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General Solution of Thick Plates on Winkler-Pasternak Foundation Using Orthogonal Polynomial Displacement Functions

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ABSTRACT

A solution for thick plates on Winkler-Pasternak foundation using characteristic orthogonal polynomials (COPs) displacement functions was presented in this study. A shear deformation function was developed from Soldatos' (1992) trigonometric function and the Ritz energy approach was used to determine the total potential energy of the thick plate on elastic foundation. This method presents a simple and efficient method where only definite integration is required to obtain solutions. Results for non-dimensional in-plane and out-of-plane displacements for simply supported (SSSS) thick plates were presented and was observed to converge with that of other third-order shear deformation theories (TSDT) available in literature. From the study, it was observed that the in-plane and out-of-plane displacements reduced generally as the foundation modulus of the soil increased and decreases as the span-depth ratio increased for each aspect ratio. Both the Winkler and Pasternak foundation parameters influenced the results obtained as the foundation modulus is dependent on these parameters.

1. Introduction

Many practical engineering systems such as pavement of highways, foundation slabs of buildings, etc can be related to the solution of plates resting on an elastic foundation. The relationship between all engineering infrastructure and their foundation soils is of great importance for designers, engineers and contractors (Nwaiwu et al., 2020). In the construction such engineering structures, the idea is to transfer superstructure load safely to the foundation (soil), hence an obvious interaction between the soil and the structure itself. Such a system leads to the consideration of soil-structure interaction. Soil-structure interaction concerns the response of the soil and the structure on it, especially when subjected to dynamic loads (Gaikwad and Shingade, 2020; NIST, 2012). Therefore, the stresses induced in the soil-structure system are largely dependent on the properties and behaviour of the soil and the structure under the applied load.

A plate is said to be a flat structural component bound by two planes that are parallel to each other called faces and a cylindrical surface called an edge. Plates have wide applications in engineering structures including bridges, architectural structures, etc. They can be classified

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as thick or thin depending on their thickness relative to their lateral dimensions. Thin plates can be analysed using the *Kirchoff Classical Plate Theory* (CPT) where the effect of transverse shear deformation is neglected. However as the plate becomes thicker, the effect of shear deformation becomes more important. In order to overcome the deficiency of the CPT, shear deformation theories have been proposed including higher-order shear deformation theory (HSDT), refined theory, trigonometric shear deformation theory (TSDT) amongst others (Ibearugblem and Ezeh, 2013; Ezeh *et al*., 2014).

On the other hand soils can be analysed using any of these two approaches namely; the continuum approach (elastic, elastoplastic, anisotropic etc.) and the modeling approach (where the soil is modeled and model parameters are determined). Winkler (1867) proposed a one parameter elastic model where he considered the behavior of soils to be linear, and can be represented using independent discrete linear elastic springs. An identified problem with the Winkler model is the independent (uncoupled) behavior of the soil springs which does not represent the displacement behavior of typical soils under load (Ubani *et al*., 2020).

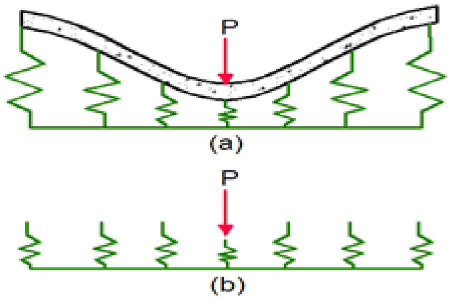

Figure 1. Schematic representation of soil spring system (a) Coupled behavior (b) Uncoupled behavior (Ubani *et al*., 2020)

Contrary to the Winkler model, Pasternak (1954) proposed a two parameter elastic model where he noted the existence of shear interaction among the spring elements, and that the load-deflection relationship is obtained by considering the vertical equilibrium of a shear layer. Both the Winkler model (Naderi and Saidi, 2011; Ozdemir, 2012; Aziz *et al*., 2013; Al-Azzawi, 2016) and the Pasternak model (Malekzadeh, 2009; Hosseini *et al*., 2010; Thai and Choi, 2013; Akavci, 2014) have been used by past researchers to analyse thick plates on elastic foundation. However the use of characteristic orthogonal polynomials (COP) has not been used to analyse thick plates on elastic foundations. This research aims to fill that gap.

2. Theoretical Framework

The shear deformation shape function was derived from the Soldatos (1992) trigonometric shape function. The soldatos (1992) trigonometric shear function can be expressed as:

$$f(z) = \left[zcosh\left(\frac{1}{2}\right) - tsinh\left(\frac{z}{t}\right)\right] \tag{1}$$

In non-dimensional form; $z = st$

Where "*s*" is a non-dimensional parameter and "*t*" is the thickness of the plate. The trigonometric shear function was plotted graphically across a plate of unit thickness ($-0.5 \leq s \leq 0.5$) with an interval of 0.025. The shear deformation shape function used in this study is the equation of the curve expressed in polynomial form; $f(z) = t\left[\left(\frac{z}{t}\right) - \frac{13}{10}\left(\frac{z}{t}\right)^3\right]$ which can be simplified as; $f(z) = z\left[1 - \frac{13}{10}\left(\frac{z}{t}\right)^2\right]$.

2.1. Assumptions

It is assumed the out-of-plane displacement *w*, is only differentiable in *x* and *y* axes. Consequently the vertical strain, $\varepsilon_z = 0$, and the effect of the out-of-plane normal stress on the response of the plate is small with respect to the other stresses. Thus, it can be neglected (i.e $\sigma_z = 0$. However, in contrast to the CPT, it is also assumed that the vertical line that is initially normal to the middle surface of the plate before bending becomes parabolic after bending.

2.2. Kinematic Relations

In-plane displacements, *u* and *v* are defined mathematically as;

$$u = u_c + u_s;\ v = v_c + v_s \tag{2}$$

u_s and v_s are the shear deformation components of the in-plane displacements and are defined mathematically as,

$$u_s = f(z) \cdot \emptyset_x\ ;\ \ v_s = f(z) \cdot \emptyset_y \tag{3}$$

Where; $\emptyset_x$ = shear rotation in *x*-direction and $\emptyset_y$ = shear rotation in *y*-direction

According to the classical plate theory (CPT) the in–plane displacements, u_c and v_c can be expressed as;

$$u_c = -z\theta_{cx} = -z\frac{dw}{dx};\qquad v_c = -z\theta_{cy} = -z\frac{dw}{dy} \tag{4}$$

Therefore,

$$u = -z\frac{\partial w}{\partial x} + f(z) \cdot \emptyset_x;\quad v = -z\frac{\partial w}{\partial y} + f(z) \cdot \emptyset_y \tag{5}$$

2.3. Strain – Displacement Relations

Strain–displacement relations suitable for an isotropic and homogenous material in three dimensions were developed by Ventsel and Krauthammer (2001) and applying the assumption that the vertical strain, $\varepsilon_z = 0$ to the strain – displacement we have;

$$\boldsymbol{\varepsilon_x} = \frac{\mathrm{d}u}{dx} = -z\frac{\partial^2 w}{\partial x^2} + f(z) \cdot \frac{\partial \emptyset_x}{\partial x}\ ;\ \boldsymbol{\varepsilon_y} = \frac{dv}{dy} = -z\frac{\partial^2 w}{\partial y^2} + f(z) \cdot \frac{\partial \emptyset_y}{\partial y} \tag{6}$$

$$\boldsymbol{\gamma_{xy}} = \frac{du}{dy} + \frac{dv}{dx} = -z\frac{\partial^2 w}{\partial x\,\partial y} + f(z) \cdot \frac{\partial \emptyset_x}{\partial y} - z\frac{\partial^2 w}{\partial x\,\partial y} + f(z) \cdot \frac{\partial \emptyset_y}{\partial x} = -2z\frac{\partial^2 w}{\partial x\,\partial y} + f(z) \cdot \frac{\partial \emptyset_x}{\partial y} + f(z) \tag{7}$$

$$\boldsymbol{\gamma_{xz}} = \frac{du}{dz} + \frac{dw}{dx} = -\frac{\partial w}{\partial x} + f(z) \cdot \frac{\partial \emptyset_x}{\partial z} + \frac{dw}{dx} = f(z) \cdot \frac{\partial \emptyset_x}{\partial z} \tag{8}$$

$$\boldsymbol{\gamma_{yz}} = \frac{dv}{dz} + \frac{dw}{dy} = -\frac{\partial w}{\partial y} + f(z) \cdot \frac{\partial \emptyset_y}{\partial z} + \frac{dw}{dy} = f(z) \cdot \frac{\partial \emptyset_y}{\partial z} \tag{9}$$

Where u, v, w are displacements in x, y, and z directions respectively.

2.4. Constitutive (Stress–Strain) Relations

Ugural (1999) expressed a relationship for stress and strain by the generalized Hooke's law for a three dimensional state of stress valid for an isotropic homogenous material, upon simplification and necessary substitution into the constitutive equations, we obtain the stress – displacement relations as below;

$$\boldsymbol{\sigma_x} = \frac{E}{1-\mu^2}\left[-Z\frac{\partial^2 w}{\partial x^2} + f(z)\cdot\frac{\partial \phi_x}{\partial x} + \mu\left(-z\frac{\partial^2 w}{\partial y^2} + f(z)\cdot\frac{\partial \phi_y}{\partial y}\right)\right] \tag{10}$$

$$\boldsymbol{\sigma_y} = \frac{E}{1-\mu^2}\left[\mu\left(-z\frac{\partial^2 w}{\partial x^2} + f(z)\cdot\frac{\partial \phi_x}{\partial x}\right) + \left(-z\frac{\partial^2 w}{\partial y^2} + f(z)\cdot\frac{\partial \phi_y}{\partial y}\right)\right] \tag{11}$$

$$\boldsymbol{\tau_{xy}} = \frac{E(1-\mu)}{2(1-\mu^2)}\left[-2z\frac{\partial^2 w}{\partial x\,\partial y} + f(z)\cdot\frac{\partial \phi_x}{\partial y} + f(z)\cdot\frac{\partial \phi_y}{\partial x}\right] \tag{12}$$

$$\boldsymbol{\tau_{xz}} = \frac{E(1-\mu)}{2(1-\mu^2)}\left[f(z)\cdot\frac{\partial \phi_x}{\partial z}\right]\ ;\ \boldsymbol{\tau_{yz}} = \frac{E(1-\mu)}{2(1-\mu^2)}\left[f(z)\cdot\frac{\partial \phi_y}{\partial z}\right] \tag{13}$$

2.5. Strain Energy (U)

The Strain energy *U*, stored in a continuum of a plate was given by Ugural (1999) as the product of stress, strain and volume of the continuum. *U* is given mathematically as shown in Equation (14);

$$U = \frac{1}{2}\int_x\int_y\left[\int_z \sigma\cdot\varepsilon dz\right]dxdy \tag{14}$$

Considering strain and stress in *x*, *y*, *z* direction of an elastic body, the strain energy is expressed as;

$$U = \frac{1}{2}\int_x\int_y\left[\int_{-t/2}^{t/2}\left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}\right)dz\right]dxdy \tag{15}$$

Let the sum of the components of the strain energy of the thick plate be U_1. U_1 can therefore be expressed as;

$$U_1 = \sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz} \tag{16}$$

Substituting the stress and strain components and let $\propto = \frac{a}{t}$ and $p = \frac{b}{a}$, upon simplification, we obtain;

$$U = \frac{1}{2}\int_x\int_y\left[\int_{-\frac{t}{2}}^{\frac{t}{2}}\left(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}\right)dz\right]dxdy =$$
$$\frac{D}{2}\int_x\int_y\Bigg[\left[\Delta_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2\Delta_2\cdot\frac{\partial\phi_x}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}\right) + \Delta_3\cdot\left(\frac{\partial\phi_x}{\partial x}\right)^2\right] + \left[2\Delta_1\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 - 2\Delta_2\cdot\frac{\partial\phi_x}{y}\frac{\partial^2 w}{\partial x\partial y} -\right.$$
$$\left.2\Delta_2\cdot\frac{\partial\phi_y}{\partial x}\frac{\partial^2 w}{\partial x\partial y}\right] + \left[\Delta_1\left(\frac{\partial^2 w}{\partial y^2}\right)^2 - 2\Delta_2\frac{\partial\phi_y}{\partial y}\frac{\partial^2 w}{\partial y^2} + \Delta_3\left(\frac{\partial\phi_y}{\partial y}\right)^2\right] + (1+\mu)\left[\Delta_3\cdot\frac{\partial\phi_y}{\partial x}\frac{\partial\phi_x}{\partial y}\right] +$$
$$\frac{(1-\mu)}{2}\left[\Delta_3\left(\frac{\partial\phi_x}{\partial y}\right)^2 + \Delta_3\left(\frac{\partial\phi_y}{\partial x}\right)^2\right] + \frac{(1-\mu)}{2}\cdot\frac{\propto^2}{a^2}\Delta_4\cdot\left[\phi_x^2 + \phi_y^2\right]dxdy \tag{17}$$

Using $f(z) = z\left[1 - \frac{13}{10}\left(\frac{z}{t}\right)^2\right]$, Δ_i values upon calculation can be summarized below;

$\Delta_1 = 1$, $\Delta_2 = 0.805$ $\Delta_3 = 0.655$; $\Delta_4 = 6.48$

2.6. External Work, V on the Thick Plate

The external work due to lateral load uniformly distributed on the plate, *q* is given as:

$$V = -q\iint_{xy} w\,dxdy \tag{18}$$

Where *q* = lateral load and *w* = deflection.

2.7. Potential Energy of the Plate

The potential energy Π_p, of the thick plate is the sum of strain energy, U and the external work, V.

$$\Pi_p = U + V = \frac{D}{2}\int_x\int_y \Bigg[\left[\Delta_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2\Delta_2\cdot\frac{\partial\phi_x}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}\right) + \Delta_3\cdot\left(\frac{\partial\phi_x}{\partial x}\right)^2\right] + \left[2\Delta_1\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 - 2\Delta_2\cdot\frac{\partial\phi_x}{\partial y}\frac{\partial^2 w}{\partial x\partial y} - 2\Delta_2\cdot\frac{\partial\phi_y}{\partial x}\frac{\partial^2 w}{\partial x\partial y}\right] + \left[\Delta_1\left(\frac{\partial^2 w}{\partial y^2}\right)^2 - 2\Delta_2\frac{\partial\phi_y}{\partial y}\frac{\partial^2 w}{\partial y^2} + \Delta_3\left(\frac{\partial\phi_y}{\partial y}\right)^2\right] + (1+\mu)\left[\Delta_3\cdot\frac{\partial\phi_y}{\partial x}\frac{\partial\phi_x}{\partial y}\right] + \frac{(1-\mu)}{2}\left[\Delta_3\left(\frac{\partial\phi_x}{\partial y}\right)^2 + \Delta_3\left(\frac{\partial\phi_y}{\partial x}\right)^2\right] + \frac{(1-\mu)}{2}\cdot\frac{\alpha^2}{a^2}\Delta_4\left[\phi_x^2+\phi_y^2\right]\Bigg]dxdy - q\int_x\int_y w\,dxdy \quad (19)$$

Where $D = \frac{Et^3}{12(1-\mu^2)}$.

3. Methodology

The model of ground behaviour proposed by Pasternak (1954) assumes the existence of shear interaction between the spring elements. This is achieved by connecting the spring elements to a layer of incompressible vertical elements that deform in transverse shear.The deformations and forces maintaining equilibrium in the shear layer are shown in Figures.2b and 2c. For an isotropic linear shear layer in the x–y plane with shear moduli $G_x = G_y = G$.

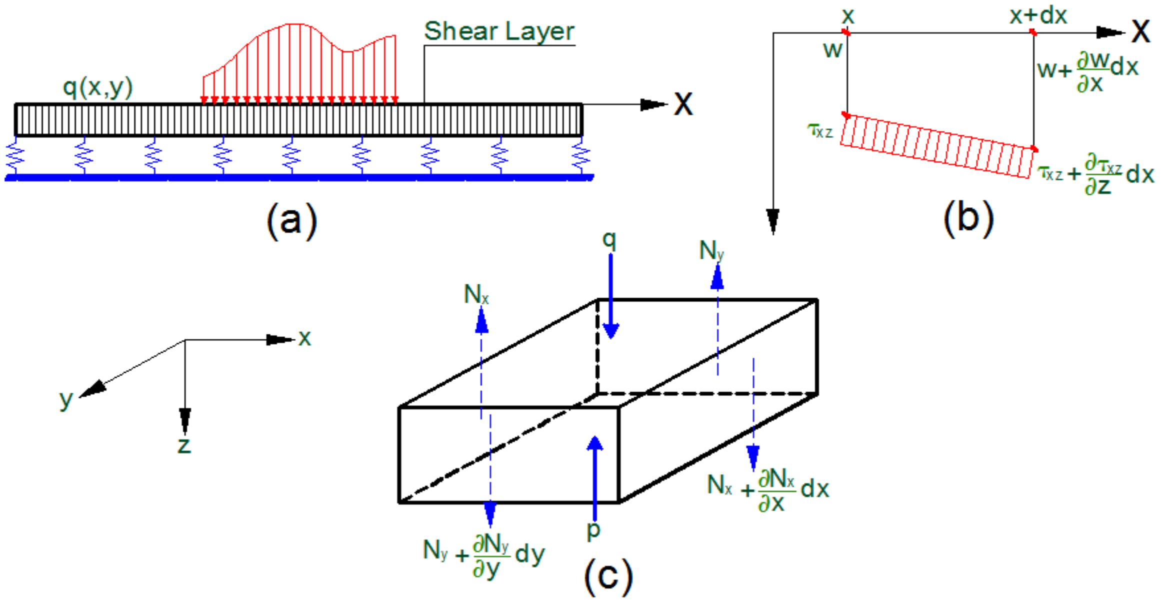

Figure 2. Pasternak model (1954): (a) basic model, (b) stress state of infinitesimal element of shear layer, and (c) forces acting on the shear layer element

The governing equation for the two-parameter Pasternak model is:

$$q(x,y) = k_w w(x,y) - k_p \nabla^2 w(x,y) \quad (20)$$

Where ∇^2 is the Laplace operator,
q = Applied load; k_w and k_p are Winkler and Pasternak foundation parameters respectively and w = deflection.
The Pasternak model is the most reasonable, generalized two-parameter model and is easily conceivable for geotechnical applications as ground exhibits compressibility and deforms in

shear (Madhav, 1998). The expression of strain energy due to Pasternak foundation can be expressed as:

$$U_F = \frac{1}{2}\int_x \int_y \left[K_w w_x^2 + K_p\left\{\left(\frac{dw}{dx}\right)^2 + \left(\frac{dw}{dy}\right)^2\right\}\right] \mathrm{d}x\mathrm{d}y \qquad (21)$$

3.1. The general governing Equations for Thick Plates on Elastic Foundation

To solve the problems of plates on elastic foundations, the usual approach is based on the inclusion of the foundation reactions into the corresponding differential equations of plates (Husain *et al.*, 2007). Hence, the equation of thick plate – soil system:

$$\Pi = U + V + U_F = \frac{D}{2}\int_x \int_y \Bigg[\left[\Delta_1\left(\frac{\partial^2 w}{\partial x^2}\right)^2 - 2\Delta_2 \cdot \frac{\partial \emptyset_x}{\partial x}\left(\frac{\partial^2 w}{\partial x^2}\right) + \Delta_3 \cdot \left(\frac{\partial \emptyset_x}{\partial x}\right)^2\right] + \left[2\Delta_1\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - 2\Delta_2 \cdot \frac{\partial \emptyset_x}{\partial y}\frac{\partial^2 w}{\partial x \partial y} - 2\Delta_2 \cdot \frac{\partial \emptyset_y}{\partial x}\frac{\partial^2 w}{\partial x \partial y}\right] + \left[\Delta_1\left(\frac{\partial^2 w}{\partial y^2}\right)^2 - 2\Delta_2 \frac{\partial \emptyset_y}{\partial y}\frac{\partial^2 w}{\partial y^2} + \Delta_3\left(\frac{\partial \emptyset_y}{\partial y}\right)^2\right] + (1+\mu)\left[\Delta_3 \cdot \frac{\partial \emptyset_y}{\partial x}\frac{\partial \emptyset_x}{\partial y}\right] + \frac{(1-\mu)}{2}\left[\Delta_3\left(\frac{\partial \emptyset_x}{\partial y}\right)^2 + \Delta_3\left(\frac{\partial \emptyset_y}{\partial x}\right)^2\right] + \frac{(1-\mu)}{2} \cdot \frac{\alpha^2}{a^2}\Delta_4\left[\emptyset_x^2 + \emptyset_y^2\right]\Bigg] dxdy + \int_x \int_y \frac{1}{2}\left[K_w w^2 + K_p\left\{\left(\frac{dw}{dx}\right)^2 + \left(\frac{dw}{dy}\right)^2\right\}\right] \mathrm{d}x\mathrm{d}y - q\int_x \int_y w dxdy \qquad (22)$$

Let the domain of x and y be; $x = aR$; $y = bQ$ respectively. Since $P = \frac{b}{a}, \therefore b = Pa, \frac{1}{P} = \frac{a}{b}$, $\frac{1}{P^2} = \frac{a^2}{b^2}$, $\frac{1}{P^4} = \frac{a^4}{b^4}, \frac{1}{P^3} = \frac{a^3}{b^3}$.

Let $w = w_x.w_y$; $\emptyset_x = \emptyset_{xy} \cdot \emptyset_{xx}$; $\emptyset_y = \emptyset_{yx} \cdot \emptyset_{yy}$

Executing closed domain integration on $\frac{d\Pi}{dw} =$ expressed in non-dimensional terms, and letting $w = J_1 h$ we obtain;

$$\emptyset_x = \emptyset_{xy} \cdot \emptyset_{xx} = \frac{1}{\mathrm{a}^2 k_2^2}\frac{\int_0^1\left(\frac{\partial^2 w_x}{\partial R^2}\right)dR}{\int_0^1\left(\frac{\partial \emptyset_{xx}}{\partial R}\right)dQ} \cdot \frac{\int_0^1\left(\frac{\partial^2 w_y}{\partial Q^2}\right)dQ}{\int_0^1\left(\frac{\partial^2 \emptyset_{xy}}{\partial Q^2}\right)dQ}\frac{\partial w}{\partial R} = C_a \cdot J_1\frac{\partial h}{\partial x} = J_2\frac{\partial h}{\partial x} \qquad (23)$$

$$\emptyset_y = \emptyset_{yx}.\emptyset_{yy} = \frac{{n_3}^2}{P^2\mathrm{a}^2 k_2^2}\frac{\int_0^1\left(\frac{\partial^2 w_y}{\partial Q^2}\right)dR}{\int_0^1\left(\frac{\partial \emptyset_{yy}}{\partial Q}\right)dQ} \cdot \frac{\int_0^1\left(\frac{\partial^2 w_x}{\partial R^2}\right)dR}{\int_0^1\left(\frac{\partial^2 \emptyset_{yx}}{\partial R^2}\right)dR}\frac{\partial w}{\partial Q} = C_\mathrm{b} \cdot J_1\frac{\partial h}{\partial y} = J_3\frac{\partial h}{\partial y} \qquad (24)$$

Inserting the values of *w*, $\emptyset_x$ and $\emptyset_y$ into equation (22) and expressing in non-dimensional terms, we obtain equation (25).

$$\Pi = \frac{Dab}{2a^4}\int_0^1 \int_0^1 \Bigg[\left(J_1^2\Delta_1 - 2J_1J_2\Delta_2 + J_2^2\Delta_3\right)\left(\frac{d^2h}{dR^2}\right)^2 + \left(J_1^2\Delta_1 - 2J_1J_3\Delta_2 + J_3^2\Delta_3\right)\frac{1}{P^4}\left(\frac{d^2h}{dQ^2}\right)^2 + \left(2J_1^2\Delta_1 - 2J_1J_2\Delta_2 - 2J_1J_3\Delta_2\right) \cdot \frac{1}{P^2}\left(\frac{d^2h}{dR^2} \cdot \frac{d^2h}{dQ^2}\right) + (1+\mu)J_2J_3\Delta_3 \cdot \frac{1}{P^2}\left(\frac{d^2h}{dR^2} \cdot \frac{d^2h}{dQ^2}\right) + \left[\left(\frac{1-\mu}{2}\right)\left(J_2^2\Delta_3 + J_3^2\Delta_3\right)\right] \cdot \frac{1}{P^2}\left(\frac{d^2h}{dR^2} \cdot \frac{d^2h}{dQ^2}\right) + \propto^2\left(\frac{1-\mu}{2}\right) \cdot \left(J_2^2\Delta_4\right)\left(\frac{dh}{dR}\right)^2 + \frac{\propto^2}{P^2}\left(\frac{1-\mu}{2}\right)\left(J_3^2\Delta_4\right)\left(\frac{dh}{dQ}\right)^2\Bigg] dRdQ + \frac{1}{2}\int_0^1 \int_0^1 \left(k_w abj_1^2 h^2 + k_p abj_1^2\left\{\left(\frac{dh}{dR}\right)^2 + \left(\frac{dh}{dQ}\right)^2\right\}\right) dRdQ - qabJ_1\int_0^1 \int_0^1 [\mathrm{h}]\, \partial R\, \partial Q \qquad (25)$$

Adopting Ritz method of total energy minimization, the general governing equation for thick plates on elastic foundations is obtained by differentiating the total potential energy with respect to the coefficients of deflection w, shear rotation in x-axis $\emptyset_x$ and shear rotation in y-axis $\emptyset_y$ to yield three simultaneous equations. i.e. $\frac{d\Pi}{dj_1} = 0$; $\frac{d\Pi}{dj_2} = 0$ and $\frac{d\Pi}{dj_3} = 0$.

$$\frac{D}{a^4}\int_0^1 \int_0^1 \left[(J_1\Delta_1 - J_2\Delta_2)\left(\frac{d^2h}{dR^2}\right)^2 + (J_1\Delta_1 - J_3\Delta_2)\frac{1}{P^4}\left(\frac{d^2h}{dQ^2}\right)^2 + (2J_1\Delta_1 - J_2\Delta_2 - J_3\Delta_2)\cdot \frac{1}{P^2}\left(\frac{d^2h}{dR^2}\cdot\frac{d^2h}{dQ^2}\right)\right] dRdQ \ + k_w j_1 \int_0^1 \int_0^1 h^2 dRdQ + k_p j_1 \int_0^1 \int_0^1 \left\{\left(\frac{dh}{dR}\right)^2 + \left(\frac{dh}{dQ}\right)^2\right\} dRdQ = q\int_0^1 \int_0^1 [h]\,\partial R\,\partial Q \tag{26}$$

$$\frac{D}{a^4}\int_0^1 \int_0^1 \left[(-J_1\Delta_2 + J_2\Delta_3)\left(\frac{d^2h}{dR^2}\right)^2 + (-J_1\Delta_2)\cdot\frac{1}{P^2}\left(\frac{d^2h}{dR^2}\cdot\frac{d^2h}{dQ^2}\right) + \frac{(1+\mu)}{2}\cdot J_3\Delta_3\frac{1}{P^2}\left(\frac{d^2h}{dR^2}\cdot \frac{d^2h}{dQ^2}\right) + \left(\frac{1-\mu}{2}\right)(J_2\Delta_3)\cdot\frac{1}{P^2}\left(\frac{d^2h}{dR^2}\cdot\frac{d^2h}{dQ^2}\right) + \propto^2\left(\frac{1-\mu}{2}\right)\cdot(J_2\Delta_4)\left(\frac{dh}{dR}\right)^2\right] dRdQ = 0 \tag{27}$$

$$\frac{D}{a^4}\int_0^1 \int_0^1 \left[(-J_1\Delta_2 + J_3\Delta_3)\frac{1}{P^4}\left(\frac{d^2h}{dQ^2}\right)^2 + (-J_1\Delta_2)\cdot\frac{1}{P^2}\left(\frac{d^2h}{dR^2}\cdot\frac{d^2h}{dQ^2}\right) + \frac{(1+\mu)}{2}J_2\Delta_3\cdot\frac{1}{P^2}\left(\frac{d^2h}{dR^2}\cdot \frac{d^2h}{dQ^2}\right) + \left[\left(\frac{1-\mu}{2}\right)(J_3\Delta_3)\right]\cdot\frac{1}{P^2}\left(\frac{d^2h}{dR^2}\cdot\frac{d^2h}{dQ^2}\right) + \frac{\propto^2}{P^2}\left(\frac{1-\mu}{2}\right)(J_3\Delta_4)\left(\frac{dh}{dQ}\right)^2\right] dRdQ = 0 \tag{28}$$

Let $Å_1 = \int_0^1 \int_0^1 \left(\frac{\partial^2 h}{\partial R^2}\right)^2 \partial R\,\partial Q$; $Å_2 = \int_0^1 \int_0^1 \left(\frac{\partial^2 h}{\partial R^2}\cdot\frac{\partial^2 h}{\partial Q^2}\right)\partial R\,\partial Q$; $Å_3 = \int_0^1 \int_0^1 \left(\frac{\partial^2 h}{\partial Q^2}\right)^2 \partial R\,\partial Q$

$Å_4 = \int_0^1 \int^1 \left(\frac{\partial \mathrm{h}}{\partial \mathrm{R}}\right)^2 \partial R\,\partial Q$; $Å_5 = \int_0^1 \int_0^1 \left(\frac{\partial \mathrm{h}}{\partial \mathrm{Q}}\right)^2 \partial R\,\partial Q$; $Å_6 = \int_0^1 \int_0^1 h^2 \partial R \partial Q$; $Å_7 = \int_0^1 \int_0^1 h\,\partial R\,\partial Q$

Equations (26), (27) and (28) can be expressed in matrix form as;

$$\begin{bmatrix}\omega_{11} & \omega_{12} & \omega_{13}\\ \omega_{21} & \omega_{22} & \omega_{23}\\ \omega_{31} & \omega_{32} & \omega_{33}\end{bmatrix}\cdot\begin{bmatrix}J_1\\ J_2\\ J_3\end{bmatrix} = \frac{a^4}{D}\begin{bmatrix}\mathrm{q}Å_7\\ 0\\ 0\end{bmatrix} \tag{29}$$

Where:

$$\omega_{11} = \Delta_1\left(Å_1 + \frac{2Å_2}{P^2} + \frac{Å_3}{P^4}\right) + k_w Å_7 + k_p(Å_4 + Å_5); \quad \omega_{12} = \omega_{21} = -\Delta_2\left(Å_1 + \frac{Å_2}{P^2}\right) \tag{30}$$

$$\omega_{13} = A_{31} = -\Delta_2\left(\frac{Å_2}{P^2} + \frac{Å_3}{P^4}\right) \ ; \quad \omega_{23} = \omega_{32} = \left(\frac{1+\mu}{2P^2}\right)Å_2\Delta_3 \tag{31}$$

$$\omega_{22} = Å_1\Delta_3 + \left(\frac{1-\mu}{2P^2}\right)Å_2\Delta_3 + \left(\frac{1-\mu}{2}\right)\propto^2 Å_4\Delta_4 \tag{32}$$

$$\omega_{33} = \left(\frac{1-\mu}{2P^2}\right)Å_2\Delta_3 + \frac{Å_3}{P^4}\Delta_3 + \left(\frac{1-\mu}{2P^2}\right)\propto^2 Å_5\Delta_4 \ ; \quad Å_7 = \int_0^1 \int_0^1 h\,\partial R\,\partial Q \tag{33}$$

4. Results and Discussion

4.1. Simply Supported (SSSS) Edge Condition

The boundary conditions for SSSS edge are as follows;

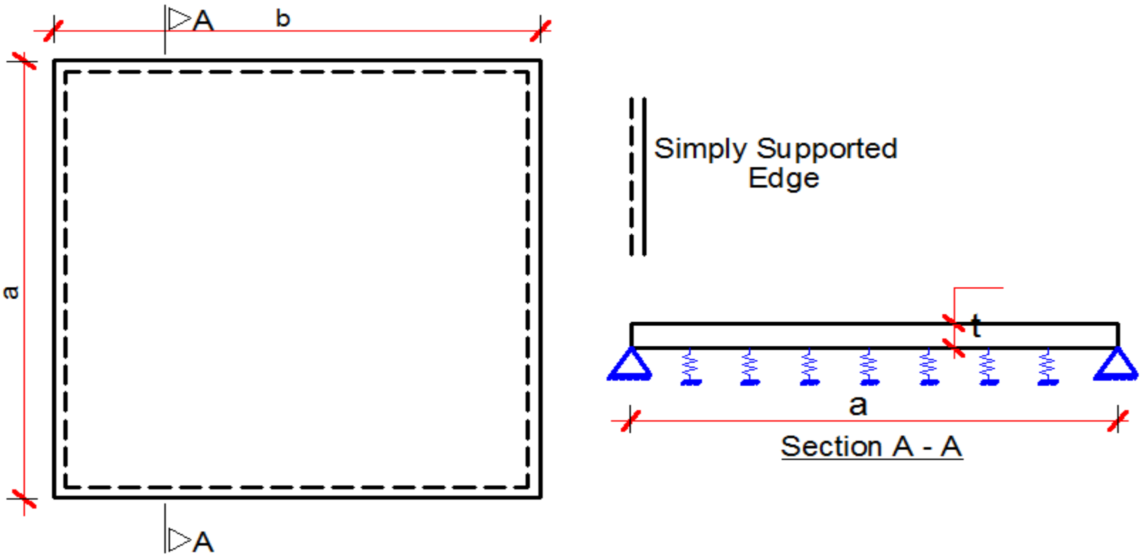

Figure 3. Simply supported edge condition

I. At $R = 0$ and at $R = 1$, the deflection $w_x = 0$

II. At $R = 0$ and at $R = 1$, Bending moment = 0, *i.e.* $\frac{\partial^2 w_x}{\partial R^2} = 0$

Substituting conditions:

$a_0 = 0;\ a_2 = 0;\ a_3 = -2a_4$ and $a_1 = a_4$

$$w_x = a_4(R - 2R^3 + R^4) \tag{34}$$

Similarly, we can obtain:

$$w_y = b_4(Q - 2Q^3 + Q^4) \tag{35}$$

4.1.1. SSSS Rectangular Plate Particular Shape Function

The deflection equation is obtained by multiplying equations (34) and (35).

$$w = a_4(R - 2R^3 + R^4) \cdot b_4(Q - 2Q^3 + Q^4) \tag{36}$$

However since; $w = Ah = A(R - 2R^3 + R^4) \cdot (Q - 2Q^3 + Q^4)$

Where; $A = a_4 b_4$ And $h = (R - 2R^3 + R^4) \cdot (Q - 2Q^3 + Q^4)$

h is the shape function for SSSS thick plate, 'A' is the coefficient of deflection. Upon substitution and simplification we obtain;

$\text{Å}_1 = 0.23619$; $\text{Å}_2 = 0.23592$; $\text{Å}_3 = 0.23619$; $\text{Å}_4 = 0.0239$; $\text{Å}_5 = 0.0239$; $\text{Å}_6 = 0.002421$; $\text{Å}_7 = 0.04$

And as already established $\Delta_1 = 1$, $\Delta_2 = 0.805$; $\Delta_3 = 0.655$; $\Delta_4 = 6.48$ and $\propto = \frac{a}{t}$ and $p = \frac{b}{a}$.

4.1.2. Numerical Example

The deflection at the center of SSSS isotropic thick plate, the in-plane stresses and the vertical shear stresses at the edges of the plate for different span to depth ratio corresponding to different values of $\frac{b}{a}$, k_w and k_p subjected to a uniformly distributed load q and μ = 0.3 is shown in Table 1.

Results are presented in the form:

$$\overline{\boldsymbol{w}} = \frac{\mathrm{E}t^3}{\mathrm{q}a^4} w \qquad (\overline{\boldsymbol{u}}\ \overline{\boldsymbol{v}}) = \frac{\mathrm{E}t^2}{\mathrm{q}a^3}(u, v);\ (\overline{\boldsymbol{\sigma}}_{\boldsymbol{x}}, \overline{\boldsymbol{\sigma}}_{\boldsymbol{y}}) = \frac{t^2}{\mathrm{q}a^2}(\sigma_x, \sigma_y)$$

$$\overline{\boldsymbol{\tau}}_{\boldsymbol{xy}} = \frac{t^2}{\mathrm{q}a^2}\tau_{xy} \quad (\overline{\boldsymbol{\tau}}_{\boldsymbol{zx}}, \overline{\boldsymbol{\tau}}_{\boldsymbol{yz}}) = \frac{t}{\mathrm{q}a}(\tau_{zx}, \tau_{yz})$$

Table 1.

Results for non-dimensional stresses for SSSS plate on Pasternak foundation (p = 1; p = 2)

$\overline{w}$ **at** $\left(x = \frac{a}{2}, y = \frac{b}{2}\right)$; u **at** $\left(x = 0, y = \frac{b}{2}\right)$; $\overline{v}$ **at** $\left(x = \frac{a}{2}, y = 0\right)$; $(\overline{\sigma}_x, \overline{\sigma}_y)$ **at** $\left(x = \frac{a}{2}, y = \frac{b}{2}\right)$; $\overline{\tau}_{xy}$ **at** $(x = 0, y = 0)$; $\overline{\tau}_{zx}$ **at** $\left(x = 0, y = \frac{b}{2}\right)$; $\overline{\tau}_{yz}$ **at** $\left(x = \frac{a}{2}, y = 0\right)$ **and** $\left(p = \frac{b}{a} = 1\right)$ ***subject to uniformly distributed load, q***

k_p	k_w	α	$\overline{w}$	$\overline{u}$	$\overline{v}$	$\overline{\sigma}_x$	$\overline{\sigma}_y$	$\overline{\tau}_{xy}$	$\overline{\tau}_{zx}$	$\overline{\tau}_{yz}$
		5	0.055350	-0.07491	-0.07491	0.321046	0.321046	-0.1844	0.388893	0.388893
		10	0.047724	-0.07294	-0.07294	0.312601	0.312601	-0.17955	0.389601	0.389601
	0	50	0.045278	-0.07231	-0.07231	0.309892	0.309892	-0.17799	0.389829	0.389829
		100	0.045202	-0.07229	-0.07229	0.309808	0.309808	-0.17794	0.389836	0.389836
		1000	0.045176	-0.07228	-0.07228	0.309780	0.309780	-0.17792	0.389838	0.389838
		5	0.001708	-0.00231	-0.00231	0.009904	0.009904	-0.00569	0.011997	0.011997
		10	0.001699	-0.00260	-0.00260	0.011130	0.011130	-0.00639	0.013871	0.013871
0	10000	50	0.001696	-0.00271	-0.00271	0.011607	0.011607	-0.00667	0.014601	0.014601
		100	0.001696	-0.00271	-0.00271	0.011623	0.011623	-0.00668	0.014625	0.014625
		1000	0.001696	-0.00271	-0.00271	0.011628	0.011628	-0.00668	0.014633	0.014633
		5	0.000176	-0.00024	-0.00024	0.001019	0.001019	-0.00059	0.001234	0.001234
		10	0.000176	-0.00027	-0.00027	0.001150	0.001150	-0.00066	0.001433	0.001433
	100000	50	0.000176	-0.00028	-0.00028	0.001201	0.001201	-0.00069	0.001511	0.001511
		100	0.000176	-0.00028	-0.00028	0.001203	0.001203	-0.00069	0.001514	0.001514
		1000	0.000176	-0.00028	-0.00028	0.001203	0.001203	-0.00069	0.001515	0.001515
		5	0.001676	-0.00227	-0.00227	0.009718	0.009718	-0.00558	0.011772	0.011772
		10	0.001667	-0.00255	-0.00255	0.010922	0.010922	-0.00627	0.013612	0.013612
	10000	50	0.001664	-0.00266	-0.00266	0.011391	0.011391	-0.00654	0.014329	0.014329
		100	0.001664	-0.00266	-0.00266	0.011406	0.011406	-0.00655	0.014353	0.014353
		1000	0.001664	-0.00266	-0.00266	0.011411	0.011411	-0.00655	0.014360	0.014360
10		5	0.000175	-0.00024	-0.00024	0.001017	0.001017	-0.00058	0.001232	0.001232
		10	0.000175	-0.00027	-0.00027	0.001148	0.001148	-0.00066	0.001430	0.001430
	100000	50	0.000175	-0.00028	-0.00028	0.001199	0.001199	-0.00069	0.001508	0.001508
		100	0.000175	-0.00028	-0.00028	0.001201	0.001201	-0.00069	0.001511	0.001511
		1000	0.000175	-0.00028	-0.00028	0.001201	0.001201	-0.00069	0.001512	0.001512
		5	0.001645	-0.00223	-0.00223	0.009539	0.009539	-0.00548	0.011555	0.011555
		10	0.001637	-0.00250	-0.00250	0.010722	0.010722	-0.00616	0.013363	0.013363
20	10000	50	0.001634	-0.00261	-0.00261	0.011182	0.011182	-0.00642	0.014067	0.014067
		100	0.001634	-0.00261	-0.00261	0.011197	0.011197	-0.00643	0.014090	0.014090
		1000	0.001634	-0.00261	-0.00261	0.011202	0.011202	-0.00643	0.014098	0.014098

	100000	5	0.000175	-0.00024	-0.00024	0.001015	0.001015	-0.00058	0.001229	0.001229
		10	0.000175	-0.00027	-0.00027	0.001145	0.001145	-0.00066	0.001427	0.001427
		50	0.000175	-0.00028	-0.00028	0.001197	0.001197	-0.00069	0.001505	0.001505
		100	0.000175	-0.00028	-0.00028	0.001198	0.001198	-0.00069	0.001508	0.001508
		1000	0.000175	-0.00028	-0.00028	0.001199	0.001199	-0.00069	0.001509	0.001509

$\bar{w}$ at $\left(x = \frac{a}{2}, y = \frac{b}{2}\right)$; u at $\left(x = 0, y = \frac{b}{2}\right)$; $\bar{v}$ at at $\left(x = \frac{a}{2}, y = 0\right)$; $(\bar{\sigma}_x, \bar{\sigma}_y)$ at $\left(x = \frac{a}{2}, y = \frac{b}{2}\right)$; $\bar{\tau}_{xy}$ at $(x = 0, y = 0)$; $\bar{\tau}_{zx}$ at $\left(x = 0, y = \frac{b}{2}\right)$; $\bar{\tau}_{yz}$ at $\left(x = \frac{a}{2}, y = 0\right)$ and $\left(p = \frac{b}{a} = 2\right)$. *subject to uniformly distributed load, q*

k_p	k_w	α	$\bar{w}$	$\bar{u}$	$\bar{v}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{zx}$	$\bar{\tau}_{yz}$
0	0	5	0.131911	-0.18921	-0.0473	0.670556	0.343075	-0.23288	0.622423	0.311212
		10	0.119703	-0.18606	-0.04651	0.659377	0.337355	-0.22899	0.623154	0.311577
		50	0.11579	-0.18505	-0.04626	0.655794	0.335522	-0.22775	0.623389	0.311694
		100	0.115668	-0.18501	-0.04625	0.655682	0.335465	-0.22771	0.623396	0.311698
		1000	0.115627	-0.185	-0.04625	0.655645	0.335446	-0.2277	0.623398	0.311699
	10000	5	0.001739	-0.00249	-0.00062	0.008838	0.004522	-0.00307	0.008204	0.004102
		10	0.001736	-0.0027	-0.00067	0.009565	0.004894	-0.00332	0.009039	0.004520
		50	0.001736	-0.00277	-0.00069	0.009829	0.005029	-0.00341	0.009344	0.004672
		100	0.001735	-0.00278	-0.00069	0.009838	0.005033	-0.00342	0.009353	0.004677
		1000	0.001735	-0.00278	-0.00069	0.009841	0.005035	-0.00342	0.009357	0.004678
	100000	5	0.000176	-0.00025	-6.3E-05	0.000894	0.000458	-0.00031	0.000830	0.000415
		10	0.000176	-0.00027	-6.8E-05	0.000969	0.000496	-0.00034	0.000916	0.000458
		50	0.000176	-0.00028	-7E-05	0.000996	0.000510	-0.00035	0.000947	0.000474
		100	0.000176	-0.00028	-7E-05	0.000997	0.000510	-0.00035	0.000948	0.000474
		1000	0.000176	-0.00028	-7E-05	0.000998	0.000510	-0.00035	0.000948	0.000474
10	10000	5	0.001705	-0.00245	-0.00061	0.00867	0.004436	-0.00301	0.008047	0.004024
		10	0.001703	-0.00265	-0.00066	0.009382	0.004800	-0.00326	0.008867	0.004433
		50	0.001702	-0.00272	-0.00068	0.009642	0.004933	-0.00335	0.009165	0.004583
		100	0.001702	-0.00272	-0.00068	0.00965	0.004937	-0.00335	0.009175	0.004588
		1000	0.001702	-0.00272	-0.00068	0.009653	0.004939	-0.00335	0.009178	0.004589
	100000	5	0.000176	-0.00025	-6.3E-05	0.000893	0.000457	-0.00031	0.000829	0.000414
		10	0.000176	-0.00027	-6.8E-05	0.000967	0.000495	-0.00034	0.000914	0.000457
		50	0.000176	-0.00028	-7E-05	0.000994	0.000509	-0.00035	0.000945	0.000473
		100	0.000176	-0.00028	-7E-05	0.000995	0.000509	-0.00035	0.000946	0.000473
		1000	0.000176	-0.00028	-7E-05	0.000996	0.000509	-0.00035	0.000947	0.000473
20	10000	5	0.001673	-0.0024	-0.0006	0.008507	0.004352	-0.00295	0.007896	0.003948
		10	0.001671	-0.0026	-0.00065	0.009206	0.004710	-0.0032	0.008701	0.00435
		50	0.001671	-0.00267	-0.00067	0.009461	0.004841	-0.00329	0.008994	0.004497
		100	0.001671	-0.00267	-0.00067	0.00947	0.004845	-0.00329	0.009003	0.004502
		1000	0.001671	-0.00267	-0.00067	0.009472	0.004846	-0.00329	0.009006	0.004503
	100000	5	0.000175	-0.00025	-6.3E-05	0.000891	0.000456	-0.00031	0.000827	0.000413
		10	0.000175	-0.00027	-6.8E-05	0.000965	0.000494	-0.00034	0.000912	0.000456
		50	0.000175	-0.00028	-7E-05	0.000992	0.000508	-0.00034	0.000943	0.000472
		100	0.000175	-0.00028	-7E-05	0.000993	0.000508	-0.00034	0.000944	0.000472
		1000	0.000175	-0.00028	-7E-05	0.000994	0.000508	-0.00035	0.000945	0.000472

From Table 1, the effect of the foundation modulus of subgrade reaction, span-depth ratio and aspect ratio of the plate can be observed. The value of in-plane quantities and that of out-of-plane quantities generally reduced as the foundation modulus increased. Similarly, it is observed that as the span-depth ratio increases, values of in-plane quantities and that of out-of-plane quantities decreases for each aspect ratio, p. It can also be observed generally that as the aspect ratio increases, the values of in-plane quantities and that of out-of-plane quantities increases. However, worthy of note is that in-plane quantities being functions of *x, y* and *z*, vary linearly with the plate thickness but the out-plane displacements, function of only *x* and *y*, do not vary linearly with the plate thickness.

Table 2.
Results for non-dimensional stresses for SSSS plate

Study	k_p k_w α	$\bar{w} = \frac{100Et^3}{qa^4}w$	$\bar{u} = \frac{Et^2}{qa^3}u$	$\bar{\sigma}_x = \frac{t^2}{qa^2}\sigma_x$	$\bar{\sigma}_y = \frac{t^2}{qa^2}\sigma_y$	$\bar{\tau}_{xy} = \frac{t^2}{qa^2}\tau_{xy}$	$\bar{\tau}_{zx} = \frac{t}{qa}\tau_{zx}$	**Plate theory**
Present study		4.772441	-0.07294	0.312601	0.312601	-0.17955	0.389601	TRDT
Ibearugbulem et al		4.7723	-0.073	0.3127	0.3127	-0.1796	0.392	TRDT
% difference		-0.00295	0.082192	0.03166	0.03166	0.02784	0.61199	
Karama		4.797	-0.0723	0.3243	0.3243	-0.1746	0.3909	TSDT
% difference		0.511966	-0.8852	3.607462	3.607462	-2.83505	0.33231	
Mindlin	0 0 10	4.6701	-0.0741	0.287	-	-0.1951	0.3331	FSDT
% difference		-2.19141	1.565452	-8.92021	-	7.970272	-16.9622	
Reddy		4.666	-0.075	0.289	-	-0.2031	0.492	HSDT
% difference		-2.2812	2.746667	-8.16644	-	11.59527	20.8128	
Kirchoff		4.436	-0.0723	0.287	0.287	-0.195	0	CPT
% difference		-7.58433	-0.8852	-8.92021	-8.92021	7.923077	100	
present		4.520167	-0.07229	0.309808	0.309808	-0.17794	0.389836	TRDT
Ibearugbulem et al		4.5203	-0.0723	0.3099	0.3099	-0.178	0.3802	TRDT
% difference		0.002942	0.013831	0.029687	0.029687	0.033708	-2.53446	
Karama		4.5201	-0.0723	0.3098	0.3098	-0.1779	0.392	TSDT
% difference		-0.00148	0.013831	-0.00258	-0.00258	-0.02248	0.552041	
Mindlin	0 0 100	4.546	-0.0714	0.3203	0.3203	-0.1725	0.3909	FSDT
% difference		0.568258	-1.2465	3.275679	3.275679	-3.15362	0.272192	
Reddy		4.546	-0.0714	0.3203	0.3203	-0.1725	0.3892	HSDT
% difference		0.568258	-1.2465	3.275679	3.275679	-3.15362	-0.16341	
Kirchhoff		4.5201	-0.0723	0.3099	0.3099	-0.179	0	CPT
% difference		-0.00148	0.013831	0.029687	0.029687	0.592179	100	

Table 2 shows a comparison of the results obtained without the influence of foundation parameters k_w *and* k_p, it is observed that results obtained in this study converges with that obtained in third order deformation theories while significant difference was observed in higher order shear deformation theory, classical plate theory and first order deformation theory. This may be due to the fact that the classical plate theory and the first order shear deformation theory do not exactly inculcate the shear deformation shape function. Worthy of note is the fact that there was approximately no difference between the out-of-plane displacements and in-plane obtained in this study and that obtained using the classical plate theory at span-depth ratios of 100 and above. This further validates the consideration of plates as thin for span-depth ratio of 100 and above.

When compared with finite element analysis, the difference between the deflection results obtained from this study and finite element analysis without the consideration for foundation parameters is shown in Table 3.

Table 3.
Comparison of values obtained in this study with FEM

α	**Thickness of plate (mm)**	**Deflection from present study (mm)**	**Deflection from FEM (mm)**	**Percentage difference**
5	1000	0.013837	0.0144	-4.068%
10	500	0.095448	0.097	-1.626%
50	100	11.3195	11.225	0.8348%
100	50	90.404	89.523	0.9745%
1000	5	90352	89429.482	1.021%

$a = b = 5$m; $E = 2.5 \times 10^7$ kN/m^2; $\mu = 0.3$

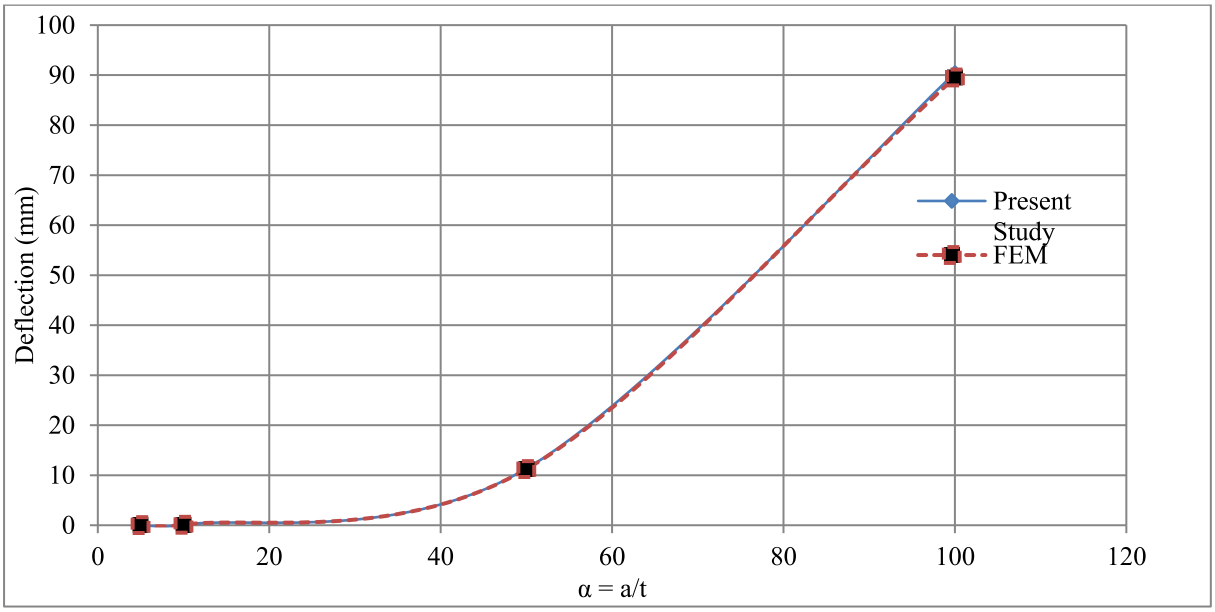

Figure 4. A graph of deflection obtained in this study and that obtained with FEM

5. Conclusion

Analysis of thick plates resting on Winkler-Pasternak two parameter foundation using characteristic orthogonal polynomials was carried out in this study. The shape function derived from the Soldatos (1992) trigonometric shape function was found to be sufficient and satisfactory for obtaining the solution to plates on elastic foundation and the resulting governing equations can be satisfied throughout the domain of the plate. The method proposed in this study provides a simplified method as only definite integration is needed to obtain solutions.

Energy approach (Ritz method) was also found to be satisfactory in determining the potential energy of the thick plate resting on Winkler-Pasternak elastic foundation under different boundary conditions. Foundation parameters greatly affected the in-plane and out-of-plane displacements of the plates. As the foundation modulus of the soil increased, the in-plane and out-of-plane displacements of the plates decreased.

At span-depth ratios of 100 and above, the in-plane and out-of-plane displacements of the plates obtained in this study were approximately the same with those obtained in the Classical plate theory (CPT). This further validates the fact that plates with span-depth ratios of 100 and above, a plate is considered to be thin.

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