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Performance Assessment of Multi-Objective Optimization Algorithms on Large-Scale Problems

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Abstract. Multiobjective optimization (MO) has been an attractive field in recent decades and many different algorithms have been proposed to solve MO problems. Although a lot of studies were focused to small-scale problems, real-world optimization problems frequently consist a large number of decision variables. In this study, the capabilities of the MO algorithms on large-scale optimization problems are investigated. For this purpose, four different techniques (NSGAII, MOCell, IBEA, and MOEA/D) known as the-state-of-art algorithms in the specialized literature are applied to solve a set of standard benchmark problems called ZDT functions for 10, 50, 100, 200, 500, and 1000 variable instances. Also, the values with standard deviations of four performance indicators (HV, SP, ε, and IGD) are calculated to promote the research. The experimental results have demonstrated that MOCell is generally able to reach the superior results than the other algorithms under the all conditions.

1 Introduction

Many real-world optimization problems involve multiple objectives conflicting each other, which have to be optimized at the same time [1]. These are called as multi-objective optimization problems (MOPs) and formally can be defined as in Eq. (1).

$$\begin{aligned} \text{Maximize/Minimize} \quad & y = f(x) = \{f_1(x), f_2(x), \dots, f_M(x)\} \\ \text{Subject to} \quad & g(x) = \{g_1(x), g_2(x), \dots, g_J(x)\} \leq 0 \\ & h(x) = \{h_1(x), h_2(x), \dots, h_K(x)\} = 0 \\ \text{where} \quad & x = \{x_1, x_2, \dots, x_N\} \in X \\ & y = \{y_1, y_2, \dots, y_N\} \in Y \end{aligned} \tag{1}$$

where x is set of the decision variable and X is the parameter space, y is the objective, Y is the objective space, g(x) and h(x) is the constraints depend on the decision variables.

Unlike single-objective optimization seeking global optimum, multi-objective optimization usually does not produce a single optimal solution since it is not always possible to obtain a single decision vector for conflicting objectives. Thus, multiobjective optimization algorithms generally obtain a set of optimal solutions by using a technique that satisfies a balance among the objectives. The techniques depend on Pareto-Optimality are the most suitable algorithms to handle objectives separately and synchronously.

According to concept of Pareto-Optimality, an optimal solution is the solution that is not the worst in any of the objectives and is better than the others in at least one objective. It is also a solution that is not dominated by any other solution in search space. This situation is mathematically defined as in Eq. (2).

$$\forall i : f_i(x) \leq f_i(y) \ \text{ and } \ \exists j : f_j(x) < f_j(y) \tag{2}$$

Such a solution is called as nondominated or Pareto-Optimal solution, and the set of such optimal solutions is called the Pareto-Optimal Set [2, 3].

Evolutionary algorithms are very convenient to obtain a set of solutions owing to their population-based nature. So, several multiobjective evolutionary algorithms (MOEAs) have been presented to solve MOPs up to now. These algorithms are generally classified in three groups: (i) dominance-based methods [3-5], (ii) decomposition-based methods [6-8], and (iii) performance indicator-based methods [9, 10].

Successful implementations of many real-world problems on mathematics, engineering, automotive, and a great number of others, have been increasing the attractiveness of multiobjective optimization field [1, 2]. However, there are some factors that adversely affect the performance of the MOEAs, such as to be too many constraint functions, the number of objective functions, and the problems involving large number of decision variables. In literature, various constraint-handling techniques have been studied [11-13]. On the other hand, the problems involving more than three objectives have been recently defined as many-objective optimization problems and some strategies have been proposed to overcome this difficulty [14, 15].

Large scale optimization problems including many science and engineering applications have a growing interest in meta-heuristic optimization field [16]. Unlike single-objective optimization literature, there is not much study for MO. One of the few studies based on large-scale multiobjective optimization is presented by Zille et al. [17], in which they have proposed an optimization framework based on problem transformation by grouping variables. Cheng et al. [18] proposed a set of generic test problems for large-scale multiobjective and many-objective optimization. In another recently presented study, Mahdavi et al. [19] made a comprehensive survey for solving high-dimensional optimization problems.

This study, as the main contribution, focuses on an investigation how the performance of MOEAs affects when there are large number of variables causing a very difficult exploration on large search space. Therefore, four successful algorithms developed based on different strategies mentioned above, are selected from literature. All of them is known as state-of-the-art algorithms. The first one is Nondominated Sorting Genetic Algorithm-II (NSGAII) that is the pioneering work for dominance-based methods [3]. The second one is Multi-Objective Cellular genetic algorithm (MOCell) that uses the neighborhood (cell) concept for recombination process allowing only corporation with adjacent solutions [20]. The third algorithm is Indicator-based Evolutionary Algorithm (IBEA) [21]. The last algorithm is Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D) [22].

In the scope of this study, all algorithms are run for 10, 50, 100, 200, 500, and 1000 variable instances. ZDT benchmark functions which were first introduced by Zitzler et al. [23], are selected for performance comparison of MOEATs. Since the number decision variables of the functions can be adjusted. Moreover, all ZDT functions contain two objectives. So, algorithms are not affected in the challenges that are associated with many-objective optimization. To assess the outcomes of the algorithms, four different indicator values are calculated. These are hypervolume [24], generalized spread [25], epsilon [26], and inverted generational distance [27].

The rest of the paper is organized as follows. Section II gives the brief descriptions of the MOEATs used in this study. Section III explains the performance indicators and presents the formulas. Section IV shows the experimental results. Finally, section V concludes the paper.

2 The Brief Explanations of The Algorithms

In this paper, four successful MOEAs in different categories are used for performance comparison on large-scale problems. All of them is the mostly-cited techniques developed to solve multiobjective continues optimization problems in literature. The short descriptions of the algorithms called NSGAII, MOCell, IBEA, and MOEA/D, are outlined in this section, respectively.

2.1 Nondominated Sorting Genetic Algorithm-II (NSGAII)

NSGAII [3] developed by Deb et al. in 1999, is the most well-known multi-objective optimization algorithm in literature. It is the dominance-based technique that presents pioneering methods for exploration and exploitation such as fast sorting strategy and crowing distance method. The algorithm begins by generating a random population. The N-sized population are separated into fronts by use of nondominated-sorting strategy. It means that all solutions are sorted according to pareto-dominance and nondominated solutions are picked to the first front and this process are repeated for remaining solutions for next front until no solutions remain in the population. Front level is used to select parent solutions as a rank in binary tournament. When the ranks are equal, crowding-distance method promoting diversity is used. N-new solutions are generated by SBX crossover and polynominal mutation. At the end of each iteration, 2N-sized population is obtained by combining current and new solutions. The best N solutions are picked to next generation by using nondominated sorting. In this way, elitism is guaranteed.

2.2 Multi-Objective Cellular genetic algorithm (MOCell)

MOCell [20] introduced by Nebro et al. in 2006, is depend on cellular model of genetic algorithms. This model uses the concept of (small) neighborhood that a candidate solution may only interact with its nearby neighbors in the breeding loop. So, overlapping small neighborhoods provides diversification, whereas intensification is satisfied by genetic operators. This strategy which is very popular in single-objective optimization, is adapted to MO by using the concept of Pareto-Optimality. MOCell uses an external archive and a feedback of solutions from the archive to the population. It also benefits the crowding distance as a density estimator and updates the archive.

2.3 Indicator-based Evolutionary Algorithm (IBEA)

IBEA was proposed by Zitzler and Künzli in 2004 [21]. It is based on a strategy that allows the arbitrary performance indicators to be used directly in the selection process. Thus, IBEA can be tailored to user preferences and does not require additional diversity protection mechanisms such as fitness sharing. The algorithm performs binary tournaments for mating selection and removes the worst individuals iteratively. Authors placed two different versions of the algorithm named basic IBEA and adaptive IBEA in their paper.

2.4 Multi-Objective Evolutionary Algorithm based on Decomposition (MOEA/D)

MOEA/D is presented by Zhang and Li in 2007, which is a decomposition-based evolutionary algorithm [22]. MOEA/D uses any decomposition approaches such as Weighted Sum, Tchebycheff, Boundary Intersection in order to separate the optimization problem into N-scalar subproblems. Then it minimizes all objectives simultaneously in a single run. The population consists of the best solution for each subproblem up to now. To optimize a sub-problem, only existing solutions of neighbour sub-problems are used in the algorithm.

3 PERFORMANCE INDICATORS

In recent years, many indicators have been proposed to compare the performance of MOEATs. The indicators are important to reflect the output quality of different algorithms and to carry out the necessary experimental work to compare various approaches. According to a study on performance indicators conducted in 2015, hypervolume (HV), generational distance (GD), epsilon indicator (ε),

inverted generational distance (IGD) and spread indicator (SP) are the most used metrics in literature, respectively [28].
The values of four indicators (HV, SP, ε, and IGD) are calculated in this paper. The descriptions and mathematical formulations of them are given below. P* is a set of points scattered uniformly along the true Pareto front (PF) in the objective space; Q is a set of points that estimates the PF.

3.1 Hypervolume (HV)

HV measures the volume of objective space that is weakly dominated by set Q. In order to calculate this volume, a bounded space has to be constructed by PF and a user-defined reference point [24].

$$HV = volume\left(\cup_{i=1}^{|Q|} v_i\right) \tag{3}$$

The larger the indicator value, the greater the size of the dominated area, the better front is obtained. For this reason, it uses the normalized objective values. The best value for the indicator is one.

3.2 Spread (SP)

Spread indicator measures the distribution of obtained set Q along PF [25]. There are two versions of the indicator. The first one is used for only two objectives, while the other one is generalized to be able to use for all multiobjective problems, which is named as generalized spread and denoted as Δ. It is formulated as follows.

$$\Delta = \frac{\sum_{i=1}^{n} d(e_i, Q) + \sum_{X\in S^*} |d(X,Q)| - \bar{d}}{\sum_{i=1}^{n} d(e_i, Q) + |P^*|.\bar{d}} \tag{4}$$

$$d(X,Q) = \min_{Y\in Q, Y\neq X} \|f(X) - f(Y)\|^2$$

$$\bar{d} = \frac{1}{|P^*|} \sum_{X\in P^*} d(X,Q)$$

Here, $(e_1, \ldots, e_n)$ refers to the m extreme points in set P^*; $d(e_i, Q)$ refers to the minimum Euclid distance between e_i and the set Q; m refers the number of the objective function. Δ is based on calculating the distance between two consecutive solutions closest to each other in the normalized objective space. It is desired to approximate zero.

3.3 Epsilon (ε)

When an obtained front for a problem is Q, epsilon is the indicator measuring the smallest distance that is required to convert every solution in Q so that it is able to dominate PF. It is desired to approximate zero [26].

$$E(Q, P^*) = inf_{\epsilon\in\mathbb{R}^+}\{\forall \vec{p} \in P^*, \exists \vec{q} \in Q: \vec{q} \prec_\epsilon \vec{p}\} \tag{5}$$

Here, $\vec{q} = (q_1, \ldots, q_m)$, $\vec{p} = (p_1, \ldots, p_m)$, m refers the number of the objectives and ϵ means a small positive number.

$$\vec{q} \prec_\epsilon \vec{p} \; if \; and \; only \; if \; \forall 1 \leq i \leq m: q_i < \epsilon + p_i$$

3.4 Inverted General Distance (IGD)

IGD is used to measure accuracy by calculating the average distances between Q and P^* [27]. The desired ideal value is zero. Mathematically, it is defined as in equation (6).

$$IGD(Q, P^*) = \frac{\sum_{p \in, P^*} d(p, Q)}{|P^*|} \quad (6)$$

Here, $d(p, Q)$ is the minimum Euclid distance between p and the set Q.

4 EXPERIMENTAL RESULTS

In this study, four well-known MOEAs (NSGAII, MOCell, IBEA and MOEA/D) have been obtained from the jMetal 4.5 software package [29]. All algorithms were applied to ZDT problem family that consists of five functions (named ZDT1, ZDT2, ZDT3, ZDT4, and ZDT6) for performance assessment on large-scale problems. All functions in ZDT problems have two objectives and dimensions may be set any number according to user-demand. In order to evaluate the effects of problem dimension, the number of decision variables of ZDT functions are set to 10, 50, 100, 200, 500, and 1000.

In the literature, ZDT functions are often run for 30 variables and 25.000 maximum function evaluation number (maxFES). It means just about 800 maxFES per each variable. This value was assumed as a coefficient of decision variables in the study and the algorithms were run for $(800 \times nvar)$ maxFES in functions involving totally $nvar$ variables. Also, population size was set to 100 for all algorithms.

All algorithms have been run 50 times for each function and six different number of variable instances. Then the mean values with standard deviations of HV, SPREAD, EPSILON and IGD indicators are calculated by obtained outcomes. The results are given in tables below. To increase readability of the tables, the best and the second-best values have highlighted with dark-gray and light-gray background color, respectively.

The tables from 1 to 4 shows the results of HV, SPREAD, ε, and IGD for functions with 10 variables, respectively.

Table 1: HV results for functions with 10 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	6.59e-01	4.5e-04	6.61e-01	9.3e-04	6.61e-1	7.3e-04	6.55e-01	2.9e-03
ZDT2	3.24e-01	4.2e-03	3.27e-01	1.6e-03	3.20e-1	1.7e-02	3.17e-01	8.2e-03
ZDT3	5.14e-01	8.8e-04	5.14e-01	3.0e-03	5.06e-1	1.3e-02	4.95e-01	8.9e-03
ZDT4	2.43e-02	6.0e-02	4.04e-01	1.5e-01	1.38e-2	4.7e-02	0.00e+00	0.0e+00
ZDT6	9.91e-03	1.2e-02	3.20e-01	9.9e-03	5.22e-2	2.7e-02	1.57e-01	1.5e-01

Table 2: SP results for functions with 10 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	3.78e-01	2.8e-02	1.29e-0	13.1e-2	3.55e-01	2.9e-02	3.65e-01	6.1e-02
ZDT2	3.84e-01	4.8e-02	1.82e-0	17.5e-2	4.83e-01	1.1e-01	3.74e-01	1.4e-01
ZDT3	7.51e-01	1.4e-02	7.16e-01	2.2e-02	1.15e+00	9.5e-02	1.01e+00	3.0e-02
ZDT4	9.25e-01	8.2e-02	9.68e-01	1.6e-01	9.71e-01	6.1e-02	1.15e+00	6.5e-02
ZDT6	8.35e-01	6.6e-02	5.73e-01	1.6e-01	8.18e-01	4.9e-02	1.00e+00	3.3e-01

Table 3: EPSILON results for functions with 10 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	1.43e-02	2.5e-03	1.00e-02	1.3e-02	1.13e-02	1.0e-02	1.55e-02	5.8e-03
ZDT2	2.48e-02	5.6e-02	2.61e-02	4.1e-02	8.11e-02	1.3e-01	2.69e-02	2.2e-02
ZDT3	3.36e-02	8.3e-02	8.30e-02	1.4e-01	1.33e-01	1.9e-01	5.15e-02	2.7e-02

ZDT4	1.50e+00	7.2e-01	4.81e-01	1.9e-01	1.47e+00	4.4e-01	6.49e+00	2.3e+00
ZDT6	7.76e-01	8.6e-02	9.62e-02	1.5e-02	5.70e-01	8.7e-02	5.27e-01	3.8e-01

Table 4: IGD results for functions with 10 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	1.95e-04	9.8e-06	1.80e-04	1.9e-04	1.96e-4	1.5e-04	2.44e-04	7.3e-05
ZDT2	2.85e-04	4.3e-4	2.25e-04	2.0e-04	1.05e-3	1.3e-03	3.18e-04	1.8e-04
ZDT3	3.27e-04	6.5e-04	7.02e-04	1.1e-03	1.91e-3	1.4e-03	4.62e-04	1.5e-04
ZDT4	4.02e-02	2.2e-02	1.20e-02	5.0e-03	4.03e-2	1.3e-02	1.95e-01	7.4e-02
ZDT6	1.73e-02	2.2e-03	1.87e-03	2.5e-04	1.22e-2	2.0e-03	1.34e-02	9.7e-03

Table 5: HV results for functions with 50 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	6.59e-01	3.1e-04	6.60e-01	2.6e-04	6.62e-01	7.1e-05	6.25e-01	1.1e-02
ZDT2	3.26e-01	3.2e-04	3.28e-01	5.0e-04	3.27e-01	9.9e-05	3.08e-01	9.0e-03
ZDT3	5.15e-01	1.4e-04	5.14e-01	1.2e-03	5.10e-01	2.5e-04	4.01e-01	2.3e-02
ZDT4	0.00e+00	0.0e+00	3.63e-03	2.0e-02	0.00e+00	0.0e+00	0.00e+00	0.0e+00
ZDT6	1.11e-01	1.8e-02	2.46e-01	9.6e-03	1.80e-01	1.7e-02	0.00e+00	0.0e+00

Table 6: SP results for functions with 50 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	3.65e-01	2.7e-02	7.46e-02	8.7e-03	2.96e-01	1.5e-02	3.95e-01	5.5e-02
ZDT2	3.77e-01	3.6e-02	7.96e-02	1.4e-02	3.29e-01	2.2e-02	3.01e-01	7.8e-02
ZDT3	7.48e-01	1.5e-02	7.05e-01	4.1e-03	1.21e+00	3.4e-02	9.84e-01	2.7e-02
ZDT4	9.60e-01	2.2e-02	9.88e-01	5.1e-02	9.94e-01	3.7e-02	1.27e+00	1.0e-01
ZDT6	7.20e-01	4.7e-02	4.25e-01	3.3e-02	7.87e-01	4.7e-02	9.62e-01	2.1e-02

Table 7: EPSILON results for functions with 50 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	1.37e-02	2.1e-03	6.53e-03	2.9e-04	8.84e-03	8.0e-04	3.48e-02	1.0e-02
ZDT2	1.31e-02	1.9e-03	5.73e-03	3.7e-04	1.64e-02	1.0e-03	4.95e-02	2.3e-02
ZDT3	8.79e-03	1.7e-03	4.99e-02	1.1e-01	3.50e-02	5.7e-02	1.76e-01	2.1e-02
ZDT4	6.75e+00	1.4e+00	1.91e+00	5.8e-01	1.07e+01	2.6e+00	4.32e+01	1.1e+01
ZDT6	4.51e-01	3.9e-02	2.13e-01	1.5e-02	3.12e-01	3.0e-02	1.73e+00	2.6e-01

Table 8: IGD results for functions with 50 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	1.90e-04	9.8e-06	1.41e-04	1.9e-06	1.64e-4	4.2e-06	8.74e-04	2.5e-04
ZDT2	1.94e-04	9.8e-06	1.42e-04	2.9e-06	5.44e-4	2.4e-05	5.06e-04	2.1e-04
ZDT3	1.34e-04	8.4e-06	4.45e-04	8.5e-04	1.27e-3	2.9e-04	2.90e-03	7.6e-04
ZDT4	2.03e-01	4.4e-02	5.17e-02	1.8e-02	3.29e-1	8.1e-02	1.36e+00	3.5e-01
ZDT6	9.29e-03	8.9e-04	4.05e-03	3.0e-04	6.19e-3	6.5e-04	4.52e-02	8.3e-03

Table 9: HV results for functions with 100 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	6.59e-01	3.2e-04	6.60e-01	2.8e-04	6.62e-01	7.2e-05	6.09e-01	1.5e-02
ZDT2	3.26e-01	3.1e-04	3.28e-01	4.8e-04	3.27e-01	3.3e-04	3.03e-01	1.2e-02
ZDT3	5.15e-01	1.5e-04	5.14e-01	8.0e-04	5.10e-01	2.4e-04	3.47e-01	2.5e-02
ZDT4	0.00e+00	0.0e+00	0.00e+00	0.0e+00	0.00e+00	0.0e+00	0.00e+00	0.0e+00
ZDT6	1.38e-01	1.1e-02	2.05e-01	8.9e-03	2.01e-01	1.2e-02	0.00e+00	0.0e+00

Table 10: SP results for functions with 100 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std

ZDT1	3.78e-01	3.0e-02	6.03e-02	1.0e-02	2.94e-01	1.4e-02	3.84e-01	4.3e-02
ZDT2	3.68e-01	3.5e-02	6.96e-02	1.3e-02	3.34e-01	2.4e-02	2.55e-01	6.1e-02
ZDT3	7.49e-01	1.4e-02	7.04e-01	4.2e-03	1.21e+00	4.4e-02	9.87e-01	2.1e-02
ZDT4	9.75e-01	1.1e-02	9.90e-01	2.9e-02	9.93e-01	8.8e-03	1.26e+00	1.2e-01
ZDT6	5.72e-01	2.7e-02	4.30e-01	2.1e-02	7.45e-01	4.9e-02	9.45e-01	3.2e-02

Table 11: EPSILON results for functions with 100 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	1.38e-02	1.9e-03	6.32e-03	2.1e-04	8.84e-03	8.0e-04	4.52e-02	1.4e-02
ZDT2	1.32e-02	1.8e-03	5.56e-03	2.7e-04	1.65e-02	1.4e-03	5.75e-02	2.5e-02
ZDT3	8.66e-03	1.9e-03	2.51e-02	7.4e-02	4.09e-02	6.9e-02	2.33e-01	4.2e-02
ZDT4	1.48e+01	2.2e+00	4.05e+00	9.0e-01	6.39e+01	1.9e+01	8.30e+01	1.4e+01
ZDT6	3.99e-01	2.2e-02	2.79e-01	1.5e-02	2.79e-01	2.0e-02	1.68e+00	2.1e-01

Table 12: IGD results for functions with 100 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	1.91e-04	8.7e-06	1.41e-04	1.9e-06	1.63e-04	4.0e-06	1.25e-03	3.5e-04
ZDT2	1.93e-04	8.6e-06	1.41e-04	2.6e-06	5.43e-04	3.3e-05	6.14e-04	2.8e-04
ZDT3	1.35e-04	9.0e-06	2.50e-04	5.8e-04	1.31e-03	3.4e-04	4.43e-03	7.8e-04
ZDT4	4.59e-01	6.9e-02	1.18e-01	2.8e-02	2.01e+00	6.1e-01	2.62e+00	4.3e-01
ZDT6	8.07e-03	5.0e-04	5.44e-03	3.2e-04	5.47e-03	4.2e-04	4.36e-02	6.7e-03

Table 13: HV results for functions with 200 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	6.59e-01	2.8e-04	6.60e-01	2.5e-04	6.62e-01	7.0e-05	5.08e-01	3.1e-02
ZDT2	3.26e-01	3.1e-04	3.28e-01	5.3e-04	3.11e-01	5.9e-02	2.88e-01	1.4e-02
ZDT3	5.15e-01	1.2e-04	5.14e-01	2.1e-04	5.10e-01	1.9e-04	2.90e-01	1.9e-02
ZDT4	0.00e+00	0.0e+00	0.00e+00	0.0e+00	0.00e+00	0.0e+00	0.00e+00	0.0e+00
ZDT6	1.32e-01	1.0e-02	1.57e-01	7.5e-03	2.19e-01	1.2e-02	0.00e+00	0.0e+00

Table 14: SP results for functions with 200 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	3.82e-01	3.5e-02	5.42e-02	8.8e-03	2.87e-01	1.4e-02	4.65e-01	4.2e-02
ZDT2	3.73e-01	2.9e-02	5.99e-02	1.0e-02	4.09e-01	1.4e-01	2.48e-01	4.8e-02
ZDT3	7.51e-01	1.5e-02	7.03e-01	3.2e-03	1.22e+00	1.0e-02	9.92e-01	1.8e-02
ZDT4	9.84e-01	6.2e-03	9.89e-01	1.8e-02	9.98e-01	5.8e-03	1.27e+00	1.2e-01
ZDT6	5.61e-01	2.1e-02	4.51e-01	1.5e-02	7.01e-01	3.8e-02	9.26e-01	3.4e-02

Table 15: EPSILON results for functions with 200 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	1.40e-02	2.0e-03	6.13e-03	1.9e-04	8.78e-03	8.6e-04	1.28e-01	2.8e-02
ZDT2	1.35e-02	2.5e-03	5.50e-03	2.8e-04	6.04e-02	2.0e-01	8.48e-02	2.8e-02
ZDT3	8.59e-03	1.4e-03	6.12e-03	3.6e-04	2.33e-02	1.0e-03	3.39e-01	4.1e-02
ZDT4	3.67e+01	5.2e+00	8.96e+00	1.7e+00	2.99e+02	8.8e+01	1.64e+02	2.4e+01
ZDT6	4.13e-01	2.1e-02	3.65e-01	1.4e-02	2.53e-01	1.9e-02	1.71e+00	2.4e-01

Table 16: IGD results for functions with 200 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	1.90e-04	9.5e-06	1.40e-04	1.8e-06	1.62e-04	4.0e-06	3.65e-03	7.8e-04
ZDT2	1.92e-04	8.4e-06	1.41e-04	2.3e-06	1.34e-03	3.9e-03	9.68e-04	3.5e-04
ZDT3	1.35e-04	8.2e-06	1.02e-04	1.4e-06	1.22e-03	3.0e-05	6.06e-03	5.2e-04
ZDT4	1.15e+00	1.6e-01	2.73e-01	5.3e-02	9.46e+00	2.8e+00	5.18e+00	7.7e-01

ZDT6	8.42e-03	4.7e-04	7.32e-03	3.1e-04	4.90e-03	4.0e-04	4.44e-02	7.5e-03

Table 17: HV results for functions with 500 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	6.59e-01	2.4e-04	6.61e-01	2.6e-04	6.62e-01	8.0e-05	2.27e-01	3.1e-02
ZDT2	3.25e-01	2.6e-04	3.27e-01	6.9e-04	2.98e-01	8.8e-02	2.46e-01	1.8e-02
ZDT3	5.15e-01	1.2e-04	5.15e-01	2.4e-04	5.10e-01	2.0e-04	1.80e-01	1.9e-02
ZDT4	0.00e+00	0.0e+00	0.00e+00	0.0e+00	0.00e+00	0.0e+00	0.00e+00	0.0e+00
ZDT6	9.30e-02	8.0e-03	7.38e-02	5.5e-03	2.09e-01	6.9e-03	0.00e+00	0.0e+00

Table 18: SP results for functions with 500 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	3.88e-01	3.2e-02	5.43e-02	9.2e-03	2.89e-01	1.5e-02	6.00e-01	2.5e-02
ZDT2	3.90e-01	3.2e-02	5.85e-02	8.8e-03	3.81e-01	1.8e-01	2.92e-01	4.8e-02
ZDT3	7.51e-01	1.6e-02	7.02e-01	2.1e-03	1.22e+00	1.1e-02	9.87e-01	7.3e-03
ZDT4	9.91e-01	4.0e-03	9.86e-01	9.4e-03	9.99e-01	2.2e-03	1.25e+00	9.7e-02
ZDT6	6.07e-01	1.8e-02	5.19e-01	1.2e-02	6.32e-01	3.4e-02	8.93e-01	3.9e-02

Table 19: EPSILON results for functions with 500 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	1.41e-02	2.1e-03	5.96e-03	1.9e-04	8.72e-03	7.4e-04	4.06e-01	3.9e-02
ZDT2	1.41e-02	2.1e-03	5.71e-03	3.5e-04	1.52e-01	4.9e-01	1.62e-01	3.9e-02
ZDT3	8.85e-03	1.7e-03	5.96e-03	2.8e-04	2.34e-02	1.2e-03	6.03e-01	5.2e-02
ZDT4	1.13e+02	1.1e+01	2.74e+01	2.6e+00	1.52e+03	4.1e+02	3.95e+02	4.3e+01
ZDT6	4.98e-01	1.9e-02	5.46e-01	1.5e-02	2.73e-01	1.2e-02	1.64e+00	1.6e-01

Table 20: IGD results for functions with 500 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	1.91e-04	7.9e-06	1.40e-04	1.9e-06	1.63e-04	5.1e-06	1.17e-02	1.1e-03
ZDT2	1.98e-04	1.0e-05	1.44e-04	2.9e-06	3.59e-03	1.1e-02	2.16e-03	5.5e-04
ZDT3	1.34e-04	8.2e-06	1.02e-04	1.7e-06	1.22e-03	3.2e-05	9.00e-03	6.1e-04
ZDT4	3.58e+00	3.3e-01	8.57e-01	8.4e-02	4.81e+01	1.3e+01	1.25e+01	1.4e+00
ZDT6	1.04e-02	4.5e-04	1.16e-02	3.5e-04	5.31e-03	2.5e-04	4.24e-02	4.9e-03

Table 21: HV results for functions with 1000 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	6.58e-01	2.8e-04	6.60e-01	1.8e-04	6.62e-01	6.2e-05	7.45e-02	1.2e-02
ZDT2	3.24e-01	2.7e-04	3.27e-01	7.8e-04	3.27e-01	2.5e-04	1.91e-01	1.1e-02
ZDT3	5.14e-01	1.3e-04	5.15e-01	1.7e-04	5.10e-01	1.4e-04	1.15e-01	1.1e-02
ZDT4	0.00e+00	0.0e+00	0.00e+00	0.0e+00	0.00e+00	0.0e+00	0.00e+00	0.0e+00
ZDT6	4.68e-02	5.0e-03	1.68e-02	3.6e-03	1.84e-01	6.6e-03	0.00e+00	0.0e+00

Table 22: SP results for functions with 1000 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	3.95e-01	3.0e-02	5.68e-02	7.3e-03	2.90e-01	1.3e-02	6.77e-01	8.9e-03
ZDT2	3.88e-01	3.6e-02	5.77e-02	8.3e-03	2.98e-01	1.7e-02	3.60e-01	3.2e-02
ZDT3	7.51e-01	1.8e-02	7.01e-01	1.4e-03	1.22e+00	9.1e-03	9.90e-01	1.2e-02
ZDT4	9.94e-01	2.7e-03	9.83e-01	5.7e-03	9.99e-01	1.1e-03	1.26e+00	8.5e-02
ZDT6	6.55e-01	1.9e-02	5.84e-01	9.9e-03	6.12e-01	2.9e-02	8.65e-01	3.7e-02

Table 23: Epsilon results for functions with 1000 decision variables

	NSGAII	MOCell	IBEA	MOEA/D

	mean	std	mean	std	mean	std	mean	std
ZDT1	1.47e-02	2.1e-03	5.96e-03	1.2e-04	8.59e-03	7.5e-04	6.52e-01	2.7e-02
ZDT2	1.45e-02	2.3e-03	6.06e-03	3.8e-04	1.65e-02	1.3e-03	2.67e-01	2.5e-02
ZDT3	9.23e-03	2.0e-03	5.85e-03	2.8e-04	2.32e-02	7.0e-04	8.10e-01	3.7e-02
ZDT4	2.70e+02	1.7e+01	6.51e+01	7.3e+00	4.56e+03	1.4e+03	7.99e+02	7.2e+01
ZDT6	6.24e-01	1.6e-02	7.44e-01	1.9e-02	3.15e-01	1.1e-02	1.63e+00	1.9e-01

Table 24: IGD results for functions with 1000 decision variables

	NSGAII		MOCell		IBEA		MOEA/D	
	mean	std	mean	std	mean	std	mean	std
ZDT1	2.00e-04	8.9e-06	1.41e-04	1.3e-06	1.63e-04	3.8e-06	1.85e-02	7.4e-04
ZDT2	2.04e-04	1.1e-05	1.48e-04	4.4e-06	5.41e-04	3.1e-05	3.99e-03	4.2e-04
ZDT3	1.36e-04	9.2e-06	1.01e-04	1.0e-06	1.21e-03	2.1e-05	1.17e-02	5.3e-04
ZDT4	8.53e+00	5.4e-01	2.05e+00	2.3e-01	1.44e+02	4.5e+01	2.53e+01	2.3e+00
ZDT6	1.35e-02	4.0e-04	1.66e-02	4.9e-04	6.22e-03	2.5e-04	4.20e-02	5.9e-03

First of all, it can be expressed that this study supports the NFL theorem [30] and there is no single algorithm that gives the best results for all functions in all conditions. On the other hand, when examined the table 25 that shows the number of the best and the second-best for each indicator in all conditions, MOCell algorithm is superior to other algorithms, although the results are very close to each other.

Table 25: Superiority results over indicators

	NSGAII		MOCell		IBEA		MOEA/D	
	best	second	best	second	best	second	best	second
HV	4	8	12	12	10	4	0	1
SP	4	12	26	4	0	9	0	5
ε	3	15	24	3	3	10	0	2
IGD	3	18	24	3	3	8	0	1
total	14	53	91	22	16	31	0	9

Examining the obtained results given Table-1 to Table-24, MOCell, NSGAII, IBEA and MOEA / D achieved the best results, respectively. While the IBEA has slightly better results than NSGAII in small dimensions, in general, MOCell has achieved the best successful values in all cases. The MOEA / D has performed worse performance than other algorithms under these operating conditions. It can be concluded that for these four algorithms, the problem size can be tolerated by the number of function evaluations and they can maintain their running stability in this direction.

5 CONCLUSIONS AND FUTURE WORK

In this study, a research over the number of decision variables in multiobjective optimization was conducted and investigated the effect of large-scale problems to MOEAs. On the other hand, algorithms were run by increasing the number of function evaluations in parallel to dimension. Four well-known algorithms, NSGAII, MOCell, IBEA, and MOEA/D, were applied to ZDT functions for 10, 50, 100, 200, 500, and 1000 variable instances. Four different performance indicator (HV, SP, ε, and IGD) values were calculated for the obtained results afterwards 30 independent runs. MOCell algorithm was reached to the best results and NSGAII followed it as second best. As a result, it can be said that increasing the maxFes according to dimension of the problem supported to algorithms' running character.

For the future work, the effect of population size on large-scale optimization problems can be examined by keeping the number of evaluations as constant.

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