*Corresponding Author's Email: ismailyenilmez@eskisehir.edu.tr
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A New Approach Using the Lindley Distribution in Stochastic Frontier Analysis

İsmail Yenilmez
Eskişehir Technical University, Science Faculty, Department of Statistics, Türkiye

Abstract

Stochastic Frontier Analysis plays a crucial role in assessing technical efficiency and modelling production processes across various disciplines. Traditionally, SFA assumes specific error distributions, such as the normal distribution for random effects and the half-normal distribution for technical efficiency. However, the choice of error distributions can significantly impact model estimation and interpretation. This study proposes a novel approach by incorporating the Lindley distribution as a flexible error distribution in SFA, termed Lindley Stochastic Frontier Analysis (L-SFA). This extension offers a more detailed representation of the error structure, potentially enhancing the accuracy of efficiency estimates. The derivation and solution of maximum likelihood estimators for the theoretical foundations of L-SFA are provided. Furthermore, a simulation study demonstrates the advantages of L-SFA over traditional SFA. The findings underscore the importance of flexible error distributions in capturing the complexities of production processes with this new SFA extension.

Keywords: Efficiency, Error distribution, Lindley stochastic frontier analysis, Maximum likelihood estimation, Simulation

1. Introduction

For benchmarking, two dominant methods are Data Envelopment Analysis (DEA) and Stochastic Frontier Model (SFM). DEA is non-parametric, while SFM is a parametric method. In the parametric approach, the production function has a specific functional form with unknown parameters, as represented in Eq. 1:

$$f(x) = f(x;\beta) \tag{1}$$

For $v_i \in R$ and $u_i \in R^+$, additional forms are obtained and represented as follows:

$$f(x) = f(x;\beta) + v \tag{1.a}$$

$$f(x) = f(x;\beta) - u \tag{1.b}$$

$$f(x) = f(x;\beta) + v - u \tag{1.c}$$

These forms correspond to the regression, deterministic, and stochastic models, respectively. In this study, we focus on SFM, represented by equation (1.c). SFM was introduced by Aigner, Lovell, and Schmidt in 1977, and independently proposed by Meeusen and van den Broeck (1977). SFM is used to estimate technical efficiency and model production (Battese and Coelli, 1992). The model includes an error term with two components, as represented in Eq. 2:

$$\varepsilon_i = v_i - u_i \tag{2}$$

where v_i and u_i represent noise and inefficiency, respectively. It is assumed that $v_i \sim N(0, \sigma_v^2)$ and $u_i \sim |N(0, \sigma_u^2)|$. In this study, it is assumed that v_i is distributed as the Lindley distribution (LD), resulting in a more flexible model for SFM. Moreover, the relationship between the Lindley distribution and other distributions commonly used in the literature (such as exponential and gamma) provides significant motivation for exploring LD in the context of SFM.

In the literature, half-normal and exponential distributions have been employed by Aigner et al. (1977) for modeling u_i, and these models are frequently utilized for efficiency analysis. Models using half-normal and exponential distributions for the one-sided error (u_i) represent inefficiency, while the two-sided error (v_i) representing noise follows a normal distribution. There are numerous studies that focus on the distribution of u_i, including truncated-normal (Stevenson, 1980), gamma (Greene, 1990), binomial (Carree, 2002), Weibull (Tsionas, 2007), mixture (Kumbhakar et al., 2013), and double truncated normal (Almanidis et al., 2014) distributions. These studies generally assume that the two-sided error (v_i) is normally distributed. However, the Laplace distribution has also been used in SFM for the two-sided error (Horrace and Parmeter, 2014).

In this study, we propose that the two-sided error (v_i) representing noise is normally distributed, while the one-sided error (u_i) representing inefficiency follows a one-parameter Lindley distribution. This leads to the introduction of the Lindley-Stochastic Frontier Model (L-SFM) as an alternative approach for efficiency measurement. In this context, the second section briefly discusses the one-parameter Lindley distribution, introduces the L-SFM based on LD, and outlines its estimation methods. In the third section, the estimators commonly used in the literature are compared with the proposed estimator through a simulation study. Finally, the findings are presented in the results section.

2. Literature

The stochastic frontier model (SFM) has evolved as a critical tool in econometrics for analysing technical efficiency across various sectors. The foundational work by Aigner, Lovell, and Schmidt (1977) laid the groundwork by formulating and estimating stochastic frontier production function models. This approach was pivotal in introducing the concept of a composed error term, where one component captures inefficiency while the other captures random noise. Meeusen and van den Broeck (1977) further contributed to this field by applying the stochastic frontier approach to the Cobb-Douglas production function. Their work emphasized efficiency estimation in the presence of composed error terms, reinforcing the robustness of SFMs in economic research. Subsequent research has expanded on these initial models. Battese and Coelli (1992) introduced panel data into the stochastic frontier framework, allowing for more dynamic analysis of technical efficiency over time, particularly in agricultural settings. This was a significant advancement, enabling the study of efficiency in the context of longitudinal data.

Stevenson (1980) and Greene (1990) both contributed to the generalization of the stochastic frontier model by exploring different distributions for the inefficiency term. Stevenson focused on likelihood functions for generalized stochastic frontier estimation, while Greene proposed a gamma-distributed inefficiency component, offering more flexibility in modelling. Further innovations include Carree's (2002) investigation into the technological inefficiency and the skewness of the error component in SFM, and Tsionas' (2007) application of the Weibull distribution in efficiency measurement. These studies highlight the ongoing refinement of the SFM to better capture the complexities of inefficiency in various economic contexts.

The development of the zero-inefficiency stochastic frontier model by Kumbhakar, Parmeter, and Tsionas (2013) represents another significant advancement, allowing for the possibility that some firms operate on the frontier with no inefficiency. Almanidis, Qian, and Sickles (2014) extended this by introducing bounded inefficiency in SFM, which provides a more realistic approach by setting a lower bound on inefficiency.

Recent studies continue to explore the frontiers of efficiency analysis using SFM. Makieła and Mazur (2022) examined model uncertainty and its implications for efficiency measurement, addressing the challenges posed by generalized error distributions. The applications of SFM in specific contexts, such as the assessment of technical efficiency in Turkish banks (Kantar & Yenilmez, 2017) and universities (Yenilmez et al., 2022; Yenilmez, 2024), demonstrate the versatility and adaptability of SFM in different economic environments.

3. An Alternative for Stochastic Frontier Analysis

This study represents the first known instance of employing the Lindley distribution (LD) to model the one-sided error component in Stochastic Frontier Analysis (SFA). The LD's relationship with other distributions commonly used in SFA makes it a compelling alternative. Accordingly, the Lindley distribution is introduced, and the SFA model based on LD is derived.

3.1. Lindley Distribution

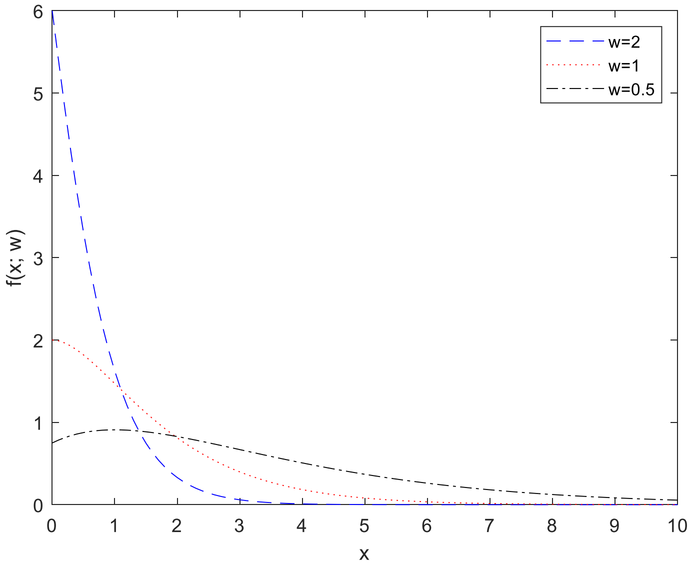
The Lindley distribution is used to model u_i. The probability density function (PDF) $f_u(.)$ and cumulative density function (CDF) $F_u(.)$ of the random variable uuu with parameter www are given as follows:

$$f(u;w) = \frac{w^2}{1+w}(1+u)e^{-wu} \qquad u > 0, \;\; w > 0 \tag{3}$$

$$F(u;w) = 1 - \frac{e^{-wu}(1+w+w^2)}{1+w} \qquad u > 0, \;\; w > 0 \tag{4}$$

Figure 1 illustrates the PDF of the Lindley distribution for selected values of w.

Figure 1: The PDF of the LD for selected values of w



3.2. Lindley Stochastic Frontier Model and Estimation

To estimate the unknown parameters in a Stochastic Frontier Model (SFM), the Maximum Likelihood Estimation (MLE) procedure can be employed. For this, the probability density function (PDF) of the combined error term must be known. In the seminal study by Aigner et al. (1977), the error components v and u are assumed to follow normal and half-normal distributions, respectively. Aigner et al. (1977) addressed the distribution of $\varepsilon_i = v_i - u_i$ and derived the log-likelihood function for the likelihood estimation procedure.

In this study, it is assumed that $v_i \sim N(O, \sigma_v^2)$ and $u_i \sim$Lindley(w). Under these assumptions, the probability density function (PDF) of the combined error term $\varepsilon_i = t_i = v_i - u_i$ is derived as follows:

$$f_t(t) = \int_{-\infty}^{\infty} f_{v,u}(u = v - t, v)du = \int_{-\infty}^{\infty} f_{v,u}(u, v = t + u)du \tag{5}$$

It is assumed that the random variables v and u are independent. Therefore, the joint PDF is found as: $f_{v,u}(v,u) = f_v(v)f_u(u)$. $v_i \sim N(O,\ \sigma_v^2) = N(O,\ s^2)$ and $f_v(v) = \frac{1}{s\sqrt{2\pi}} e^{-\frac{(t+u)^2}{2s^2}}$; $u_i \sim$Lindley(w) and $f_u(u) = \frac{w^2}{1+w}(1+u)e^{-wu}$. If the problem, initially solved as an indefinite integral up to this stage, is now evaluated using specific boundaries as a definite integral, the solution will take the following form:

$$= -\frac{w^2 e^{\frac{s^2w^2}{2}+tw}}{2\sqrt{\pi}(w+1)}\left(\sqrt{\pi}\, erf\left(\frac{\sqrt{2}s^2w+\sqrt{2}t}{2s}\right) + \Gamma\left(\frac{1}{2}, \frac{s^4w^2+2s^2tw+t^2}{2s^2}\right) s^2 w\right)$$

$$-\frac{w^2 e^{\frac{s^2w^2}{2}+tw}}{2\sqrt{\pi}(w+1)}\left(\Gamma\left(\frac{1}{2}, \frac{s^4w^2+2s^2tw+t^2}{2s^2}\right) t - \sqrt{2}\Gamma\left(1, \frac{s^4w^2+2s^2tw+t^2}{2s^2}\right) s - \sqrt{\pi}\right) \tag{6}$$

where $\Gamma(\alpha,\beta)$ is upper incomplete gamma functions and it is assumed thats$^2w + t > 0s > 0$.

To use Maximum Likelihood Estimation (MLE) in Stochastic Frontier Analysis (SFA), the likelihood function $L = f(t_1, t_2, \ldots, t_n) = \prod_{i=1}^{n} f(t_i)$ must be determined. The first step in this process is to find the logarithm of the density function of t; ($log\ f\ (t)$).

$$log\,f\,(t) = log\left(-\frac{w^2 e^{\frac{s^2w^2}{2}+tw}}{2\sqrt{\pi}(w+1)}\left(\sqrt{\pi}\,erf\left(\frac{\sqrt{2}s^2w+\sqrt{2}t}{2s}\right)+\Gamma\left(\frac{1}{2},\frac{s^4w^2+2s^2tw+t^2}{2s^2}\right)s^2w\right)+\ldots\right.$$

$$\left.-\frac{w^2 e^{\frac{s^2w^2}{2}+tw}}{2\sqrt{\pi}(w+1)}\left(\Gamma\left(\frac{1}{2},\frac{s^4w^2+2s^2tw+t^2}{2s^2}\right)t-\sqrt{2}\Gamma\left(1,\frac{s^4w^2+2s^2tw+t^2}{2s^2}\right)s-\sqrt{\pi}\right)\right) \quad (7)$$

The log of the joint density function, also known as the log-likelihood, is expressed as:

$$log\,L = log\,f\,(t_1, t_2, \ldots, t_n) = \sum_{i=1}^{n} log\,f\,(t_i) \quad (8)$$

After taking the partial derivatives of the $log\,L$ function with respect to the parameters of interest, the likelihood equations are obtained.

4. Analysis and Results

A simulation study is conducted to compare the relative bias and Mean Squared Error (MSE) of Maximum Likelihood Estimates (MLEs) under different distributions. The simulation of the Stochastic Frontier Model (SFM) for cross-sectional data of firms is based on production functions. Given the advantages of linear transformation in econometric models, both Cobb-Douglas and trans-log production functions are utilized. The Cobb-Douglas production function and the trans-log production function are presented in equations (30) and (31), respective.

$$log(y_i) = \beta_0 + \sum_{j=1}^{n} \beta_j x_{ijt} \quad (9)$$

$$log(y_i) = \beta_0 + \sum_{j=1}^{n} \beta_j x_{ij} + \sum_{k=1}^{n} \beta_{jk} x_{ijt} x_{ikt} \quad (10)$$

In this context, the Cobb-Douglas production function is utilized due to its ease of implementation and interpretation. The parameters of the Cobb-Douglas production frontier are estimated using the following equation:

$$log(y_i) = \beta_0 + \beta_1\,log(x_i) + \varepsilon_i \qquad \text{for } i = 1,2,\ldots,n \quad (11)$$

where $\varepsilon_i = v_i - u_i$.

In the simulation procedure, 1,000 datasets are generated, with sample sizes of 100, 250, 500, and 750. The error term v_i is assumed to follow a normal distribution, while u_i is distributed as half-normal, exponential, gamma, Weibull, log-normal, and Lindley distributions. The formulas for bias and Mean Squared Error (MSE) are presented as follows:

$$Bias(\hat{y}) = \left(\frac{1}{1000}\sum_{i=1}^{1000} \hat{y}_i\right) - y \quad (12)$$

$$MSE(\hat{y}) = \left(\frac{1}{1000}\sum_{i=1}^{1000} (\hat{y}_i - y)^2\right) \quad (13)$$

The simulation results are presented in Tables 1-3. Table 1 indicates that for the Half-Normal error distribution, the MLE based on the Half-Normal distribution ($MLE_{Half\text{-}Normal}$) generally exhibits lower bias and MSE values as the sample size increases. This is particularly evident in larger samples (n = 500 and n = 750), where the $MLE_{Half\text{-}Normal}$ achieves the lowest MSE (0.003) compared to $MLE_{Exponential}$ and $MLE_{Lindley}$. Under the Exponential error distribution, the MLE based on the Exponential distribution ($MLE_{Exponential}$) consistently shows lower MSE values across all sample sizes compared to $MLE_{Half\text{-}Normal}$ and $MLE_{Lindley}$. This trend is particularly strong for larger sample sizes (n = 500 and n = 750), where $MLE_{Exponential}$ achieves the lowest MSE (0.020) and relatively low bias. The $MLE_{Lindley}$, while performing reasonably well under the Exponential distribution for larger sample sizes, shows higher MSE values under the Half-Normal error distribution, indicating its reduced effectiveness when the error term does not align with the Lindley distribution.

Table 2 reveals that for the Gamma error distribution, the results show that $MLE_{Half\text{-}Normal}$ and $MLE_{Exponential}$ tend to have lower bias and MSE in smaller sample sizes (n = 100 and n = 250). However, as the sample size increases to n = 500 and n = 750, $MLE_{Exponential}$ performs slightly better in terms of MSE. Under the Weibull error distribution, $MLE_{Lindley}$ demonstrates strong performance with lower MSE values, particularly in larger samples (n = 500 and n = 750). This suggests that the Lindley distribution's shape flexibility may offer advantages in capturing the distributional characteristics of the Weibull error term.

Table 1: Simulation results for estimators under Half-Normal and Exponential error distributions for u_i

	$\varepsilon_i = t_i = v_i - u_i$	v_i	u_i	v_i	u_i
	Distribution	**Normal**	**Half-Normal**	**Normal**	**Exponential**
n	**Estimators**	**Bias**	**MSE**	**Bias**	**MSE**
100	$MLE_{Half\text{-}Normal}$	0.074	0.083	0.895	0.272
	$MLE_{Exponential}$	0.190	0.080	0.078	0.332
	$MLE_{Lindley}$	1.532	0.217	-0.077	0.230
250	$MLE_{Half\text{-}Normal}$	0.145	0.017	-0.121	0.073
	$MLE_{Exponential}$	0.014	0.027	-0.017	0.069
	$MLE_{Lindley}$	0.446	0.178	0.710	0.065
500	$MLE_{Half\text{-}Normal}$	0.296	0.010	0.032	0.050
	$MLE_{Exponential}$	0.033	0.010	-0.090	0.045
	$MLE_{Lindley}$	0.835	0.151	0.875	0.058
750	$MLE_{Half\text{-}Normal}$	0.191	0.003	-0.015	0.025
	$MLE_{Exponential}$	0.008	0.004	-0.004	0.020
	$MLE_{Lindley}$	1.560	0.058	0.741	0.030

Table 2: Simulation results for estimators under Gamma and Weibull error distributions for u_i

	$\varepsilon_i = t_i = v_i - u_i$	v_i	u_i	v_i	u_i
	Distribution	**Normal**	**Gamma**	**Normal**	**Weibull**
n	**Estimators**	**Bias**	**MSE**	**Bias**	**MSE**
100	$MLE_{Half\text{-}Normal}$	-0.413	0.253	-0.353	0.140
	$MLE_{Exponential}$	-0.323	0.224	-0.175	0.140
	$MLE_{Lindley}$	1.834	0.719	0.589	0.253
250	$MLE_{Half\text{-}Normal}$	0.452	0.089	0.096	0.076
	$MLE_{Exponential}$	0.071	0.083	-0.002	0.069
	$MLE_{Lindley}$	0.791	0.327	0.942	0.065
500	$MLE_{Half\text{-}Normal}$	0.070	0.058	0.014	0.059
	$MLE_{Exponential}$	0.117	0.054	0.040	0.055
	$MLE_{Lindley}$	1.065	0.043	0.975	0.050
750	$MLE_{Half\text{-}Normal}$	-0.033	0.034	-0.049	0.032
	$MLE_{Exponential}$	-0.030	0.034	-0.031	0.032

	$MLE_{Lindley}$	0.781	0.029	1.118	0.024

Table 3: Simulation results for estimators under Log-Normal and Lindley error distributions for u_i

	$\varepsilon_i = t_i = v_i - u_i$	v_i	u_i	v_i	u_i
	Distribution	**Normal**	**Log-Normal**	**Normal**	**Lindley**
n	**Estimators**	**Bias**	**MSE**	**Bias**	**MSE**
100	$MLE_{Half\text{-}Normal}$	0.193	0.240	0.226	0.601
	$MLE_{Exponential}$	-0.045	0.176	-0.502	0.361
	$MLE_{Lindley}$	2.034	0.187	-0.087	0.360
250	$MLE_{Half\text{-}Normal}$	0.116	0.197	-0.037	0.362
	$MLE_{Exponential}$	0.054	0.147	0.094	0.176
	$MLE_{Lindley}$	0.116	0.155	0.040	0.168
500	$MLE_{Half\text{-}Normal}$	-0.011	0.141	0.207	0.129
	$MLE_{Exponential}$	0.193	0.050	0.069	0.105
	$MLE_{Lindley}$	0.126	0.041	0.032	0.100
750	$MLE_{Half\text{-}Normal}$	0.001	0.038	-0.039	0.087
	$MLE_{Exponential}$	-0.324	0.017	-0.039	0.023
	$MLE_{Lindley}$	0.003	0.016	-0.043	0.012

Table 3 shows that for the Log-Normal error distribution, $MLE_{Exponential}$ tends to outperform $MLE_{Half\text{-}Normal}$ and $MLE_{Lindley}$, especially in smaller sample sizes (n = 100 and n = 250). The $MLE_{Exponential}$ achieves the lowest MSE (0.017) in the largest sample size (n = 750), indicating its robustness in modeling Log-Normal errors. When the error term follows the Lindley distribution, $MLE_{Lindley}$ is the most effective estimator, achieving the lowest MSE values across all sample sizes. This is especially evident in larger samples (n = 500 and n = 750), where $MLE_{Lindley}$ outperforms both $MLE_{Half\text{-}Normal}$ and $MLE_{Exponential}$. This confirms the model's superiority when the error structure aligns with the Lindley distribution.

5. Conclusion

The Stochastic Frontier Analysis (SFA) remains a vital methodology for assessing technical efficiency. This study advances the classical SFA framework by introducing the Lindley Stochastic Frontier Model (L-SFA), which incorporates the Lindley error distribution to enhance flexibility and performance in efficiency estimation. Through a comprehensive simulation study, the effectiveness of different Maximum Likelihood Estimators (MLEs) under various error distributions was examined.

Across all models and distributions, the accuracy and reliability of the estimators improve with increasing sample sizes, as evidenced by the decreasing Mean Squared Error (MSE) values. This trend highlights the benefit of larger datasets in obtaining precise efficiency estimates, particularly for practitioners utilizing the L-SFA model. The ability of $MLE_{Lindley}$ to produce more reliable results with larger samples is especially beneficial, confirming that the L-SFA model is well-suited for extensive datasets, thereby enhancing the robustness and precision of the efficiency estimates as more data becomes available.

The $MLE_{Lindley}$ demonstrates considerable robustness and competitive performance across various error distributions, not just under its native Lindley distribution. While it excels when the true error distribution aligns with the Lindley form, $MLE_{Lindley}$ also provides relatively low bias and MSE under Exponential and Weibull distributions. This versatility suggests that the L-SFA model can be effectively applied in diverse contexts where the underlying error structure is complex, uncertain, or varies across firms or industries. Such adaptability reduces the need for overly restrictive assumptions about the error distribution, thereby broadening the model's applicability in practical settings.

The simulation results underscore the importance of selecting the appropriate estimator based on the assumed or known error distribution. When comparing the performance of $MLE_{Lindley}$ with other estimators, such as $MLE_{Half\text{-}Normal}$ and $MLE_{Exponential}$, $MLE_{Lindley}$ shows superior performance in scenarios where the true error distribution closely aligns with the Lindley or Exponential forms. This superior performance is reflected in lower MSE values and reduced bias, making $MLE_{Lindley}$ a preferred choice for efficiency analysts seeking to minimize estimation error. The study's results also suggest that $MLE_{Lindley}$ could serve as a flexible and effective alternative in situations where the true error structure is complex or not well-defined, enhancing the robustness of efficiency estimates across varying distributional assumptions.

The introduction of the Lindley error distribution within the SFA framework, as evidenced by the L-SFA model, represents a significant advancement in efficiency analysis. For practitioners and researchers, adopting a flexible model like L-SFA with the Lindley distribution offers more reliable and accurate estimates, especially in complex empirical settings. The model's demonstrated accuracy under the Lindley distribution assumption makes it a valuable tool for econometric modeling across various fields, including economics, operations research, and management science.

The integration of the Lindley error distribution within the SFA framework, as demonstrated by the L-SFA model, provides a robust and flexible alternative to traditional SFA models. The ability of the L-SFA model to maintain strong performance across different distributional assumptions further establishes its utility in diverse empirical contexts. Future research could extend this work by exploring the application of L-SFA in panel data settings, integrating it with other stochastic modeling techniques, or applying it to real-world datasets to further enhance its utility and applicability in complex empirical contexts.

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