A dynamic programming model for solving a capital rationing problem in capital budgeting[1]: An applied study on Ras Lanuf oil & gas processing company.

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Abstract

This study aims to determine the extent to which the dynamic programming can be used in the preparation of capital budgeting to rationalize investment decisions under limited resources. Ras Lanuf Oil & gas processing company is chosen as a case study, due to the importance of the company's investment activity and its impact on the future of the company's business. The importance of this paper is that it illustrates how to link the managerial accounting tools and mathematical models to create a new synthesis that can be used in planning and control of investment expenditures. This paper presents a mathematical model that contributes to raising the efficiency of the capital expenditure decisions implemented by Ras Lanuf Company. A Dynamic programming (DP) is one of the important mathematical models which is suggested in order to select the optimal investment alternatives under the available funds. The DP is suitable for problems that require sequential decisions or that can be divided into a number of sub-problems (called stages). The results obtained from the application of the proposed mathematical model in the applied study on investment projects contribute to the determination of the optimal plan for implementing the investment projects in Ras Lanuf Company, leading to achieving the highest possible return within the available budget for investment.

Key words: Dynamic Programming; Capital Budgeting; Optimal Investment Plan.

Abbreviations:

Accounting Rate of Return	ARR
Capital Budgeting	CB
Discounted Cash Flow Techniques	DCFT
Dynamic Programming	DP
Goal Programming	GP
Integer Programming	IP
Internal Rate of Return	IRR

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Linear Programming	LP
Mathematical Programming	MP
Maximization	Max
Net Present Value	NPV
Operations Research Techniques	ORT
Payback Period	PP
Profitability Index	PI

1. Introduction:

Capital budgeting (CB) is one of the important developments in the twenty-first century. Not only it is used in the planning and control of investment expenditure but also plays an essential role in the allocation of limited resources. In textbooks, the term "capital budgeting" is usually used in the same way as " capital investment appraisal" (Drury, 2012 and McLaney, 2009). Capital budgeting techniques are used in appraising the investment opportunities in order to choose the best one. It is not easy to determine the best opportunities in using the available resources in economic reality which is characterized by technological development and intensive competition. In Global Market, It is generally considered that there is a close correlation between economic growth and the allocated method of available resources; economic growth does not depend only on the size of the available resources, but also depends on how to allocate these limited resources and their uses. Therefore, the allocation of limited resources as a rational basis requires the use of operations research models. However, the capital rationing exists whenever a firm's capital budget is not adequate to fund all profitable projects. Beraldi et al., (2012) stated that capital rationing is a significant issue in making investment decisions.

Consequently, the administration of Ras Lanuf gas & oil processing company has substantial interests in decisions which are associated with investing in new projects or developing the existent projects; that is because their objectives have been impacted on the company's profitability in the short or long-term. These decisions reflect on the management's interest in this kind of strategic decisions by using extensive studies which are known as "feasibility studies". Accordingly, several plans have been implemented to develop Ras Lanuf oil complex based on the available technology and market requirements.

This paper presents a mathematical model which contributes to raising the efficiency of investment decisions in Ras Lanuf company. We aim to address the capital rationing problem in capital budgeting by using a dynamic programming model; It is one of the mathematical programming techniques which is sought to maximize the profitable function The dynamic programming (DP) technique is appropriate to the problems that require sequential decisions or divide into a number of sub-problems called stages.

2. Literature review

In literature, there are three categories of capital budgeting techniques (CBT): financial appraisal, risk analysis and operations research techniques (Pike, 1996; Pike and Sharp, 1989). Most of researchers concentrate on the financial appraisal techniques used as the criteria for making CB decisions (Graham and Harvey, 2001; Holmen and Pramborg, 2009; Mohammed, 2013).

In textbooks, the net present value (NPV), internal rate of return (IRR), profitability index (PI) payback period (PP) and accounting rate of return(ARR) are often studied as the investment appraisal techniques (Drury 2012; McLaney 2009; Seal & Garrison & Noreen 2006).The link between the discounted cash flow techniques, which are the NPV, IRR & PI, and capital rationing is based on the purpose of capital rationing which aims to maximize the NPV under available budget (Manalo and Manalo, 2010).

It can be seen from empirical literature that the PP method is the most common method used amongst the UK, Indian, Malaysian, Sudanese and Nigerian firms (Pike, 1996; Verma et al., 2009; Anuar, 2005; Eljelly and AbuIdris, 2001; Obi & Adeyemo 2014). On the other hand, discounted cash flow (DCF) methods are perceived as being of more interest in the US, Canadian, Swedish and Australian firms (Ryan and Ryan, 2002; Bennouna et al., 2010; Daunfeldt and Hartwig, 2014; Truong et al., 2008).

In Libya, the PP and ARR methods are also commonly used in Libyan companies (AlWakil, 2000; Mohammed, 2013), whereas discounted cash flow techniques (DCFT) are of less interest than in developed countries. PB and ARR methods are applied based on the concept of traditional income, whereas DCFT have been based on the concept of the time value of money. In this regard, Alhouderi (1997) and Eloubedi (1993) stated that Libyan companies did not apply the scientific methods to evaluate the investment alternatives. On the other hand, Ehssona (1994) verified that 30% of Libyan industrial enterprises partly adopted operations research techniques to help in making decisions.

Even though, there is progressing towards the use of ORT in appraising the investment projects and solving the problems related to capital budgeting (Pike 1996, Klammer, 1972).The main reason is that ORT can help executives to make optimal capital budgeting decisions.

However, there are several mathematical programming models proposed in solving the capital rationing problem in capital budgeting decisions. Weingartner (1963) presented the original mathematical formulation to solve the capital rationing problem. Most studies focus on linear, integer and goal program models in making CB decisions (Khan, 2008). In terms of this orientation, Ras Lanuf Oil & Gas processing Company used linear programming in selecting the processing units for phase III of the Ras Lanuf project (Linsley and Fotouh, 1979).

Nevertheless, the conventional and linear programming techniques have faced a lot of criticism as a method used in preparation of the capital budgeting, where these methods are used in static situations. Whereupon the practical reality has required to search the new model which can apply to solve the current problems. Bellman and Dreyfus (2015) assume that the practice of doing DP is more effective than the LP. Hence, dynamic programming (DP) can be used in the preparation of capital budgeting under capital rationing; it can be provided the optimal investment plan within the available funds (Mohamed, 1991). DP can apply in linear and non-linear situations, as well as DP technique provides a solution to the problem of divisibility in investment projects rather than LP models.

Khan (2008) states that the mathematical programming models used in solving the capital budgeting problems are perceived to be more common in the private sector rather than the government sector. Accordingly, the capital rationing problem in the public sector has occurred in the following situations: the presence of limited funds, the legal constraints of borrowing, the limitation of public expenditures and the concentration of sample projects rather than complex projects requiring large amount of budget. (Ibid).

Dynamic programming (DP) history is associated with the research scientist called "Richard Bellman". Where he published his research on DP during the fifties of the last century, and summarized his contribution in the DP book published in 1957 (Bellman, 1957). Since then DP has become one of the most important quantitative methods that can be used in many Industrial, economic and military fields. DP was also used to solve lots of the accounting and administrative problems facing decision makers in different businesses.

Bellman's research has been based on solving problems related to the allocation of resources (Bellman & Dreyfus, 2015). In line with this aspect, Khalesi et al (2011) used dynamic programming in the electricity distribution system for customers to attain the optimal allocation of this energy.

Recently, dynamic programming research focused on solving the problems associated with optimal control whether to apply for applied sciences or human sciences (Pham & Wei, 2017; Bertsekas, 2017; Sundström et al 2010).

In terms of the applications of DP in accounting, they initially focused on a deterministic or stochastic inventory routing problem, where stochastic modeling is due to asymmetrical data in accounting environment. Kleywegt et al. (2004) used DP to determine the amount of inventory to be replenished or provided for each customer to meet the sudden shortage of stores. In the case of asymmetrical data of the inventory quantity movement from time to time, it is required to use a multistage approach, such as DP. Lee & Wolpin (2010) used a DP in accounting for changes in the structure of wages and employment in the USA during the period 1968-2000.

Moreover, accounting practices have provided robust evidence that DP can be used in the preparation of capital budgeting under limited resources, leading to the selection of optimal oil fields within the available funds (Mohamed, 1991).

However, the most prominent studies, which deal with the subject of the study, are summarized as follows:

Mohamed (1991, pp.148-149):

Mohamed's Study aims to identify the available techniques for capital budgeting, and emerges their difficulties and criticisms. It highlights the role of DP in the preparation of capital budgeting. The main findings can be summarized as follows:

- The DP can be used in capital budgeting processes for the development activity within the available funds.
- The application of the dynamic programming method enables to identify the optimal production fields (oil production fields) that will be developed at the level of each production area of GAPCO[4] company.

Özlem Ince, (2006, pp.77-80):

This study aims to evaluate the investments of railway systems; it employs probabilistic dynamic programming to determine the "optimal investment policy and the optimal timing of the investment". Özlem Ince (2006) developed new DP software by used "Microsoft Visual Studio C++ 6.0 language". The results derived from processing the DP model are determined the "number of trains required to be purchased in each investment epoch", and decide which years should be undertaken to consider as the optimal timing of the investments. In addition, the sensitivity analysis was made to determine the significant effects on the recursive equation. Where, the changes in interest rate or discounted rate are the most important variable.

Silva et al. (2017):

This study employs fuzzy goal programming (FGP) to improve the capital budgeting process under uncertain circumstances. In processing the FGP model, "the GAMS software - 23.6.5 with CPLEX" was running for obtaining the optimal solutions. The results confirmed that the FGP is preferred for preparing the CB and provides simple model for solving the multiobjectives problem in CB decisions, such as increased profitability index, reduced payback and capital rationing. Moreover, the FGP model is efficient in practical applications for the situations that deal with uncertain conditions.

Stensland and Tjostheim (1989, p.119):

They used a dynamic programming algorithm to build time series models in order to "generate transition probabilities from one price level to another". The results of this study confirmed that the optimal investment policies are based on "the amount of the present correlation in the price process and on additional features of scenario type" that can be adopted.

[4] GAPCO: Gulf of Alsues Petroleum Company

Pike & Sharp, 1989 (A survey of operations research techniques):

This study examines the trends in the use of ORT for capital budgeting decisions in the UK large companies. It explores the extent of using the operational research techniques (ORT) for capital budgeting. It was based on 3 surveys conducted on the 100 large UK companies during the period 1975-1986. The study also applied a predictive logistic model to forecast the use of ORT in 1991. The findings of the study reported that mathematical programming (MP) are rarely used in the UK companies, whereas there is an increasing tendency to the use of program evaluation & review technique (PERT) and critical path analysis (CPA).

For more details, Table 1 illustrates the extent of using the ORT for solving the capital budgeting problems in the Libyan, UK and US businesses.

Table 1: Percentage of using the operations research techniques by the Libyan, UK and US businesses.

No	Author	Country and year of survey	The sample	The type of answer	% The use of the operations research techniques*		
					MP*	DT*	PERT/CPT*
1	Alsharif (2016)	Libya 2008-2010	69 firms	Moderate, high and essential priority	8.7	17	12
2	Pike and Sharp (1989) (large firms)	UK 1986	100 firms	Rarely, often, almost and always used	21	34	49
3	Ryan and Ryan (2002)	USA 2002	205 firms	Always, often or sometimes used	16.8	31	31
4	Khan (2008)	USA 2008	Case study 9 projects	Optimal projects	LP, IP & GP...	NA	NA

*MP: Mathematical programming: linear Programming (LP), Integer Programming (IP) and Goal Programming (GP)
DT: Decision Tree. PERT: Program Evaluation & Review Technique. CPA: Critical path analysis

3. The proposed mathematical Model.

3.1 A Dynamic Programming Model.

Dynamic programming (DP) is a recursive procedure which is sought to reach the optimal solution of the problem in sequential stages, and each stage should be used information obtained from the previous stages. Where, the solution of the problem is not conducted in one step or one decision, but the problem is solved in sub-stages. In each stage, it can be found a sub-solution for that stage, with the sub-solution for each stage being used to solve the next stages (Al-Attar, 1991, p. 350). Each stage of dynamic programming is sequentially linked to the previous stages. A dynamic programming (DP) is a quantitative analysis technique applied extensively to solve the problems that require sequential decisions (Render &Stair, 2000 & Lieberman & Hillier, 2001). The application of DP is based on the principle of optimality that formed by Richard Bellman, as follows:

> An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. (Bellman, 1957).

The application of the principle of optimality depends on the relationship between the optimal values of the objective function in the successive stages. Therefore, the optimal solution is determined by a certain stage, either by the optimal policy in the previous stages or by the optimal policy in the subsequent stages. Accordingly, the solution method consist of two ways to solve the problems of dynamic programming: the front and back computation methods (Yousef, 1979).

Forward computation method:

This method is based on the evaluation of investment alternatives and take decisions in successive stages on the starting point is the first stage and then the next stage, until the user reaches the final stage (Mohamed, 1991, p. 52). According to this method, sequential functions of stages are determined in the following order:

$$f_1 \square\ f_2 \square\ f_3 \square \ldots\ldots\ldots\ldots\ldots\ldots\ldots \square f_n$$

That is, the forward method depends on the principle of progress in work. Where, the value of the first function (f_1) is calculated and then move to the second function (f_2) and so on ... (f_n).

Backward computation method:

This is the opposite of the previous method. The sequential relationship is used to find the optimal solution by moving from the end to the beginning. In each stage, the optimal plan for each stage is found until the user reach the first stage. As shown in the following diagram (Al-Naimi et al., 1999, p. 319):

$$f_n \rightarrow f_{n-1} \rightarrow f_{n-2} \rightarrow \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \rightarrow f_1$$

3.2 The formulation of the mathematical model[5]:

The mathematical model (Recursive Equation) can be formulated as follows (Hastings, 1973, pp.15-16 & Kwak, 1973, pp.241-242):

$$\mathbf{F_J(X_J) = \underset{d_J}{Max}\{R_J(d_J) + F_{J-1}(X_{J-1})\}}$$

Subject to :

$$\mathbf{X_{J-1} = X_J - C_J(d_J)}$$

$$0 \le C_J(d_J) \le X_J$$

$$X_J \le B \quad , J=1,2,3,4 \; d_J=0,1,2,3$$

Where:

J: Represents the stage that refers to the proposed investment projects in Ras Lanuf complex, which are four projects.

F_J (X_J): The aggregate objective function of stage J that indicates the optimal return of the best alternative in stage J under X_J when is applied the sequential solution.

X_J: The funds allocated to the investment in stage J.

d_J: A possible alternative to implement in stage J, where there are four alternatives proposed in each stage.

R_J (d_J): The net present value (NPV) that refers to the net profit from the implementation of alternative d in stage J.

F_{J-1}: Objective function in the previous stage.

CJ (dJ): The investment cost required to implement the alternative d in stage J.

[5] For more details about the computations of each stage, you can contact to the researcher: ali.alsharif61@gmail.com.

B: The available budget for investment.

X_{J-1}**:** The amount allocated to the investment in the previous stage that means the output of the system status at the previous stage.

However, the computations of dynamic programming model resulting from all stages depend on each other. For example, the calculations in the fourth stage depend on the calculations of the revenue equation of the third stage. The solution of this model includes the optimal returns for all stages that must be taken into account before the current stage. It is also noted that we have not interested in the decisions undertaken in the previous stages, where the optimal decisions are selected for the current stage without reference to the optimal decisions of the previous stages.

4. The matrix of mathematical model coefficients

The results of feasibility studies of the investment projects, which had been prepared for the development of Ras Lanuf complex, are the basic data for the mathematical model. Table 2 can be summarized the essential data for running the mathematical model by the following mathematical model coefficients:

Table No: 2
Mathematical model coefficients matrix

(Numbers in Million $ (

Polypropylene plant		Petrochemical units/ Benzene Extraction		Petrochemical units/ Mixed-C_4		Refinery development project		Investment projects "J " (Stages) / d_j* Alternatives
$R_4(d_4)$ "NPV"	$C_4(d_4)$ Investment outlays	$R_3(d_3)$ "NPV"	$C_3(d_3)$ Investment outlays	$R_2(d_2)$ "NPV"	$C_2(d_2)$ Investment outlays	$R_1(d_1)$ "NPV"	$C_1(d_1)$ Investment outlays	
٠	٠	٠	٠	٠	٠	٠	٠	Alternative"٠"
66	١٦٢	٢,٤	٤٨	٦٨,٨	٦٥	٢٩٢	١٧٥٥	Alternative"١"
٦٠	87	١,٧	٢٣	٦٨,٧	٣٣	٢٦٩	٩١٦	Alternative"٢"
-	-	-	-	-	-	٢٩٥	٧٣٧	Alternative"٣"

*Source: Ras Lanuf Oil & Gas Processing Company, Projects Management - Studies Department (2003): Feasibility studies carried out by foreign consultants.

5. The empirical results

According to the objectives of this paper, the findings can be summarized as follows:

- The results of this study confirmed that dynamic programming is an appropriate method for preparing the capital budgeting and planning investment projects. In line with these results, Mohamed's study (1991) confirmed that the application of dynamic programming (DP) enables users to identify the optimal oil production fields that Alsues gulf petroleum company seek to develop them in each of production area.

*d_j: The possible alternatives are basically three alternatives and one alternative is 0 choice that can be allocated to each project/stage.

- The proposed mathematical model applied in this study addressed only the factors that can be quantified. As to the factors that are unquantifiable, they are left for the official management to assess the ability of management and their efficiency in estimating these factors.

- It can be noticed by looking at the feasibility studies for investment projects that the Ras Lanuf company used linear programming to determine the optimal production selection. Whereas, LP is not applied in evaluating the available investment alternatives, as a tool used in rationing the company's investment decisions.
- The results obtained from the application of our proposed model on the investment projects of Ras Lanuf Oil and Gas Processing Company determine the optimal plan for the investment alternatives that achieves the maximum possible return within the constraint of the available funds. The results can be shown in table 3.

Table No: 3

The optimal plan for the implementation of the investment projects in the RasLanuf company within the constraint of the available investment budget[6].

($ Million)

NPV/ Optimal profit	Investment cost/outline	The optimal plan for the implementation of investment projects in Ras Lanuf company				The available capital budget	Case
		Polypropylene plant	Petrochemical units / Benzene	Petrochemical units /Mixed-C_4	Refinery development project		
432.2	1012	First alternative	First alternative	First alternative	Third alternative	2030	1
432.2	1012	First alternative	First alternative	First alternative	Third alternative	1015	2
425.4	880	Second alternative	Second alternative	Second alternative	Third alternative	880	3
137.2	275	First alternative	First alternative	First alternative	Nonimplementation	275	4

Table 3 shows the following findings:

I. The investment alternatives have been chosen for the implementation of investment projects, and the company achieves the maximum return within the limitations of the allocated budget for Ras Lanuf complex; on the bases of the cases studied.

II. If the allocated amount exceeds $1012 million, it is unfeasible in economic terms. Whatever the increase over the amount$1012million; the outcomes are the same.

[6] Data which are collected from the company - the practical approach-(cost-benefit matrix) that it is adopted in processing of mathematical model.

III. The company should imply the banking finance (second alternative - third case) for the implementation of the petrochemical units and polypropylene factory, rather than the implementation based on self-financing (first alternative - second case), because the opportunity cost resulting from selecting the first alternative equals 125.2 $million.

6. Recommendations

Based on the findings of the study, the recommendations can be suggested as follows:

- In recent years, it is certain that the scientific research requires the use of dynamic programming and its various applications, in order to reach algorithms applied for solving the problems similar to the multiple stage applications. In spite of the importance of this technique (DP) in solving of the economic and administrative problems, the present studies in Arabic literature may not have reached the required level.
- With regard to the mathematical model used in the applied study, there is not available to obtain a software package in processing the dynamic programming models. It is certain that the existing programs at Libyan universities and institutes are not suitable for this case study. Where, this study applied Microsoft Excel (2000) to run the mathematical model (DP). Therefore, we recommend that the academic institutions should provide accessible software for applications of dynamic programming. On the contrary, software for linear programming (LP) applications is available in most universities and can be downloaded from multiple sites over the internet, such as QM for Windows; LINDO; LINGO ... etc.

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