Hurst Methods for Fractal Analysis of Electrocardiographical Signals

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Abstract

This article is devoted to the fractal analysis of the intervals between heart beats (RR intervals) obtained from electrocardiographical signals. The following methods are used to determine the fractal behavior of the studied signals by the Hurst exponent: Rescaled range, wavelet method, Detrended Fluctuation Analysis. The Hurst exponent value determined by the proposed methods depends on a number of factors: the estimation method, the size of the data, the type of wavelet function, etc. To solve the problem associated with finding the optimal Hurst method, fractal Gaussian noise (FGN) was simulated with different inputs of the Hurst exponent (0.6, 0.7, 0.8, 0.9) and with different data lengths (1000, 10000, 100000). The testing results of the accuracy of the Hurst exponent when applying those three methods is that at a data length of 100000 points, the relative error of the Hurst exponent is the smallest. The Detrended Fluctuation Analysis and wavelet method for estimating the Hurst exponents with respect to the precision parameter have a relative error of less than 1.4%. These two methods have been applied to examine the Holter recordings of two groups of people: healthy and unhealthy subjects. The results show that the Hurst values in healthy and diseased individuals differ. Another marker used to distinguish between the two groups is the generalized Hurst exponent, with diseased subjects having monofractal behavior and healthy subjects-multifractal. In the conclusion, based on the obtained results, it follows that fractal analysis is appropriate for estimating the fuction state of the human body.

Keywords: Detrended Fluctuation Analysis, MultiFractal Detrended Fluctuation Analysis, Fractal Gaussian Noise, Rescaled range method, wavelet-based method.

1. Introduction

A number of studies conducted in recent years show that many systems in nature generate time series with complex, fractal behavior. To study these processes, approaches based on methods for the statistical analysis of random variables and functions are most commonly used. These traditional methods of analysis define characteristics such as mathematical expectation, variance, autocorrelation functions, spectral densities, and more. Along with these methods of analysis, some less well-known methods of signal analysis, such as fractal methods (Pilgrim&Taylor, 2018), have become spread in recent years. Their distinguishing feature is that together with the global characteristics of the studied processes, they allow to reveal the peculiarities of their local structure. An important feature of fractal methods of analysis is that they are fundamental and can be applied to different types of time series, such as: packet data arrival times on the Internet (Feldmann et al., 1998), (Sheluhin et al., 2007); time series between successive heart beats (Peng at al., 1996), (Ernst, 2014); economic data (Naiman, 2009); electricity prices (Cheng&Jiao, 2019); geophysical data (Dimri, 2005), etc.

Time series with fractal behavior are characterized by the following basic properties: irregularity, self-similarity, fractal dimension. The assessment of the degree of self-similarity (fractality) plays an important role in the study of processes having fractal properties. The degree of self-similarity is determined by the Hurst exponent and is an indicator that determines the complexity of the dynamics and the correlation function of the time series. The Hurst exponent accepts values in the range 0 to 1 that show the following (Mielniczuk&Wojdyllo, 2009):

- H = 0.5 means that the time series is received by random and independent (uncorrelated) increments. This is the so-called "white noise", which is characterized by maximum randomness and minimal predictability. The correlation determined by the formula: $C=2^{2h-1}-1$, is zero, indicating that the current state does not affect the future and lacks long-range dependence.
- At $0 \leq H < 0.5$, the time series is called "pink noise". If the time series has increased in the previous period, it is likely that it will decrease in the next period and vice versa. This behavior is anti-persistent. The closer H is to 0, the closer the correlation measure is to -0.5.
- If $0.5 < H < 1$, the time series is called "black noise" and has a long-term memory. If the value of the Hurst exponent is greater than 0.5 and if the time series increases or decreases in the previous period, it is more likely that it continues to do so in the next period. When H tends to 1, the noise becomes smaller and the order approaches a state of determination and full predictability. At H=1, the measure of correlation is C=1.

Another basic parameter characterizing fractal series is the fractal dimension, which takes fractional values (Mandelbrot, 1967). Determining the fractal dimension of a signal is important in examining their fluctuations in order to obtain information on long-term correlations and, accordingly, to predict the behavior of these series. Mandelbrot establishes a relation between the fractal dimension (D) and the Hurst exponent (H), which is: D = 2-H (Mandelbrot, 1967).

The analysis of fractal properties of time series is one of the promising areas for data analysis. Applying this type of analysis to the dynamic series of cardio intervals (distances between the R-tips of digital electrocardiograms) is an objective and non-invasive way of obtaining information about the functional state of the human body (Ernst, 2012). The analysis of the RR interval series using statistical methods is based on the assumption that the dynamic range of cardio intervals, reflecting a random process, is subject to normal (Gaussian) distribution and is stationary. The necessary conditions for stationarity are: independence of the mean and the dispersion from the time parameter, as well as dependence of the autocorrelation functions on the difference in time moments. The dynamic order of cardio intervals does not always meet the stationary requirements, even when the electrocardiogram is taken at rest. Filtering the non-stationary component of cardio intervals can result in the loss of important information related to the patient's health. In the traditional mathematical analysis there are statistical parameters such as skewness and kurtosis, which allow us to determine only the presence of non-stationarity in the studied process (Rivera et al., 2016). These parameters have low information content. More promising in the study of dynamic changes in the RR series are the nonlinear methods that take into account not only the stationary but also the non-stationary components (fractal components). Fractal analysis methods can evaluate functional and pathological changes in the human body, as well as predict changes in patients' health. These methods are considered to be informative in assessing the non-stationarity of the studied process and in determining the complexity of its dynamics (Ernst, 2014), (Golińska, 2012), (Rivera et al., 2016), (Peng et al., 1996) .

The purpose of this article is to make a comparative analysis of the following 3 methods: the R/S method, the wavelet method based on the Daubechies algorithm (4, 8, 10, 12, and 20 coefficients), and the Detrended Fluctuation Analysis (DFA), to evaluate the H exponents in terms of parameter accuracy. These methods will be applied to the simulated time series based on fractal Gaussian noise (FGN) of different lengths (1000, 10000,100000 data points) with preset values of the H exponents (0.6, 0.7, 0.8, 0.9). The highest accuracy methods will be applied for fractal analysis of real electrocardiological data: 24-hour holter recordings of 2 groups of people: healthy controls and patients with arrhythmia. The MultiFractal Detrended Fluctuation Analysis (MFDFA) method will be used to additionally differentiate the two study groups by determine the generalized Hurst exponent.

2. Data and Methods

2.1. Data

The article are used 2 types of data: simulated and real. The simulated data are used to determine the optimal methods to estimate the Hurst exponents with respect to the precision parameter.

2.1.1. Simulation of fractal (self-similar) process, based on the FGN

One of the known models of stochastic dynamics with fractal properties is FGN. In order to determine the optimal method for estimating the Hurst parameter, FGN-based fractal processes are simulated with given Hurst parameter inputs.

The article uses an algorithm proposed by Paxon and Floyd (Paxson&Floyd, 1997) to simulate FGN-based fractal processes. FGN is a process of incremental of the fractal Brownian motion:

$$X_k = B_H(k+1) - B_H(k). \quad (1)$$

Where X_k there is a normal distribution for each value of k.

The power spectrum *f (λ, H)* of the FGN process is related to the Hurst exponent (is related to the exponent H) and is determined by the following equation (Paxson&Floyd, 1997):

$$f(\lambda,H) = A(\lambda,H)\left[|\lambda|^{-2H-1} + B(\lambda,H)\right] \quad 0<H<1 \text{ and } -\pi<\lambda<\pi \quad (2)$$

Where:

$$A(\lambda,H)=2\sin(\pi H)\Gamma(2H+1)(1-\cos\lambda) \quad (3)$$

$$B(\lambda,H) = \sum_{j=1}^{\infty}\left[(2\pi j+\lambda)^{-2H-1} + (2\pi j-\lambda)^{-2H-1}\right] \quad (4)$$

Γ(.) - Gamma function

The major difficulty in calculating the power spectrum of FGN is related to the calculation of the infinite sum in the expression *B (λ, H)*. The algorithm offers an accurate calculation of the first three expressions of the infinite sum and approximates the other expressions as follows:

$$B(\lambda,H) = B_{1:3}(\lambda,H) + B_{4:\infty}(\lambda,H) \quad (5)$$

Where:

$$B_{1:3}(\lambda,H) = \sum_{j=1}^{3}(2\pi j+\lambda)^{-2H} + (2\pi j-\lambda)^{-2H} \quad (6)$$

$$B_{4:\infty}(\lambda,H) \approx \frac{1}{8H\pi}\sum_{j=3}^{4}(2\pi j+\lambda)^{-2H} + (2\pi j-\lambda)^{-2H} \quad (7)$$

The basic steps of the algorithm are the following:

Step 1: A sequence of values $\{f_1,f_{2,\dots,}f_{n/2}\}$ is created where $f_j = \hat{f}(\frac{2\pi j}{n},H)$ corresponds to the power spectrum of FGN for frequencies from *2π /n* to *π*.

Step 2: Noise spectrum is created by multiplying each value of the generated sequence *{fi}* by an exponentially distributed random variable with an average of one.

Step 3: The spectrum of complex numbers $\{z_1,z_2,\dots,z_{n/2}\}$ is created with module $|z_i| = \sqrt{\hat{f}_i}$ and argument uniformly distributed in the interval (0, 2π). Using a random argument makes the generated sequence normally distributed, which is a requirement for FGN.

Step 4: The sequence $\{z_0^1, z_1^1, ..., z_{n-1}^1\}$ is created, which an extended version is of $\{z_1, z_2, ..., z_{n/2}\}$ under the following conditions:

$$z_j^1 = \begin{cases} 0 & \text{if } j = 0; \\ z_j & \text{if } 0 < j \le n/2; \\ \overline{z_{n-j}} & \text{if } n/2 < j < n. \end{cases} \quad (8)$$

Step 5: In order to represent the generated sequence from frequency in the time domain, the inverse Fourier transform is used.

2.1.2. Real cardiological data

The real cardiological data are 24-hour Holter records (RR intervals) that consist of approximately 100 000 RR intervals. The RR interval series of two groups of people were studied: 25 healthy controls and 25 unhealthy subjects (patients with arrhythmia). The cardiological data were obtained from a cardiology hospital in connection with the implementation of the project "Investigation of the application of new mathematical methods for the analysis of cardiac data", funded by the National Science Fund of Bulgaria.

2.2. Hurst methods

The following methods for fractal analysis are used in this work: R/S method, wavelet-based method, DFA method and MFDFA method. The MFDFA method is applied only to investigate whether RR time series have mono- or multi-fractal behavior by determining the generalized Hurst exponent.

2.2.1. R/S method

One of the popular methods for time series analysis based on the calculation of the Hurst exponent is the Rescaled Range Analysis (R/S) method. The algorithm of the method is as follows (Sheluhin et al., 2007):

1. The time series is divided into d sub-series of length n. For each sub-series $m=1,...,d$ are calculated:

- The range $R(m)$, which is defined as the difference between the maximum and minimum value of the sum of the deviation W_j from the arithmetic mean of the data for an area of n.

$$R(m) = max(0, W_1, W_2, ..., W_n) - min(0, W_1, W_2, ..., W_n) \quad (9)$$

where $W_j = (X_1 + X_2 + ... + X_j) - j\overline{X(n)}$ $j=1,2,...,n$.

- Standard deviation: $S(m) = \sqrt{E(X_j - \mu)^2}$, where μ is the average value of $X_1, X_2, ..., X_j$.
- Rescale the range $R(m)/S(m)$.
- The mean value $(R/S)_n$ of the rescaled range for all sub-series of length n. Therefore, the statistics is defined as:

$$\left(\frac{R}{S}\right)_n = \left[\frac{1}{d}\sum_{m=1}^{d}\frac{R(m)}{S(m)}\right] \approx c\,n^H \tag{10}$$

where c is a constant.

2. The log-log diagram dependence of $(R/S)_n$ on n represents a line approximated by the least square method. The exponent H is the slope of the regression line which represents the dependence $log\left(\frac{R}{S}\right)_n$ on *log (n)*.

The R/S method is restricted to stationary processes, such as FGN.

2.2.2. DFA method

The DFA method (Peng et al., 1994), is suitable for estimating the H parameter for fractal time series. Fractal series are characterized by the property of scale invariance, that is, they appear to be approximately the same at different scales (Peng et al., 1995). By this method, the cumulative series is constructed for the initial time series x (t): $y(t) = \sum_{i=1}^{n} x(t)$. This series is decomposed into N segments of length s. The fluctuation function is calculated for each segment:

$$F^2(s) = \frac{1}{s}\sum_{t=1}^{s}\left(y(t) - Y_m(t)\right)^2 \tag{11}$$

Where $Y_m(t)$ – local m-polynomial trend for a given segment.

The function *F(s)* is averaged over the entire time series *y(t)*. Such calculations are repeated for different segments sizes in order to obtain the dependency *F(s)* in a wide range for the parameter *s*. For processes with fractal properties with increasing the value of parameter *s*, the function *F(s)* also increases. If the dependence of *logF (s)* on *log (s)* is linear, this indicates the presence of the property the scale invariance, i.e. the condition is fulfilled:

$$F(s) \sim s^{\alpha} \tag{12}$$

When $\alpha \le 1$, the value of alpha (α) is the value of the Hurst exponent.

2.2.3. Generalized Hurst exponent

For quantitative description of fractal series, only one Hurst exponent value (or fractal dimension), which characterizes the invariance of the degree of fractality (self-similarity) as the time scale changes, is sufficient. At the same time, there are time series that are non-homogeneous, and for their complete description, not only one value of the Hurst exponent is sufficient, but a spectrum of Hurst values. The MFDFA method (Kantelhardt et al., 2004), (Biswas et al., 2012) may be used to study such time series. This method determines the dependence of the fluctuation function *Fq (s)* on the parameter *q*:

$$F_q(s) = \left\{\frac{1}{N}\sum_{i=1}^{N}\left[F^2(s)\right]^{\frac{q}{2}}\right\}^{\frac{1}{q}} \tag{13}$$

Changing the time scale s at a fixed value of q is found the fluctuation function *Fq(s)*. If the investigated time series has multifractal behavior, then the fluctuation function *Fq(s)* has the following form:

$$F_q(s) \sim s^{h(q)} \quad (14)$$

Where *h(q)* is called the generalized Hurst exponent. It follows from Eq.11 and Eq.13 that for $q = 2$, this indicator reduces to a simple value of Hurst exponent.

Multifractals are heterogeneous fractal processes, for the complete description of which a spectrum of Hurst exponents (fractal dimensions) are required, while ordinary fractal processes (monofractals) are described with only one value of the Hurst exponent.

2.2.4. Wavelet-based method

An effective tool for the study of non-stationary time series is wavelet analysis, which allows to determine the structural components of the order (Kirichenko et al., 2011). In particular, methods based on discrete wave transformations allow one to estimate the degree of self-similarity by determining the Hurst exponents of the non-stationary time series ((Sheluhin et al., 2007).

The stochastic process *X(t)* with the Hurst exponent (*H*) is self-similar if the process $a^{-H}X(at)$ is described with the same distribution as $X(t)$: $Law\{X(t)\}=Law\{a^{-H}X(at)\}$, for each $a>o$ and $t>0$. In this case, the method of estimating the degree of self-similarity is based on the properties of the detailed wavelet coefficients obtained as a result of the decomposition of the study signal. If the random process *X(t)* is self-similar, then the detailed coefficients at each decomposition level also possess the self-similarity property and the condition is fulfilled:

$$Law\{det(j,k)\}=Law\{2^{j(H+1/2)}det(0,k)\} \quad (15)$$

where:

- $det(j,k)$- k[th] detail coefficient at decomposition level j, $k=1,2,\ldots N_j$;
- N_j- the number of wavelet coefficients at the decomposition level j;
- H –Hurst exponent.

The algorithm for determining the Hurst exponents by the wavelet-based method is as follows:

1. A discrete wavelet transformation is performed according to the studied process using a wavelet algorithm and the values of E_j are determined by the following formula:

$$E_j = \frac{1}{N_j}\sum_{k=1}^{N_j} det^2(j,k) \quad (16)$$

2. The graphical dependence of $log_2(E_j)$ on j is constructed. If the relationship between the two variables is linear, then the process under study is self-similar.
3. The slope of the straight line β is determined by the least squares method. The slope β has a value between 0 and 1.
4. The Hurst exponent is defined by the formula: $H=1/2(1+\beta)$.

The choice of wavelet algorithm is an important issue related to the implementation of a discrete wave transformation. In this work, the Daubeshies algorithm with 4, 8, 10, 12 and 20 wavelet coefficients is used.

3. Results and Discussion

The all results in this article were obtained on Matlab.

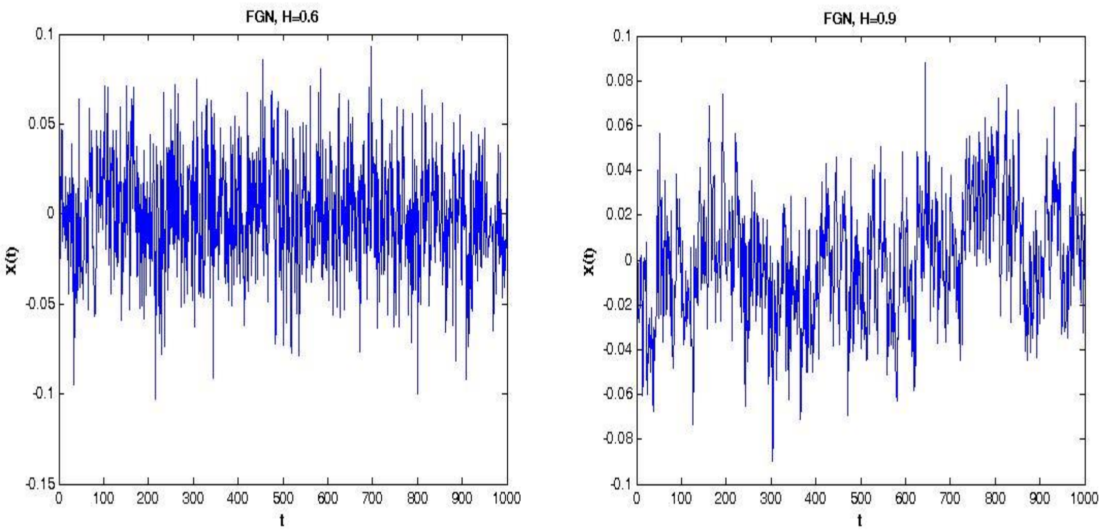
3.1. FGN simulation

FGNs are used to simulate fractal series with the following input values:

- H= 0.6, 0.7, 0.8 and 0.9;
- Simulated sequence length: 1 000, 10 000 and 100 000 data points.

Fig. 1 shows the graphs of simulated FGN time series by applying the above algorithm at 2 Hurst exponent values: H = 0.6 and H = 0.9 with the duration of the simulated processes: 1000 data points.

Figure 1: Simulating FGN processes with H=0.6 and H=0.9



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3.2. Comparative analysis of Hurst methods

Based on the simulated fractal series, a comparative analysis of the following three Hurst exponent estimation methods is made: R/S, DFA, and a wavelet method with the Daubeshies wavelets of order 4, 8, 10, 12 and 20. These methods are applied to simulated time series (FGNs) of different lengths (1000, 10 000 and 100 000 data points) and with preset Hurst exponent values (H = 0.6; H = 0.7; H = 0.8; H = 0.9). The analysis is made with respect to the precision parameter by determining the relative error of the Hurst exponent with the following formula:

$$Relative\ error\ [\%] = \frac{\hat{H} - H}{H} * 100\% \qquad (17)$$

Where: H is the input value of the Hurst parameter, and $\hat{H}$ is the determined value by the statistical methods.

The results are shown in Table1 as mean± td, where mean is the Hurst average of 10 simulated FGN time series, std is the standard deviation and the relative error(%) of the Hurst parameter determined by Eq.17.

From the comparative analysis of the 3 methods, it follows:

- As the length of the simulated process increases, the relative error of the Hurst parameter decreases in all three methods tested, being the smallest at a time series length of 100 000 data points;
- The relative error is less than 1.4% for the DFA method and less than 1.1% for the wavelet-based method with the Daubeshies wavelets of order 10 for a simulated process length of 100 000 data points for the four Hurst parameter values tested.

Based on this comparison in the present work are selected the DFA method and the wavelet-based method for the analysis of real cardiac data for the following reasons:

- These are methods with a small relative error in determining the Hurst exponent.
- These two methods are suitable for the study of both stationary and non-stationary time series, such as cardiac RR time series, while the R/S method is suitable for the study only of stationary time series.

Table 1: Comparative analysis of the investigated methods for estimating the Hurst exponent

Method	**Estimated Hurst exponent**	**Theoretical Hurst exponent**			
		H=0.6	H=0.7	H=0.8	H=0.9
R/S method	time series of 1000 points				
	mean±std	0.646±0.075	0.722±0.071	0.828±0.073	0.846±0.075
	relative error %	7.67 %	3.11 %	3.48 %	-6.05 %
	time series of 10 000 points				
	mean±std	0.620±0.061	0.718±0.062	0.782±0.059	0.861±0.069
	relative error %	3.37 %	2.57 %	-2.25 %	-4.41 %
	time series of 100 000 points				
	mean±std	0.622±0.058	0.714±0.051	0.819±0.053	0.877±0.050
	relative error %	3.33 %	2.0 %	2.37 %	-2.56 %
DFA method	time series of 1 000 points				
	mean±std	0.626±0.040	0.716±0.039	0.822±0.038	0.916±0.030
	relative error %	4.37 %	2.319 %	2.71 %	1.81 %
	time series of 10 000 points				
	mean±std	0.614±0.037	0.71±0.035	0.81±0.032	0.908±0.033
	relative error %	2.41 %	1.40 %	0.80 %	0.88 %
	time series of 100 000 points				
	mean±std	0.608±0.026	0.707±0.021	0.807±0.019	0.907±0.017
	relative error %	**1.33 %**	**1.02 %**	**0.88 %**	**0.77 %**
W ave	time series of 100 000 points and db4				
	mean±std	0.630±0.033	0.714±0.024	0.819±0.027	0.912±0.022

	relative error %	5.05 %	1.94 %	2.38 %	1.33 %
	time series of 100 000 points and db8				
	mean±std	0.591±0.012	0.674±0.019	0.776±0.018	0.871±0.020
	relative error %	-1.59 %	-3.64 %	-2.97 %	-3.26%
	time series of 100 000 points and db10				
	mean±std	0.606±0.009	0.697±0.004	0.791±0.008	0.896±0.007
	relative error %	**1.08 %**	**-0.43 %**	**-1.13 %**	**-0.49 %**
	time series of 100 000 points and db12				
	mean±std	0.619±0.025	0.689±0.029	0.787±0.019	0.870±0.016
	relative error %	3.15 %	-1.57 %	-1.62 %	-1.52 %
	time series of 100 000 points and db20				
	mean±std	0.583±0.023	0.682±0.018	0.779±0.017	0.879±0.016
	relative error %	-2.90 %	-2.50 %	-2.58 %	-2.33 %

3.3. Analysis of electrocardiological data

The following two types of analysis are performed to study real RR time series for healthy subjects and unhealthy subjects (patients with arrhythmia) obtained from 24-hour Holter recordings:

1. Fractal analysis of the RR time series, by determining the Hurst exponents with the methods: DFA and wavelet-based Hurst estimator with the Daubeshies wavelets of db10.
2. Study of the analyzed groups, whether they have monofractal or multifractal properties, by determining the generalized Hurst exponents using the MFDFA method.

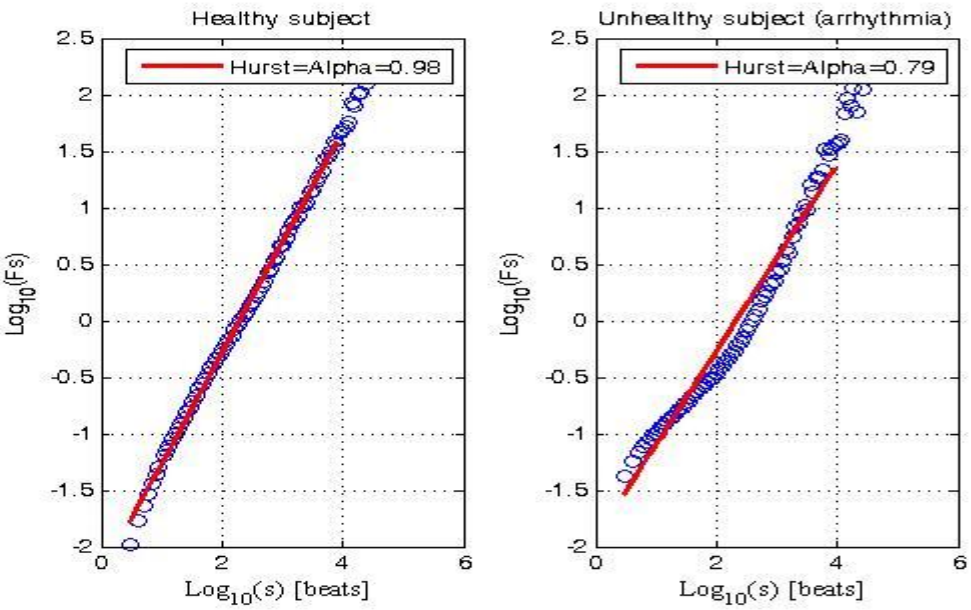
3.3.1. Fractal analysis

In Fig. 2 and Fig. 3 are showed the graphs of the RR time intervals studied for a healthy subject and unhealthy subject using the DFA and wavelet-based Hurst methods. The Hurst parameter is equal to the alpha parameter value determined by the DFA method. In Table 2 are showed the Hurst parameter values for the two types of signals tested using these methods. Both methods produce identical results, such as for unhealthy subjects, the value of this parameter is lower than that of the healthy controls. The determined values of the Hurst parameter with both methods are statistically significant, since the p-value is less than 0.0001. Therefore, these two methods can distinguish healthy from unhealthy subjects.

Table 2: Hurst methods for analysis of healthy subjects (Group1) and unhealthy subjects (Group 2)

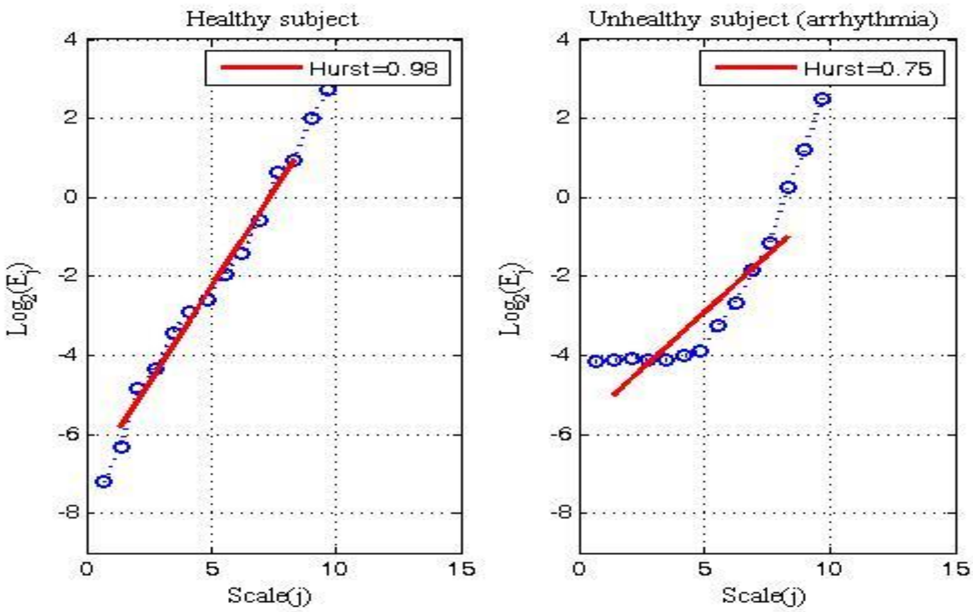
Method	**Group1 (n=25)**	**Group 2 (n=25)**	**p-value**
	mean±std	mean±std	
DFA	0.97±0.07	0.71±0.05	<0.0001
Wavelet-based	0.91±0.03	0.68±0.04	<0.0001

Figure 2: DFA analysis of the RR time series



Source: (Authors)

Figure 3: Wavelet-based Hurst estimator for analysis of the RR time series



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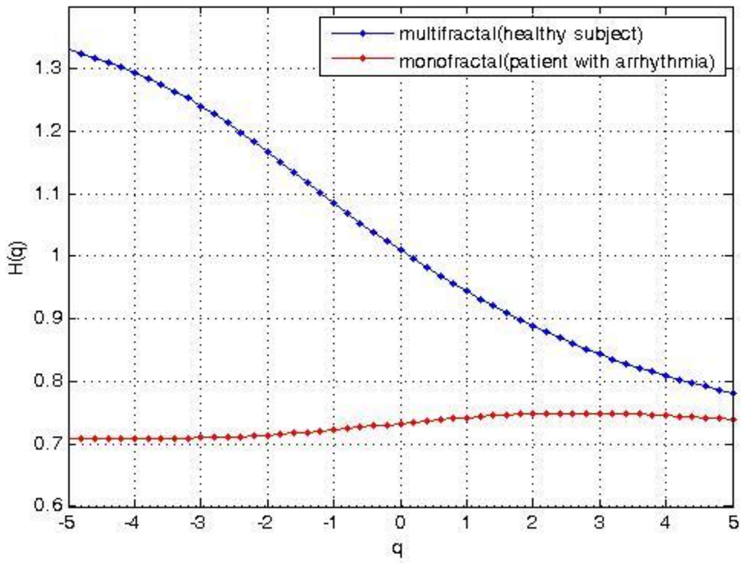
3.3.2. Multifractal analysis using generalized Hurst exponents

The multifractal concept provides additional information to the fractal analysis of complex stochastic processes, such as actual cardiac data. The multifractal analysis applied to the RR time series is done by determining the generalized Hurst exponents using the MFDFA method.

Fig. 4 show how the Hurst exponents change depending on the *q* parameter for healthy subject and unhealthy subject (with arrhythmia).

Figure 4: Generalized Hurst exponent vs. q for the RR time series



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The behavior of a healthy subject is multifractal because the Hurst exponent value changes significantly (from 1.32 to 0.78) with the change of the *q* parameter. The Hurst value for the unhealthy subject is almost constant when *q* changes, indicating that this signal has monofractal behavior. The behavior of the two types of signals studied is different and can be used to distinguish them.

Conclusion

In this study are showed the numerical results of the comparative analysis of three different methods of estimating the Hurst exponent: R/S method, DFA method and wavelet-based method. These methods are applied to simulated time series based on FGN of different lengths. It follows from the analysis that the three methods are highly dependent on the length of the time series, with the relative error of the Hurst parameter as well as their standard deviations decreasing as their length increases. The time series with length of 100 000 data points, the relative error at the DFA is less than 1.4%, and at the wavelet-based estimator with the Daubeshies wavelets of order 10, the error is less than 1.1%. These two methods, due to their small relative error, are suitable for examining real 24-hour Holter records containing about 100 000 RR time intervals.

Summarizing the results of the fractal analysis of electrocardiological data, it follows that this type of analysis is a useful tool for obtaining additional information about the health status of the subjects studied. In the study of healthy subjects and patients with arrhythmia, the measured Hurst parameter values support the usefulness of this type of analysis to distinguish healthy from unhealthy individuals. In addition to the fractal analysis of the RR time series of healthy controls and patients with arrhythmias, generalized Hurst exponents were analyzed by applying the MFDFA method. The results show that healthy controls have multifractal behavior and patients with arrhythmia-monofractal. The monofractal behavior as determined by of the generalized Hurst exponent in patients with arrhythmia reveals a loss of complexity (ie, less nonlinearity) of the RR time series, which confirms the hypothesis of a disease state.

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