Method of order reduction in second order differential equations line with GeoGebra

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Abstract.
Various authors have released the wonderful animated applets that can be built from Several authors have shown the wonderful animated applets that can be built from certain content for teaching and learning, as for example Tom Walsh (1), Dan MacIsaac (2). These authors in their work tell us how friendly it is to have GeoGebra applets in favoring the understanding of the content that one intends to teach. In this papers, the GeoGebra applets will show how the second algebraic and graphical solution of a second order linear equation is, when order reduction is applied, which already has a known solution. Reducing the order itself is of great help in ODEs, since in some cases it is not so easy to calculate the solutions of the homogeneous part of an ED, so the GeoGebra software gives us all its potential and dynamism to to be able to visualize these solutions through their applets and instruction commands.

All these applets were built as teaching support material for the differential equations course of engineering careers at the University of Antofagasta, during the year 2020.

Keywords: applets; differential equation; GeoGebra; order reduction; teaching.

1. Introduction
As we mentioned at the beginning the authors Tom Walsh (1), Dan MacIsaac (2). They show us the advantages of using GeoGebra, especially Tom Wash (1) who tells us that GeoGebra allows us to have “dynamic mathematics for teaching and learning ” Thus, we will visualize the solutions obtained by using order reduction to second order linear differential equations using GeoGebra applets.

This paper is a continuation and extension of animations and interactive creations in first order linear differential equations: the case of GeoGebra (3)

2. GeoGebra and Order reduction in differential equations
Below are several examples of the solutions obtained by order reduction and by GeoGebra applets, available at https://www.geogebra.org/m/w5vsmxrk
Example 1

Be

\[ \frac{d^2 y}{dx^2} - y = 0, \ y_1 = e^x \]

In figure 1 the second solution obtained by reduction of order \( e^{-x} \) is given in red and the general solution in blue. Where \( c_1 \) and \( c_2 \) move between -5 and 5.

![Figure 1](image1.png)

Example 2

Be

\[ \frac{d^2 y}{dx^2} \cdot 9y = 0, \ y_1 = e^{3x} \]

In figure 2 the second solution obtained by reduction of order \( e^{-3x} \) is given in red and the general solution in blue. Where \( c_1 \) and \( c_2 \) move between -5 and 5.
Example 3

Be

\[ \frac{d^2y}{dx^2} - 16y = 0, \quad y_1 = e^{4x} \]

In figure 3 the second solution obtained by reduction of order \( e^{-4x} \) is given in red and the general solution in blue. Where \( c_1 \) and \( c_2 \) move between -5 and 5.

Example 4

Be

\[ \frac{d^2y}{dx^2} - 25y = 0, \quad y_1 = e^{5x} \]
In figure 4 the second solution obtained by reduction of order \( e^{-5x} \) is given in red and the general solution in blue. Where \( c_1 \) and \( c_2 \) move between -5 and 5.

Example 5

Be

\[
\frac{d^2y}{dx^2} - 36y = 0, \quad y_1 = e^{6x}
\]

In figure 5 the second solution obtained by reduction of order \( e^{-6x} \) is given in red and the general solution in blue. Where \( c_1 \) and \( c_2 \) move between -5 and 5.
Example 6

Be

\[ \frac{d^2y}{dx^2} + 169y = 0, \quad y_1 = \text{sen}13x \]

In figure 6 the second solution obtained by reduction of order cos13x is given in red and the general solution in blue. Where \( c_1 \) and \( c_2 \) move between -5 and 5.

![Figure 6](image)

Example 7

Be

\[ \frac{d^2y}{dx^2} + 9y = 0, \quad y_1 = \text{sen}3x \]

In figure 7 the second solution obtained by reduction of order cos3x is given in red and the general solution in blue. Where \( c_1 \) and \( c_2 \) move between -5 and 5.
Example 8

Be

$$\frac{d^2y}{dx^2} + 100y = 0, \quad y_1 = \sin(10x)$$

In figure 8 the second solution obtained by reduction of order $\cos(10x)$ is given in red and the general solution in blue. Where $c_1$ and $c_2$ move between -5 and 5.

Example 9

Be

$$\frac{d^2y}{dx^2} + y = 0, \quad y_1 = \sin(x)$$

In figure 9 the second solution obtained by reduction of order $\cos(x)$ is given in red and the general solution in blue. Where $c_1$ and $c_2$ move between -5 and 5.
Example 10

Be

\[
\frac{d^2y}{dx^2} + \frac{y}{4} = 0, \quad y_1 = \text{sen}(x/2)
\]

In figure 10 the second solution obtained by reduction of order \(\cos(x/2)\) is given in red and the general solution in blue. Where \(c_1\) and \(c_2\) move between -5 and 5.
Example 11

Be

\[
\frac{d^2y}{dx^2} + \frac{y}{16} = 0, \quad y_1 = \text{sen}(x/4)
\]

In figure 11 the second solution obtained by reduction of order \(\cos(x/4)\) is given in red and the general solution in blue. Where \(c_1\) and \(c_2\) move between -5 and 5.

![Figure 11](image)

Example 12

Be

\[
\frac{d^2y}{dx^2} + \frac{y}{81} = 0, \quad y_1 = \text{sen}(x/9)
\]

In figure 12 the second solution obtained by reduction of order \(\cos(x/9)\) is given in red and the general solution in blue. Where \(c_1\) and \(c_2\) move between -5 and 5.
Example 13

Be

\[ \frac{d^2y}{dx^2} + \frac{y}{49} = 0, \quad y_1 = \sin\left(\frac{x}{7}\right) \]

In figure 13 the second solution obtained by reduction of order \(\cos(x/7)\) is given in red and the general solution in blue. Where \(c_1\) and \(c_2\) move between -5 and 5.
Example 14

Be

\[ x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0, \quad y_1 = \ln x \]

By order reduction, the second solution is \( y_2 = -1 \) which is given in red in Figure 14.

Where the delinkers \( c_1, c_2 \) move between -5 and 5 in the general solution \( y_2 \).
Example 15

Be

\[ \frac{d^2y}{dx^2} + \frac{y}{256} = 0, \quad y_1 = \sin(x/16) \]

In figure 15 the second solution obtained by reduction of order \( \cos(x/16) \) is given in red and the general solution in blue. Where \( c_1 \) and \( c_2 \) move between -5 and 5.

Example 16

Be

\[ \frac{d^2y}{dx^2} + \frac{y}{100} = 0, \quad y_1 = \sin(x/10) \]

In figure 16 the second solution obtained by reduction of order \( \cos(x/10) \) is given in red and the general solution in blue. Where \( c_1 \) and \( c_2 \) move between -5 and 5.
Example 17

Be

\[ \frac{d^2y}{dx^2} + \frac{y}{10000} = 0, \quad y_1 = \sin\left(\frac{x}{100}\right) \]

In figure 17 the second solution obtained by reduction of order \( \cos(x/100) \) is given in red and the general solution in blue. Where \( c_1 \) and \( c_2 \) move between -5 and 5.

3. Conclusion

Our purpose in this work has been to share the importance of using GeoGebra to see the solutions obtained by order reduction in differential equations.
All these applets designed by GeoGebra correspond to teaching support material, help for students in their learning, these applets that are easy to access can be viewed or downloaded from any computer, tablet or cell phone.

References