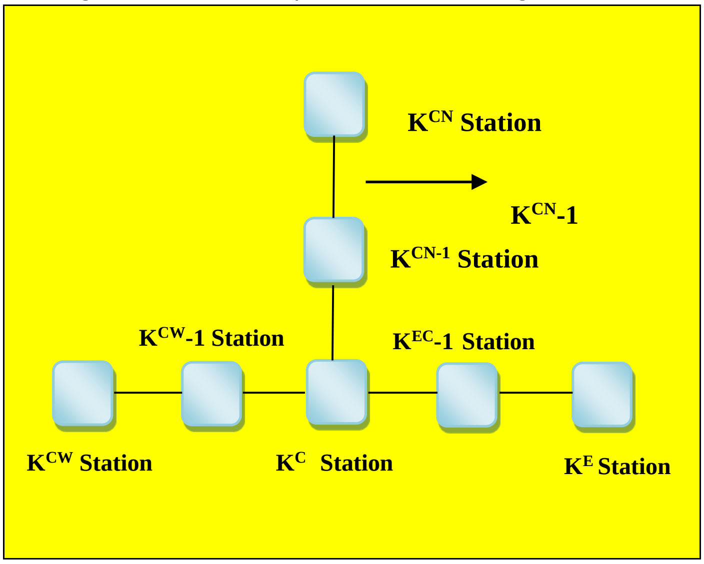
In the train timetable modeling minimizing train travel time and deviation from the ideal arrival time to destination point is our purpose. We also considered reduce costs associate with scheduling, including costs of unscheduled stops and delays as well as reducing fuel consumption by taking into account the optimal train moving from the origin to the destination point. The following are the model assumptions:

1. The route is single-path with an intersection including origin and destination points and the trains have mutual route. The trains travel from the origin to the destination point and vice versa. A train that travels from the origin or destination point must arrive to the last station.

2. The priority is considered for each train.
3. At a certain time in each block only one train can move and the others wait at the stations until the block becomes empty, at that moment with respect to the trains priority, the blocks passed one by one.
4. The trains are only allowed to stop at stations.
5. For each delay time unit on each train type, a fixed cost is considered as a coefficient.
6. For each time unit when the train is moving upper than the optimal range, the fixed cost is considered as a coefficient.
7. Trains in each block can move at the speed between the minimum and maximum allowed block.
8. Start time for each train from the primary and final stations are Predetermined.
9. Station capacity is unlimited.
10. If the train arrives at the station within the specified time interval for emergency stop, the train stop time at the station is added.

As mentioned, the problem is considered with two East-North and East-West single-path routes. These two paths are considered perpendicularly. Stations are named from East to North and East to West respectively. The distance between the two stations is called BLOCK, and each block is named based on previous station. According to the Mentioned routes, there will be four sets of trains such as East-North, North-East, East-West and West-East respectively. At any time interval, only one train can be present in each block. The East-North and East-West trains should also be avoided at the intersection of two lines.

Figure 1: Schematic of two intersection single-rail lines



3.2. Parameters, Variables and indices

In this section: sets, parameters, variables including continuous and Integer variables that used in mathematical modeling are introduced.

Sets:

EN	A set of trains moving from East to North.
NE	A set of trains moving from North to East.
EW	A set of trains moving from East to West.
WE	A set of trains moving from West to East.
T	Set of times for emergency stop (prayer)
B	A set of blocks
K *	destination block
ST	Stations Set

Parameters:

W_i^{EN}	East-North i-th train Priority coefficient
W_i^{NE}	North-East i-th train Priority coefficient
W_i^{EW}	East-West i-th train Priority coefficient
W_i^{WE}	West-East i-th train Priority coefficient
d_i	Destination station of i-th train
$S_{i,m,k}^{EN}$	Scheduled stop time for East-North i-th train type m at station k
$S_{i,m,k}^{NE}$	Scheduled stop time for North-East i-th train type m at station k
$S_{i,m,k}^{EW}$	Scheduled stop time for East-West i-th train type m at station k
$S_{i,m,k}^{WE}$	Scheduled stop time for West-East i-th train type m at station k
$Vmin_{i,m,k}^{EN}$	The minimum limited speed of East-North i-th train type m in the block k
$Vmax_{i,m,k}^{EN}$	Maximum limited speed of East-North i-th train type m in the block k
$Vmin_{i,m,k}^{NE}$	Minimum limited speed of North-East i-th train type m in the block k
$Vmax_{i,m,k}^{NE}$	Maximum limited speed of North-East i-th train type m in the block k
$Vmin_{i,m,k}^{EW}$	The minimum limited speed of East-West i-th train type m in the block k
$Vmax_{i,m,k}^{EW}$	Maximum limited speed of East-West i-th train type m in the block k
$Vmin_{i,m,k}^{WE}$	Minimum limited speed of West-East i-th train type m in the block k
$Vmax_{i,m,k}^{WE}$	Maximum limited speed of West-East i-th train type m in the block k
ds_k	Length of block k
do_i^{EN}	Minimum departure time for East-North i-th train from origin point
do_i^{NE}	Minimum departure time for North-East i-th train from origin point
do_i^{EW}	The minimum departure time for East-West i-th train from origin point
do_i^{WE}	The minimum departure time for East-West i-th train from the origin point
$TL_{i,m}^{EN}$	The minimum travel time for entire route East-North i-th train, type-m
$TU_{i,m}^{EN}$	The maximum travel time for entire route East-North i-th train, type-m
$TL_{i,m}^{NE}$	The minimum travel time for entire route East-North i-th train, type-m
$TU_{i,m}^{NE}$	The maximum travel time for entire route East-North i-th train, type-m
$TL_{i,m}^{EW}$	The minimum travel time for entire route East-North i-th train, type-m
$TU_{i,m}^{EW}$	The maximum travel time for entire route East-North i-th train, type-m
$TL_{i,m}^{WE}$	The minimum travel time for entire route East-North i-th train, type-m
$TU_{i,m}^{WE}$	The maximum travel time for entire route West-East i-th train, type-m
$H_{i,m}^{EN}$	Allowed time of the whole route for East-North i-th train, type-m
$H_{i,m}^{NE}$	Allowed time of the whole route for North-East i-th train, type-m
$H_{i,m}^{EW}$	Allowed time of the whole route for West-East i-th train, type-m
$H_{i,m}^{WE}$	Allowed time of the whole route for West-East i-th train, type-m
LTP	Minimum train stop time if arrived in allowed time interval
LL_t	lower bound of allowed stop interval in turn t
LU_t	upper bound of allowed stop interval in turn t

C_{Z_1}	Cost per unit time for a stopping train
C_{Z_2}	Fixed cost of moving too much per unit time
M	Big number

Continuous variables

$X_{i,m,k}^{EN}$	The arrival time of East-North i-th train, type-m ,to the end of the k-th block
$Y_{i,m,k}^{EN}$	Starting time of East-North i-th train, type-m, to the end of the k-th block
$X_{i,m,k}^{NE}$	Arrival time of North-East i-th train, type-m, to the end of the k-th block
$Y_{i,m,k}^{NE}$	Starting time of North-East i-th train, type-m, to the end of the k-th block
$X_{i,m,k}^{EW}$	Arrival time of East-West i-th train, type-m, to the end of the k-th block
$Y_{i,m,k}^{EW}$	Starting time of East-West i-th train, type-m, to the end of the k-th block
$X_{i,m,k}^{WE}$	Arrival time of West-East i-th train, type-m, to the end of the k-th block
$Y_{i,m,k}^{WE}$	Starting time of West-East i-th train, type-m,to the end of the k-th block

Integer variables

$A_{i,j,m,k}^{EN}$ { 1 If East-North j-th train type- m is faster than East-North i-th train type-m leaves the k-th block
0 otherwise

$A_{i,j,m,k}^{NE}$ { 1 If North-East j-th train type-m is faster than North-East i-th train type-m leaves the k-th block
0 otherwise

$A_{i,j,m,k}^{EW}$ { 1 If East-West j-th train type-m is faster than East-West i-th train type-m leaves the k-th block
0 otherwise

$A_{i,j,m,k}^{WE}$ { 1 If west- East j-th train type-m is faster than East-West i-th train type-m leaves the k-th block
0 otherwise

$A_{i,j,m,k}^{EN-EW}$ { 1 If the East-North j-th train type-m is faster than East-North i-th train type-m leaves the k-th block
0 otherwise

$A_{i,j,m,k}^{NE-WE}$ { 1 If the North-East j-th train type-m is faster than North-East i-th train type-m leaves the k-th block
0 otherwise

$A_{i,j,m,k}^{EN-NE}$ { 1 If the East-North j-th train type-m is faster than East-North i-th train type-m leaves the k-th block
0 otherwise

$A_{i,j,m,k}^{EW-WE}$ { 1 If the East-West j-th train type-m is faster than East-West i-th train type-m leaves the k-th block
0 otherwise

$A_{i,j,m,k}^{EN-WE}$ { 1 If the East-North j-th train type-m is faster than East-North i-th train type-m leaves the k-th block
0 otherwise

$A_{i,j,m,k}^{NE-EW}$ { 1 If the North-East j-th train type-m is faster than East-North i-th train type-m leaves the k-th block
0 otherwise

$NU_{i,m,k,t}^{EN}$ { 1 If East-North i-th train type-m before LU's time arrives to the station k.
0 otherwise

$NL_{i,m,k,t}^{EN}$ { 1 If East-North i-th train type-m, after LL's time, arrives to the station k
0 otherwise

$N_{i,m,k,t}^{EN}$ { 1 If East-North i-th train, type-m, before the LU time and after LL time , arrives to the station k
0 otherwise

$NU_{i,m,k,t}^{NE}$ { 1 If North-East i-th train type-m, before the LU's time, arrives to the station k.
0 otherwise

$NL_{i,m,k,t}^{NE}$ { 1 If North-East i-th train type-m, after LL time, arrives to the station k.
0 otherwise

$N_{i,m,k,t}^{NE}$ { 1 If North-East i-th train type-m, before the LU time and after LL time arrives to the station k.
0 otherwise

$NU_{i,m,k,t}^{EW}$ { 1 If the East-West i-th train type-m, before the LU's time arrives to the station k.
0 otherwise

$NL_{i,m,k,t}^{EW}$ { 1 If East-West i-th train type-m, after LL time, arrives to the station k.
0 otherwise

{

$N^{EW}_{i,m,k,t}$ 1 If East-West i-th train, type-m, before the LU time and after LL time, arrives to station k.
0 otherwise

$NU^{WE}_{i,m,k,t}$ 1 If West-East i-th train type-m, before the LU time, arrives to the station k.
0 otherwise

$NL^{WE}_{i,m,k,t}$ 1 If West-East i-th train type-m, after LL time, arrives to the station k.
0 otherwise

$N^{WE}_{i,m,k,t}$ 1 If West-East i-th train type-m, before the LU time and after LL time arrives to the station k.
0 otherwise

$V^{EN}_{i,m,k,t}$ 1 If the station k for East-North i-th train type-m selected for emergency stop.
0 otherwise

$V^{NE}_{i,m,k,t}$ 1 If the station k for North-East i-th train type-m selected for emergency stop.
0 otherwise

$V^{EW}_{i,m,k,t}$ 1 If the station k for East-West i-th train type-m, selected for emergency stop
0 otherwise

$V^{WE}_{i,m,k,t}$ 1 If the station k for West-East i-th train type-m, selected for emergency stop
0 otherwise

3.3. The objective function

The first objective function minimizes train delays and the second objective function minimizes unscheduled stop cost as well as minimizing total time when the train is moving.

$$\mathrm{Min}Z_1: \sum_{i\epsilon EN}(W_i^{EN}\sum_{m=1}^{n}(X^{EN}_{i,m,k*} - TL^{EN}_{i,m})\ ) + \sum_{i\epsilon NE}(W_i^{NE}\sum_{m=1}^{n}(X^{NE}_{i,m,k*} - TL^{NE}_{i,m})\ ) + \sum_{i\epsilon EW}(W_i^{EW}\sum_{m=1}^{n}(X^{EW}_{i,m,k*} - TL^{EW}_{i,m}) + \sum_{i\epsilon WE}(W_i^{WE}\sum_{m=1}^{n}(X^{WE}_{i,m,k*} - TL^{WE}_{i,m})\ )\ ) \tag{1}$$

$$\mathrm{Min}Z_2: C_{Z_1}(\sum_{i\epsilon EN}(W_i^{EN}\sum_{k\epsilon K}\sum_{m=1}^{n}(Y^{EN}_{i,m,k+1} - X^{EN}_{i,m,k} - S^{EN}_{i,m,k} + \sum_{i\epsilon NE}(W_i^{NE}\sum_{k\epsilon K}\sum_{m=1}^{n}(Y^{NE}_{i,m,k+1} - X^{NE}_{i,m,k} - S^{NE}_{i,m,k}) + \sum_{i\epsilon EW}(W_i^{EW}\sum_{k\epsilon K}\sum_{m=1}^{n}(Y^{EW}_{i,m,k+1} - X^{EW}_{i,m,k} - S^{EW}_{i,m,k})) + \sum_{i\epsilon WE}(W_i^{WE}\sum_{k\epsilon K}\sum_{m=1}^{n}(Y^{WE}_{i,m,k+1} - X^{WE}_{i,m,k} - S^{WE}_{i,m,k}))) +$$

$$C_{Z_2}\Big(\sum_{i\epsilon EN}\sum_{m=1}^{n} W_i^{EN}\big(\sum_{k\epsilon K}(X_{i,m,k}^{EN}-Y_{i,m,k}^{EN})\big)-H_{i,m}^{EN}$$
$$+\sum_{i\epsilon NE}\sum_{m=1}^{n} W_i^{NE}\big(\sum_{k\epsilon K}(X_{i,m,k}^{NE}-Y_{i,m,k}^{NE})\big)-H_{i,m}^{NE}$$
$$+\sum_{i\epsilon EW}\sum_{m=1}^{n} W_i^{EW}\big(\sum_{k\epsilon K}(X_{i,m,k}^{EW}-Y_{i,m,k}^{EW})\big)-H_{i,m}^{EW}$$
$$+\sum_{i\epsilon WE}\sum_{m=1}^{n} W_i^{WE}\big(\sum_{k\epsilon K}(X_{i,m,k}^{WE}-Y_{i,m,k}^{WE})\big)-H_{i,m}^{WE}\Big) \quad (2)$$

Constraints:
In this section, the model constraints for train timetabling are presented.
1. Minimum start time:

$$Y_{i,m,k_{o_i}}^{EN} \geq do_{i,m}^{EN} \qquad \forall i\epsilon EN, \forall m\epsilon M \qquad (3)$$
$$Y_{i,m,k_{o_i}}^{NE} \geq do_{i,m}^{NE} \qquad \forall i\epsilon NE, \forall m\epsilon M \qquad (4)$$
$$Y_{i,m,k_{o_i}}^{EW} \geq do_{i,m}^{EW} \qquad \forall i\epsilon EW, \forall m\epsilon M \qquad (5)$$
$$Y_{i,m,k_{o_i}}^{WE} \geq do_{i,m}^{WE} \qquad \forall i\epsilon WE, \forall m\epsilon M \qquad (6)$$

2- The minimum and maximum time during the block:

$$\frac{ds_k}{Vmax_{i,m,k}} \leq X_{i,m,k}^{EN}-Y_{i,m,k}^{EN} \leq \frac{ds_k}{Vmin_{i,m,k}} \qquad \forall k \in K^{EN}, \forall i\epsilon EN, \forall m\epsilon M \qquad (7)$$
$$\frac{ds_k}{Vmax_{i,m,k}} \leq X_{i,m,k}^{NE}-Y_{i,m,k}^{NE} \leq \frac{ds_k}{Vmin_{i,m,k}} \qquad \forall k \in K^{NE}, \forall i\epsilon NE, \forall m\epsilon M \qquad (8)$$
$$\frac{ds_k}{Vmax_{i,m,k}} \leq X_{i,m,k}^{EW}-Y_{i,m,k}^{EW} \leq \frac{ds_k}{Vmin_{i,m,k}} \qquad \forall k \in K^{EW}, \forall i\epsilon EW, \forall m\epsilon M \qquad (9)$$
$$\frac{ds_k}{Vmax_{i,m,k}} \leq X_{i,m,k}^{WE}-Y_{i,m,k}^{WE} \leq \frac{ds_k}{Vmin_{i,m,k}} \qquad \forall k \in K^{WE}, \forall i\epsilon WE, \forall m\epsilon M \qquad (10)$$

3- The minimum and maximum travel time on the entire route

$$TL_{i,m}^{EN} \leq X_{i,m,k*}^{EN}-Y_{i,m,k_{o_i}}^{EN} \leq TU_{i,m}^{EN} \qquad \forall i\epsilon EN, \forall m\epsilon M \qquad (11)$$
$$TL_{i,m}^{NE} \leq X_{i,m,k*}^{NE}-Y_{i,m,k_{o_i}}^{NE} \leq TU_{i,m}^{NE} \qquad \forall i\epsilon NE, \forall m\epsilon M \qquad (12)$$
$$TL_{i,m}^{EW} \leq X_{i,m,k*}^{EW}-Y_{i,m,k_{o_i}}^{EW} \leq TU_{i,m}^{EW} \qquad \forall i\epsilon EW, \forall m\epsilon M \qquad (13)$$
$$TL_{i,m}^{WE} \leq X_{i,m,k*}^{WE}-Y_{i,m,k_{o_i}}^{WE} \leq TU_{i,m}^{WE} \qquad \forall i\epsilon WE, \forall m\epsilon M \qquad (14)$$

4- Sequence constraints of two trains arrival to the block:

$$X_{i,m,k}^{EN} \leq Y_{i,m,k+1}^{EN} \qquad \forall i\epsilon EN, \forall k\epsilon K^{EN}, k < k* \qquad (15)$$

$X^{NE}_{i,m,k+1} \leq Y^{NE}_{i,m,k} \quad \forall i\epsilon NE, \forall k\epsilon K^{NE}, k < k* \quad (16)$
$X^{EW}_{i,m,k} \leq Y^{EW}_{i,m,k+1} \quad \forall i\epsilon EW, \forall k\epsilon K^{EW}, k < k* \quad (17)$
$X^{WE}_{i,m,k+1} \leq Y^{WE}_{i,m,k} \quad \forall i\epsilon WE, \forall k\epsilon K^{WE}, k < k* \quad (18)$

5- The intersection constraints of trains with the opposite direction in the same block:

$Y^{NE}_{i,m,k} + M \times (1 - A^{EN-NE}_{i,j,m,k}) \geq X^{EN}_{j,m,k} \quad \forall i\epsilon NE, j\epsilon EN, \forall k\epsilon B \quad (19)$
$Y^{EN}_{j,m,k} + M \times A^{NE-EN}_{i,j,m,k} \geq X^{NE}_{i,m,k} \quad \forall i\epsilon NE, j\epsilon EN, \forall k\epsilon B \quad (20)$
$Y^{WE}_{i,m,k} + M \times (1 - A^{EW-WE}_{i,j,m,k}) \geq X^{EW}_{j,m,k} \quad \forall i\epsilon WE, j\epsilon EW, \forall k\epsilon B \quad (21)$
$Y^{EW}_{j,m,k} + M \times A^{WE-EW}_{i,j,m,k} \geq X^{WE}_{i,m,k} \quad \forall i\epsilon WE, j\epsilon EW, \forall k\epsilon B \quad (22)$

6- Confluence constraint of trains in the same direction:

$Y^{EN}_{i,m,k} + M \times (1 - A^{EN}{}_{i,j,m,k}) \geq X^{EN}_{j,m,k} \quad \forall i, j\epsilon EN, i \neq j, \forall k\epsilon K^{EN} \quad (23)$
$Y^{EN}_{j,m,k} + M \times A^{EN}{}_{i,j,m,k} \geq X^{EN}_{i,m,k} \quad \forall i, j\epsilon EN, i \neq j, \forall k\epsilon K^{EN} \quad (24)$
$Y^{NE}_{i,m,k} + M \times (1 - A^{NE}{}_{i,j,m,k}) \geq X^{NE}_{j,m,k} \quad \forall i, j\epsilon NE, i \neq j, \forall k\epsilon K^{NE} \quad (25)$
$Y^{NE}_{j,m,k} + M \times A^{NE}{}_{i,j,m,k} \geq X^{NE}_{i,m,k} \quad \forall i, j\epsilon NE, i \neq j, \forall k\epsilon K^{NE} \quad (26)$
$Y^{EW}_{i,m,k} + M \times (1 - A^{EW}{}_{i,j,m,k}) \geq X^{EW}_{j,m,k} \quad \forall i, j\epsilon EW, i \neq j, \forall k\epsilon K^{EW} \quad (27)$
$Y^{EW}_{j,m,k} + M \times A^{EW}{}_{i,j,m,k} \geq X^{EW}_{i,m,k} \quad \forall i, j\epsilon EW, i \neq j, \forall k\epsilon K^{EW} \quad (28)$
$Y^{WE}_{i,m,k} + M \times (1 - A^{WE}{}_{i,j,m,k}) \geq X^{WE}_{j,m,k} \quad \forall i, j\epsilon WE, i \neq j, \forall k\epsilon K^{WE} \quad (29)$
$Y^{WE}_{j,m,k} + M \times A^{WE}{}_{i,j,m,k} \geq X^{WE}_{i,m,k} \quad \forall i, j\epsilon WE, i \neq j, \forall k\epsilon K^{WE} \quad (30)$

7- Intersection constraints:

$Y^{EN}_{i,m,K} + M \times (1 - A^{EW-EN}_{i,j,m,k}) \geq X^{EW}_{j,m,K} \quad \forall i\epsilon EW, j\epsilon N \quad (31)$
$Y^{EW}_{j,m,K} + M \times A^{EW-EN}_{i,j,m,K} \geq X^{EN}_{i,m,K} \quad \forall i\epsilon EW, j\epsilon N \quad (32)$
$Y^{WE}_{i,m,K} + M \times (1 - A^{WE-EN}_{i,j,m,K}) \geq X^{EN}_{j,m,K} \quad \forall i\epsilon WE, j\epsilon N \quad (33)$
$Y^{EN}_{j,m,NC} + M \times A^{WE-EN}_{i,j,m,K} \geq X^{EN}_{i,m,K} \quad \forall i\epsilon WE, j\epsilon N \quad (34)$
$Y^{WE}_{i,m,K} + M \times (1 - A^{NE-WE}_{i,j,m,K}) \geq X^{NE}_{j,m,K} \quad \forall i\epsilon NE, j\epsilon C \quad (35)$
$Y^{NE}_{j,m,K} + M \times A^{NE-WE}_{i,j,m,K} \geq X^{WE}_{i,m,K} \quad \forall i\epsilon WE, j\epsilon C \quad (36)$

8- Determination of emergency stop time interval:

$X^{EN}_{i,m,k} \leq LL_t + M \times NL^{EN}_{i,m,k,t} \quad \forall i\epsilon EN, \forall m\epsilon M, \forall k\epsilon K^{EN}, \forall t\epsilon T \quad (37)$
$X^{EN}_{i,m,k} \geq LL_t - M \times (1 - NL^{EN}_{i,m,k,t}) \quad \forall i\epsilon EN, \forall m\epsilon M, \forall k\epsilon K^{EN}, \forall t\epsilon T \quad (38)$
$X^{EN}_{i,m,k} \leq LU_t + M \times (1 - NU^{EN}_{i,m,k,t}) \quad \forall i\epsilon EN, \forall m\epsilon M, \forall k\epsilon K^{EN}, \forall t\epsilon T \quad (39)$
$X^{EN}_{i,m,k} \geq LU_t - M \times NU^{EN}_{i,m,k,t} \quad \forall i\epsilon EN, \forall m\epsilon M, \forall k\epsilon K^{EN}, \forall t\epsilon T \quad (40)$
$NL^{EN}_{i,m,k,t} + NU^{EN}_{i,m,k,t} \geq 2 - M \times (1 - N^{EN}_{i,m,k,t}) \quad \forall i\epsilon EN, \forall m\epsilon M, \forall k\epsilon K^{EN}, \forall t\epsilon T \quad (41)$
$NL^{EN}_{i,m,k,t} + NU^{EN}_{i,m,k,t} \leq 1 + M \times N^{EN}_{i,m,k,t} \quad \forall i\epsilon EN, \forall m\epsilon M, \forall k\epsilon K^{EN}, \forall t\epsilon T \quad (42)$
$V^{EN}_{i,m,k,t} \leq N^{EN}_{i,m,k,t} \quad \forall i\epsilon EN, \forall m\epsilon M, \forall k\epsilon K^{EN}, \forall t\epsilon T \quad (43)$
$\sum_k V^{EN}_{i,m,k,t} = 1 \quad \forall i\epsilon EN, \forall m\epsilon M, \forall t\epsilon T \quad (44)$

$X_{i,m,k}^{NE} \leq LL_t + M \times NL_{i,m,k,t}^{NE} \quad \forall i \epsilon NE, \forall m \epsilon M, \forall k \epsilon K^{NE}, \forall t \epsilon T \quad (45)$

$X_{i,m,k}^{NE} \geq LL_t - M \times (1 - NL_{i,m,k,t}^{NE}) \quad \forall i \epsilon NE, \forall m \epsilon M, \forall k \epsilon K^{NE}, \forall t \epsilon T \quad (46)$

$X_{i,m,k}^{NE} \leq LU_t + M \times (1 - NU_{i,m,k,t}^{NE}) \quad \forall i \epsilon NE, \forall m \epsilon M, \forall k \epsilon K^{NE}, \forall t \epsilon T \quad (47)$

$X_{i,m,k}^{NE} \geq LU_t - M \times NU_{i,m,k,t}^{NE} \quad \forall i \epsilon NE, \forall m \epsilon M, \forall k \epsilon K^{NE}, \forall t \epsilon T \quad (48)$

$NL_{i,m,k,t}^{NE} + NU_{i,m,k,t}^{NE} \geq 2 - M \times (1 - N_{i,m,k,t}^{NE}) \quad \forall i \epsilon NE, \forall m \epsilon M, \forall k \epsilon K^{NE}, \forall t \epsilon T \quad (49)$

$NL_{i,m,k,t}^{NE} + NU_{i,m,k,t}^{NE} \leq 1 + M \times N_{i,m,k,t}^{NE} \quad \forall i \epsilon NE, \forall m \epsilon M, \forall k \epsilon K^{NE}, \forall t \epsilon T \quad (50)$

$V_{i,m,k,t}^{NE} \leq N_{i,m,k,t}^{NE} \quad \forall i \epsilon NE, \forall m \epsilon M, \forall k \epsilon K^{NE}, \forall t \epsilon T \quad (51)$

$\sum_k V_{i,m,k,t}^{NE} = 1 \quad \forall i \epsilon NE, \forall m \epsilon M, \forall t \epsilon T \quad (52)$

$X_{i,m,k}^{EW} \leq LL_t + M \times NL_{i,m,k,t}^{EW} \quad \forall i \epsilon EW, \forall m \epsilon M, \forall k \epsilon K^{EW}, \forall t \epsilon T \quad (53)$

$X_{i,m,k}^{EW} \geq LL_t - M \times (1 - NL_{i,m,k,t}^{EW}) \quad \forall i \epsilon EW, \forall m \epsilon M, \forall k \epsilon K^{EW}, \forall t \epsilon T \quad (54)$

$X_{i,m,k}^{EW} \leq LU_t + M \times (1 - NU_{i,m,k,t}^{EW}) \quad \forall i \epsilon EW, \forall m \epsilon M, \forall k \epsilon K^{EW}, \forall t \epsilon T \quad (55)$

$X_{i,m,k}^{EW} \geq LU_t - M \times NU_{i,m,k,t}^{EW} \quad \forall i \epsilon EW, \forall m \epsilon M, \forall k \epsilon K^{EW}, \forall t \epsilon T \quad (56)$

$NL_{i,m,k,t}^{EW} + NU_{i,m,k,t}^{EW} \geq 2 - M \times (1 - N_{i,m,k,t}^{EW}) \quad \forall i \epsilon EW, \forall m \epsilon M, \forall k \epsilon K^{EW}, \forall t \epsilon T \quad (57)$

$NL_{i,m,k,t}^{EW} + NU_{i,m,k,t}^{EW} \leq 1 + M \times N_{i,m,k,t}^{EW} \quad \forall i \epsilon EW, \forall m \epsilon M, \forall k \epsilon K^{EW}, \forall t \epsilon T \quad (58)$

$V_{i,m,k,t}^{EW} \leq N_{i,m,k,t}^{EW} \quad \forall i \epsilon EW, \forall m \epsilon M, \forall k \epsilon K^{EW}, \forall t \epsilon T \quad (59)$

$\sum_k V_{i,m,k,t}^{EW} = 1 \quad \forall i \epsilon EW, \forall m \epsilon M, \forall t \epsilon T \quad (60)$

$X_{i,m,k}^{WE} \leq LL_t + M \times NL_{i,m,k,t}^{WE} \quad \forall i \epsilon WE, \forall m \epsilon M, \forall k \epsilon K^{WE}, \forall t \epsilon T \quad (61)$

$X_{i,m,k}^{WE} \geq LL_t - M \times (1 - NL_{i,m,k,t}^{WE}) \quad \forall i \epsilon WE, \forall m \epsilon M, \forall k \epsilon K^{WE}, \forall t \epsilon T \quad (62)$

$X_{i,m,k}^{WE} \leq LU_t + M \times (1 - NU_{i,m,k,t}^{WE}) \quad \forall i \epsilon WE, \forall m \epsilon M, \forall k \epsilon K^{WE}, \forall t \epsilon T \quad (63)$

$X_{i,m,k}^{WE} \geq LU_t - M \times NU_{i,m,k,t}^{WE} \quad \forall i \epsilon WE, \forall m \epsilon M, \forall k \epsilon K^{WE}, \forall t \epsilon T \quad (64)$

$NL_{i,m,k,t}^{WE} + NU_{i,m,k,t}^{WE} \geq 2 - M \times (1 - N_{i,m,k,t}^{WE}) \quad \forall i \epsilon WE, \forall m \epsilon M, \forall k \epsilon K^{WE}, \forall t \epsilon T \quad (65)$

$NL_{i,m,k,t}^{WE} + NU_{i,m,k,t}^{WE} \leq 1 + M \times N_{i,m,k,t}^{WE} \quad \forall i \epsilon WE, \forall m \epsilon M, \forall k \epsilon K^{WE}, \forall t \epsilon T \quad (66)$

$V_{i,m,k,t}^{WE} \leq N_{i,m,k,t}^{WE} \quad \forall i \epsilon WE, \forall m \epsilon M, \forall k \epsilon K^{WE}, \forall t \epsilon T \quad (67)$

$\sum_k V_{i,m,k,t}^{WE} = 1 \quad \forall i \epsilon WE, \forall m \epsilon M, \forall t \epsilon T \quad (68)$

9- Stop constraints:

$S_{i,m,k}^{EN} + LTP \times V_{i,m,k,t}^{EN} \leq Y_{i,m,k+1-}^{EN} X_{i,m,k}^{EN} \quad \forall i \epsilon EN, \forall k \epsilon K^{EN}, k < k_{d_i}, \forall t \epsilon T \quad (69)$

$S_{i,m,k}^{NE} + LTP \times V_{i,m,k+1,t}^{NE} \leq Y_{i,m,k-}^{NE} X_{i,m,k+1}^{NE} \quad \forall i \epsilon NE, \forall k \epsilon K^{NE}, k < k_{d_i}, \forall t \epsilon T \quad (70)$

$S_{i,m,k}^{EW} + LTP \times V_{i,m,k,t}^{EW} \leq Y_{i,m,k+1-}^{EW} X_{i,m,k}^{EW} \quad \forall i \epsilon EW, \forall k \epsilon K^{EW}, k < k_{d_i}, \forall t \epsilon T \quad (71)$

$S_{i,m,k}^{WE} + LTP \times V_{i,m,k+1,t}^{WE} \leq Y_{i,m,k-}^{WE} X_{i,m,k+1}^{WE} \quad \forall i \epsilon WE, \forall k \epsilon K^{WE}, k < k_{d_i}, \forall t \epsilon T \quad (72)$

3.4. Case study (Numerical example)

In this section, the mathematical model is solved as an example using GAMS software and simulated annealing algorithm. The specifications of a numerical example presented as follow: This problem has two East-West routes including 12 stations and East-North routes including 10 stations. At block number 6, the East-West route and at block number 6, the East-North intersection has occurred. The trains are scheduled for 24hours. Trains start to move at 6am. In the East-North direction there are 2 types of train including 3 numbers and in the North-East direction there are 2 types of trains including 2 numbers and in the East-West direction, there are 2 types of trains including 2 numbers and vice versa. We also considered two emergency stop intervals. If each station selected, 10 minutes is added to the stop time. The problem inputs are shown in Table 1 and 2. The rest of the information generated randomly and the costs per minute are as follows: Z1 = 45, Z2 = 60.

Table 1: Trains priority Coefficient

Geographical directions	trains priority Coefficient		
	Train1	Train2	Train 3
East - North	2.5	1	2.5
North - Eastern	1	1	-
Eastern Western	2	1	2
West - east	2	0.5	2

Table 2: Blocks length in each direction

East-West block length	East-North block length	Station number	East-West block length	East-North block length	Station number
1			7	24	29
2	23	23	8	27	25
3	28	28	9	24	28
4	22	22	10	25	24
5	24	24	11	-	25
6	27	27	12	-	27

Table 3: Trains speed

Geographical directions	Train type	Train 1		Train 2		Train 3	
East-North	1	50	70	43	60	53	60
	2	42	75	47	65	54	65
North east	1	50	56	37	55	-	-
	2	47	70	48	50	-	-
East- West	1	49	56	51	60	37	45
	2	64	70	58	65	62	70
West-east	1	47	65	33	45	50	61
	2	53	65	50	61	44	55

Table 4: The Model results for each objective function

Solution time	GAMS solution	Objective function
35.07	1724	First objective function
29.83	19728	Second objective function

3.4.1. Result of model solution using GAMS and SA algorithm

In this section, the problem is solved by changing the number of trains on each route and keeping the stations constant, using random data.

Table 5: Random data for model solution

Speed	Random number between 30 and 65
The distance between the two blocks	Random number between 20 to 30
stop time	Random number between 0 and 0.3
Emergency stop time	0.2
Priority coefficients	Randomly selected between 1, 1.5, 2, 2.5, 3 and 3.5.
Cost of stops	45
The cost of being excessive mobile	65

Table 6: Stations and Geographical directions

EN	NE	EW	WE	EN Stations	EW Stations
1	1	1	1	6	6
2	2	2	2	6	6
4	4	4	4	6	6
6	6	6	6	6	6
8	8	8	8	6	6
10	10	10	10	6	6
12	12	12	12	6	6
14	14	14	14	6	6
16	16	16	16	6	6
18	18	18	18	6	6
20	20	20	20	6	6

Table 7: Comparison the results of SA algorithm and GAMS software with increasing trains in each route

Simulated Annealing	SA time (Second)	GAMS	GAMS (Second)	Difference Percentage
2896	0.14	2896	0	0
6270	1.98	6259	254	0.175747
7016	3.89	7002	412	0.199943
11208	5.05	11179	843	0.259415
17512	7.08	17123	1002	2.271798
26196	8.84	-	-	-
36172	12.47	-	-	-
46182	15.59	-	-	-
59160	20.26	-	-	-
75725	26.96	-	-	-
108524	37.75	-	-	-

Figure 2: Solution time changes based on increasing the number of trains

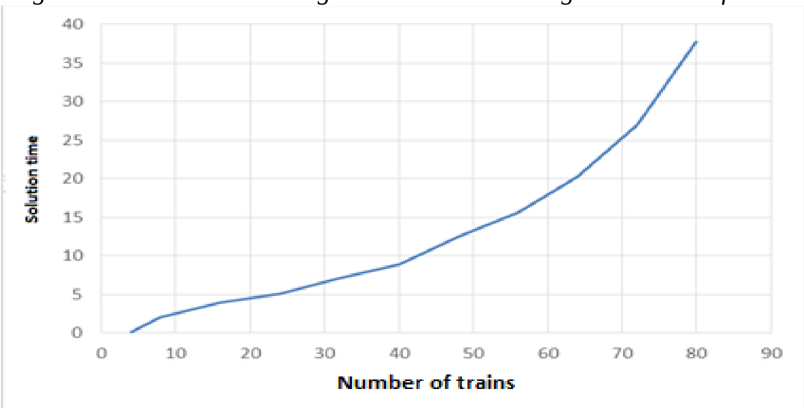
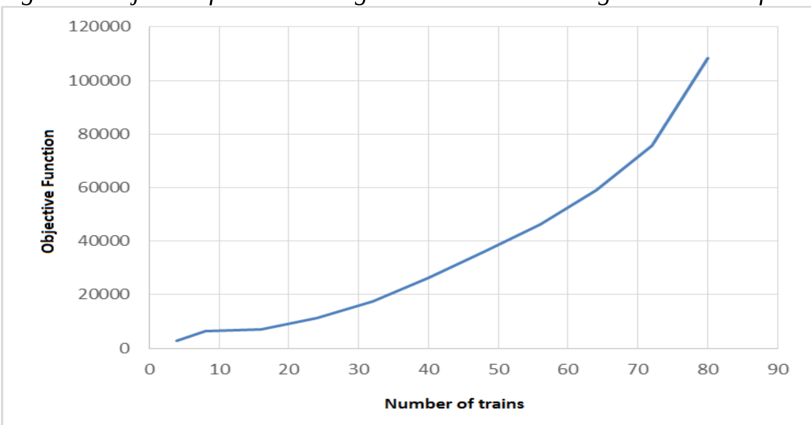


Figure 3: objective function changes based on increasing the number of trains



Solving the problem is very time consuming due to the large number of constraints, meanwhile with the small number of trains using the exact method, solving the problem with 11 trains takes more than 78 hours, while the solution time with the proposed algorithm is much lower. The solutions obtained by solving the model using both exact and meta-heuristic approaches in this paper and results show a difference of less than 1% for both methods, which due to the large size of the problem and long solution time, this value can be valid.

4. Result and discussion

In this paper, many studies from different years have been investigated on the timetabling based model. Hence the mathematical model with assumptions and variables were developed and solved based SA solution approach. One of the indicators which reveal transportation system quality is getting to the station without any delay. In this research, we enhanced transportation system efficiency by minimizing the train arrival time to destination point as well as reducing the stop costs. On the other hand the costs associated with delays and departures from stations, as well as fuel consumption are transportation managers concerns that included in the second objective function for each train with priority coefficient. In the extended model, the trains speed variable per block, the minimum start time, emergency stop situations and the consideration of the prayers time interval are taken into account. Other constraints are considered to prevent the collisions of opposite directions and the intersection of the cross blocks. In this paper, the mathematical model developed, afterward the small size model is solved by the GAMS software. Due to the number of variables and constraints the problem cannot be solved for real size by the exact method. With respect to the lack of standard problem where all the specifications of the problem are fully provided, the results of the algorithm were compared with those obtained from the mathematical model. Therefore the model was solved using a SA algorithm, then the results were compared for small dimensional problem with the GAMS software, and the results of the two methods were not significantly different. According to the model's solution, by increasing the number of trains as well as the number of stations, the exact solution time increases tremendously, while with similar examples, the model solution time is much less than the proposed algorithm. In this paper proposed scheduling model is extended based on previous research and proper constraints have been added to the model, hence the model validation has been approved by solving exact and meta-heuristic methods.

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