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Proceedings of the International Conference on Academic Research in Science, Technology and Engineering, Vol. 1, Issue. 1, 2022, pp. 21-31
DOI: https://doi.org/10.33422/icarste.v1i1.14
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ISSN: 2783-7742 online



Diamond Scientific Publishing

Numerical Methods and Approximations for the Heat Transfer Problem

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Abstract

In many studies of heat transfer problems, many researchers have studied heat transfer problems by solving partial differential equations without using approximations or finding solutions with experimental data. This paper analyzes the heat transfer problem using a heat source in a closed environment and how it transfers in the neighboring sections. Referring to mathematical concepts, this work makes possible the simplification of the complexity that associates with such thermodynamic problems. In this framework, the research group discusses such a problem as a discrete one, easily computable, rather than treating it as a continuous one. The reduction of this problem to the solution of a simple system of linear equations and differential equations gives us the possibility to obtain the desired results regarding heat distribution. In many cases, differential equations are hard and difficult to solve. So, we deal with numerical methods to approximate the differential equations to algebraic equations and solve them. Comparing the different algorithms used and showing which of them works best in our testing conditions gives us the possibility of testing and comparing the results and the proper performance. The program will simulate the heat transfer of a single heat source in a closed environment. The results of the simulations will be presented in graphs and demonstrated in visual settings. In the end, our research will provide conclusions on the performance of the numerical methods. The purpose of this article is to study and analyze heat generation and heat transport in three-dimensional space with respect to neighboring sides of a closed environment.

Keywords: transforms, decomposition method, image rendering, Poison equation, distribution

1. Introduction

In the theory of heat transfer, it is very important to understand heat transfer involves many differential equations. Considering computational runtime, we are able to apply the numerical methods and the error due to the approximation we obtain from the numerical method is acceptable. We deal with the study of algorithms that have been used to approximate the solutions to complex problems. Most of the problems do not have an analytical solution and thus the optimal use of numerical techniques becomes essential. One of the most fundamental useful mathematical constructs is the matrices. Almost all the operators can be represented by matrices including derivatives of several orders, Fourier transforms, etc.

The use of matrices is in principle computationally expensive, but the use of large computational units overcomes this barrier. In the fundamental techniques that computers operate, the only addition is possible and the rest such as subtraction, multiplication, and division are defined in elegant digital design methods. Other operations like finding the square root of a number, solving a system of linear equations, studying numerical analysis of one-dimensional heat transfer (Mahardika & Haryani, 2020) and more are made possible only through the use of numerical algorithms (Yang, 2018), that can be studied in algorithms of mathematics (Harbrecht & Multerer, 2022).

In this paper, we have discussed how to determine heat transfer in a closed environment from a heat source located at the center of the region. Here we consider three assumptions: i) analyses are conducted in a 2-dimensional space, ii) steady-state is considered and, iii) no heat transfer with the surrounding is considered (closed system). The different frequencies, amplitudes distributed energy resources (Feynman, 2018), phase angles of sinusoidal heating, and effect of multi-frequency heating are used for understanding heat behavior problems (Manna, Biswas & Mahapastra, 2019).

We first start with a basic explanation of the Laplace operator. We can perform the results using the unimodular matrix through Late\x and python (Hajrulla, Arban, Bezati & Demir, 2022). For the same, we mention that the Laplace operator is a differential operator given by the divergence of the gradient of a scalar function in Euclidean space. Using a unimodular matrix on a system of linear equations (Hajrulla, Bezati & Demir, 2021) the results of images perform better than the old algorithms used in past studies. This operator is used for describing many physical phenomena and algorithms for improving power balance (Kunzler & Lopes, 2019), which involves an efficient solving method for nonlinear convection-diffusion equations (Cui, Yuan & Shen, 2018), in a simplified way. Some of its most common usages are in describing the electrical and gravitational potential given by Poisson's equation applied as a partial differential equation (Alshmary, 2020), and in describing the heat and fluid flow given by the diffusion equation.

2. Main Theory

2.1. Laplace-Operation and Transforming the Heat Equation

In this paper, we will be based on the method using partial differential equations, which even though uses Poisson's equation, which is normally used for analyzing the electrostatic potential in a room with electrons, it is possible to approximate the heat distribution of a heat source in

a closed room on a 2-dimensional space by using same form of Poisson's equation. The equation that we will be based on will be this:

$$\Delta u(x) \coloneqq \frac{\partial^2 u}{\partial x_1^2}(x_1, x_2) + \frac{\partial^2 u}{\partial x_2^2}(x_1, x_2) \tag{2.1}$$

Our goal consists of separating the two parts of this equation and studying them in a discrete form.

$$\frac{\partial^2 u}{\partial x_2^2}(x_1, x_2) \approx \frac{u(x_1, x_2 + h) - 2u(x) + u(x_1, x_2 - h)}{h^2} \tag{2.2}$$

For an analogy, we can say that:

$$\frac{\partial^2 u}{\partial x_1^2}(x_1, x_2) \approx \frac{u(x_1 + h, x_2) - 2u(x) + u(x_1 - h, x_2)}{h^2} \tag{2.3}$$

We can select a number $n \epsilon N$ and select a unit square of Ω followed by equally distanced spaces of $h = 1/n$. The grid points are then given by:

$$x_{i,j} = h[i, j]^T \quad for\ all \quad i, j \in \{0,1 \dots n\} \tag{2.4}$$

Based on equations Eq. 2.2 and Eq. 2.3 our approximation of Poisson's equation for every value of every cell in our space will be given by the expression:

$$\frac{1}{h^2}[4u_{i,j} - u_{i-1,j} - u_{i+1,j} - u_{i,j-1} + u_{i,j+1} = f_{i,j} \tag{2.5}$$

As a result, we can transform our equation in a system of linear equations expressed as $Au = f$ where matrix $A \in \mathbb{R}^{N \times N}$ where $N = (n-1)^2$.

The system of equations is shown below.

$$Au = \begin{pmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 4 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 \end{pmatrix} \begin{pmatrix} u_{1,1} \\ u_{1,2} \\ u_{1,3} \\ u_{2,1} \\ u_{2,2} \\ u_{2,3} \\ u_{3,1} \\ u_{3,2} \\ u_{3,3} \end{pmatrix} = \begin{pmatrix} f_{1,1} \\ f_{1,2} \\ f_{1,3} \\ f_{2,1} \\ f_{2,2} \\ f_{2,3} \\ f_{3,1} \\ f_{3,2} \\ f_{3,3} \end{pmatrix} \tag{2.6}$$

In addition, the corresponding point of this equation on a graph would be:

So, our goal now has become solving this system of linear equations, so we can find the value of temperature that every cell in our grid will have in a specific period. As we can observe, we have transformed this complex problem into a problem that involves several linear equations. This is much more efficient for the computer to calculate but still, special algorithms, which we will be discussed in section 3, should be used.

2.2. The Initial State of the System

To solve our problem, we will first need to give some initial heat values to our closed system, which is an initial heat source, from which heat will be released and sent to the other parts of the system (2d room).

If we were to be based on the analyses of basic finite element method (Yang 2018), we could represent the heat source as a function of sin(x) in a 3-dimensional space, such as:

$$f(x,y) = \sin\left(\frac{\pi x}{n}\right) + \sin\left(\frac{\pi y}{n}\right) + b \tag{2.7}$$

where *b* is an arbitrary number that we have chosen to represent heat sources with different capabilities. The larger value for b indicates the larger energy released by the heat source. Nevertheless, as stated this is not a perfect representation of how the heat of a heat source would be distributed. Therefore, we go with the second alternative given by assuming that the heat is uniformly distributed along the heat energy source (Feynman 2018).

3. Application

To solve this problem, we would need to solve the system of linear equations that we have formed in section 1.2. To do so we will be using two different mathematical methods that can be used to solve such systems. The first one would be the Gauss-Seidel method and the LU-Decomposition method.

3.1. Generating the [A] Matrix and Using Numerical Method

Before starting to solve our system of linear equations, we would need to generate a pattern like a matrix A. To do so we have written a piece of code in C++. After generating matrix A, we now need to perform the calculations for solving the system of linear equations.

To do so we have used the Gauss-Seidel method. This is an iterative method, consisting in solving a system of linear equations by first getting an initial value and then performing the algorithm recursively until an initial absolute error value is reached. We also need to keep in consideration the absolute error.

We would be able to solve this system of linear equations by doing such calculations:

$$x_i = \frac{b_1 - (a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in-1}x_{n-1})}{a_{in}} \tag{3.1}$$

The method that we have implemented in the code has been modified so that it allows us to perform more iterations to get a better result. The code written in C++ is given in the appendix.

The second method that we will be discussing is LU Decomposition. It mainly consists of decomposing the A matrix and then carrying out the calculations such that it is more efficient for us to perform multiplication with L and U matrix, than it was with A matrix.

In our C++ code we have done a mixing of Gauss-Seidel method with LU Decomposition method. More specifically, we have performed the LU decomposition with a specific algorithm and after the decomposition of matrix A into matrixes L and U we have used the Gauss-Seidel method to solve the two new systems of linear equations.

3.2. Performance Comparison

After giving the code of both methods we need to analyze which one of these methods performs better in our test conditions. The measurements have been carried out in an HP laptop, Intel i3 gen 8th processor, with 8 GB DRAM. The result that we got are shown by a tabular representation.

We have considered four test cases. After performing the measurements, we ended up with the following results as shown in Table 1 and Table 2.

Table 1. For the Gauss-Seidel method

Case	1	2	3	4
Time	1999000 ns	3002000 ns	27000000 ns	9990000 ns
Number of calls	13	5	15	16

Table 2. For the LU Decomposition method

Case	1	2	3	4
Time	1004000 ns	-792300032 ns	172023186 ns	35000000 ns
Number of calls	8	2003	8	8

What we observe from the results is that the LU decomposition method has a better performance that is almost constant when we are talking about the number of times called to solve the equation.

Nevertheless, it has the risk of diverging as we see in case 2 where we have reached the number of calls limit, set by us, and the time which shows an overflow of the information.

On the other hand, we see that the Gauss-Seidel method has a better time in almost every case compared to the LU decomposition method. An important fact to be mentioned is that we have taken as a constant number of iterations the time it takes the LU decomposition algorithm to perform the decomposition process.

We have plot two graphs representing these numbers to better visualize our results as shown in Figure 1 and Figure 2.

Figure 1. Overview of methods. The number of times and the function is represented for all four cases

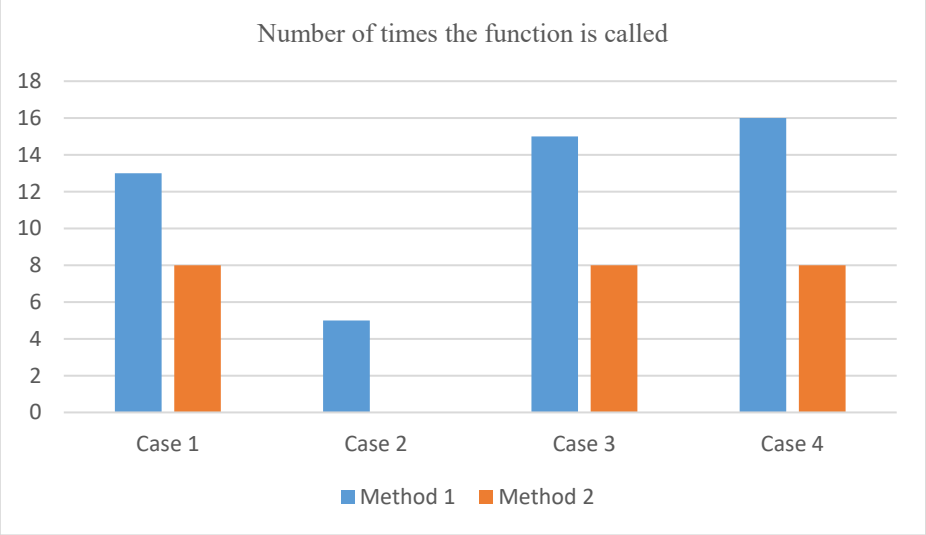
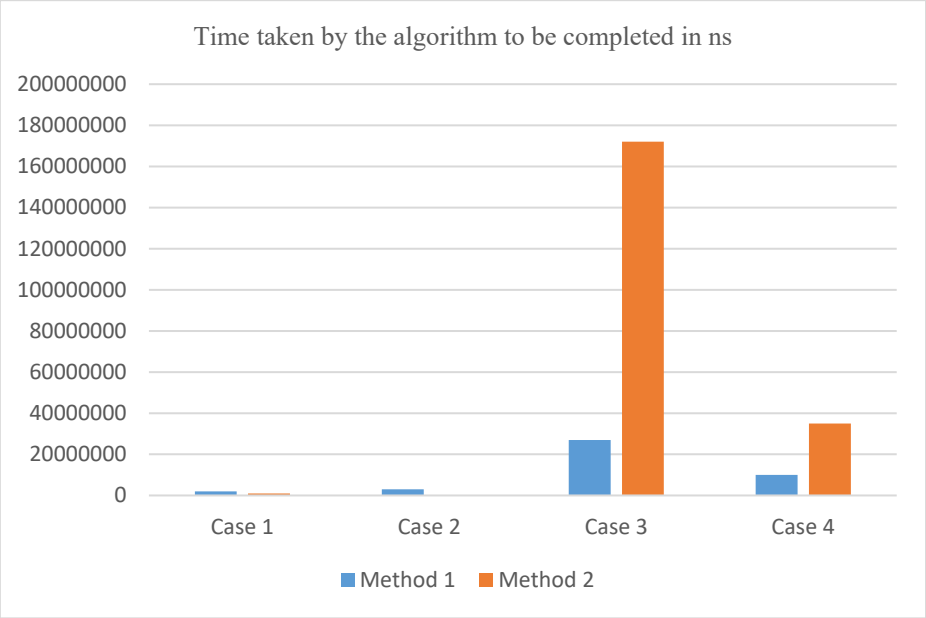


Figure 2. The number of times taken by the algorithm for all four cases

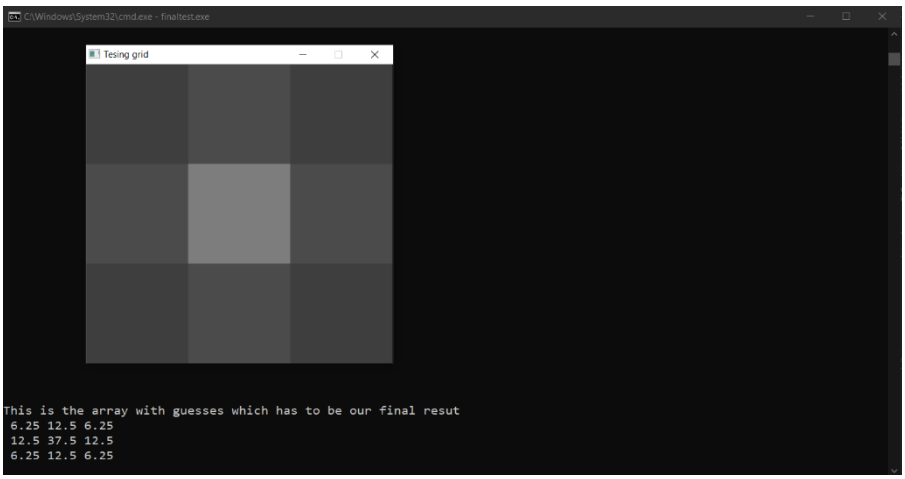


4. Simulation

In this part of our paperwork, we have given a simulation to show the result of the method that we have discussed in section 2. To perform the simulation, we have used C++ along with a specific library of it, namely SDL2. Performing 2 simulations with different heat sources and different resolution sizes, the results are received.

In the first one we have given example, which is relevant with those explained in section 2. The resolution of the image is 3 by 3. A heat source of 100 grade Celsius is placed in the center of the room. As we can see in Figure 3 and Figure 4, the heat has been released in a symmetric way in every part of our room.

Figure 3. The first simulation with a 3 by 3 matrix



In the second simulation, we have increased the number of squares to seven that will represent the room. Along with that, there are three points that represent e heat source of 100 grade Celsius in the middle of the room. As the results show, the middle of the room is much warmer than the rest of the room since the heat source is placed in a horizontal direction.

Figure 4. the second simulation using a 7 by 7 matrix

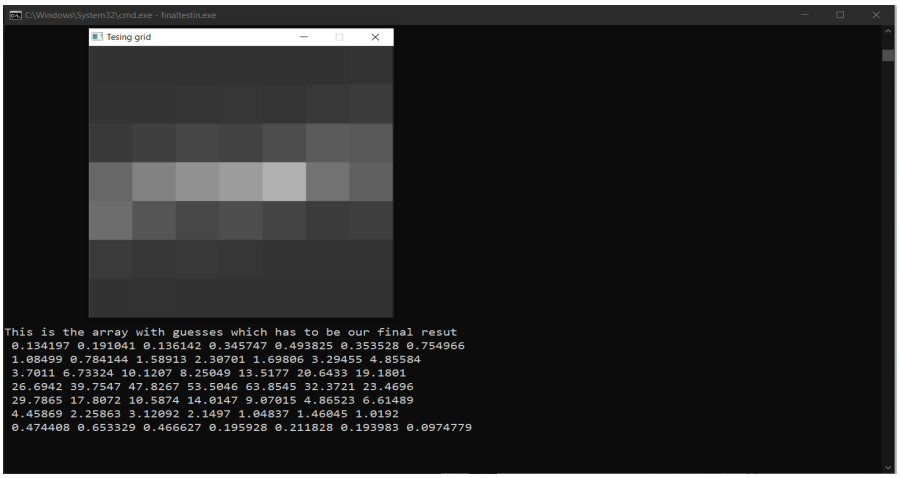


Figure 5a. Heat distribution for the first simulation for one dimensional geometry

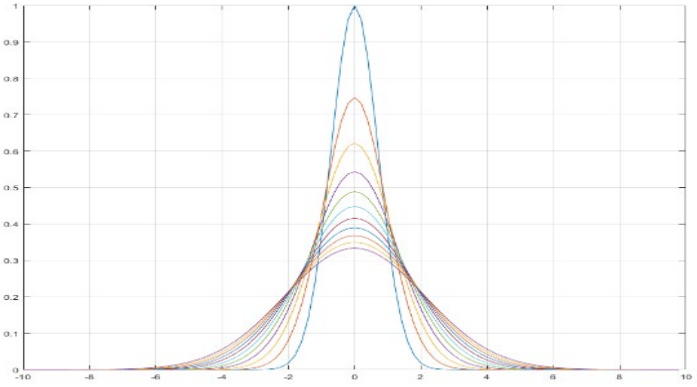


Figure 5b. Heat distribution for the second simulation smaller geometry

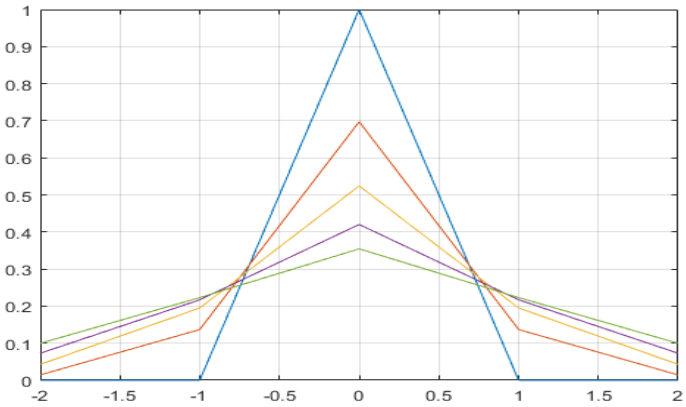


Figure 6a. Distribution of the air temperature in selected times. (first step simulation)

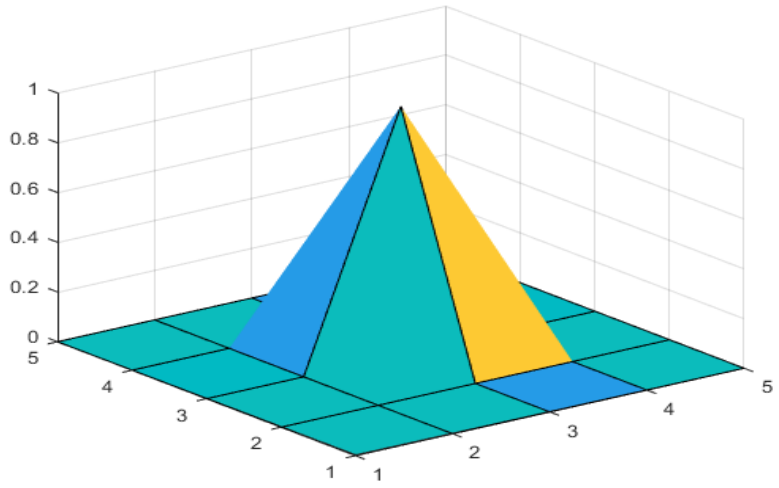


Figure 6b. Comparison of heat distribution in different conditions (second step simulation))

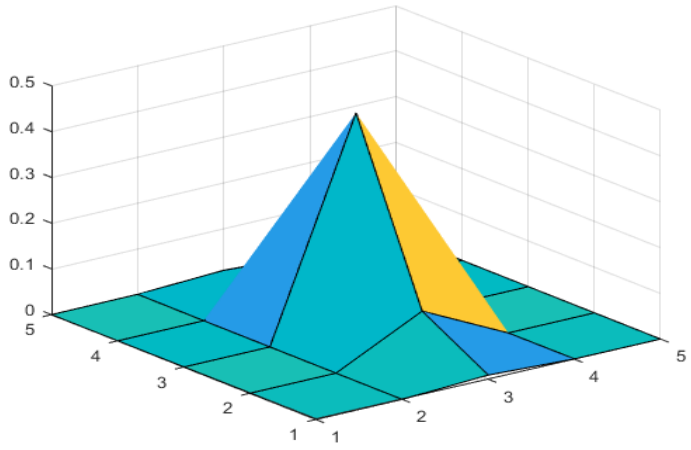


Figure 6c. Distribution of the air temperature in selected times. (third step simulation)

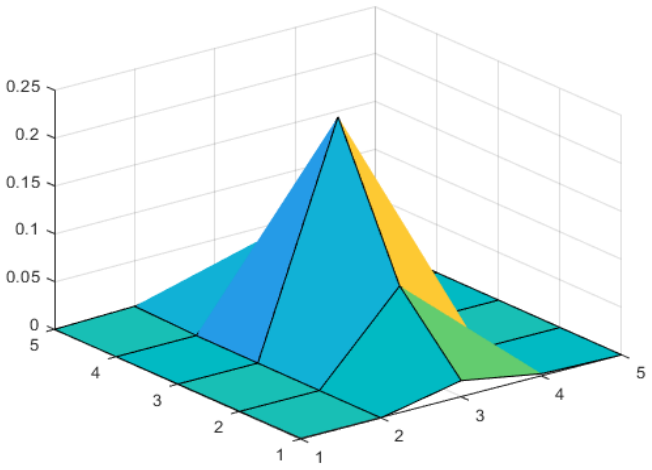


Figure 6d. Comparison of heat distribution in different conditions (forth step simulation)

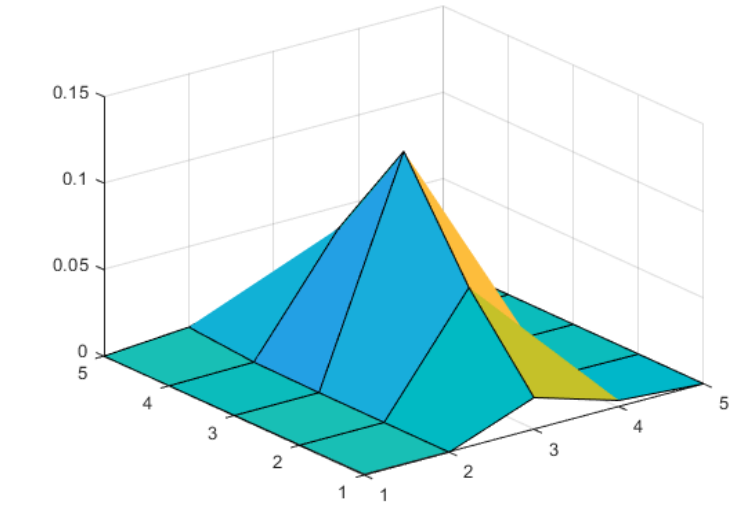
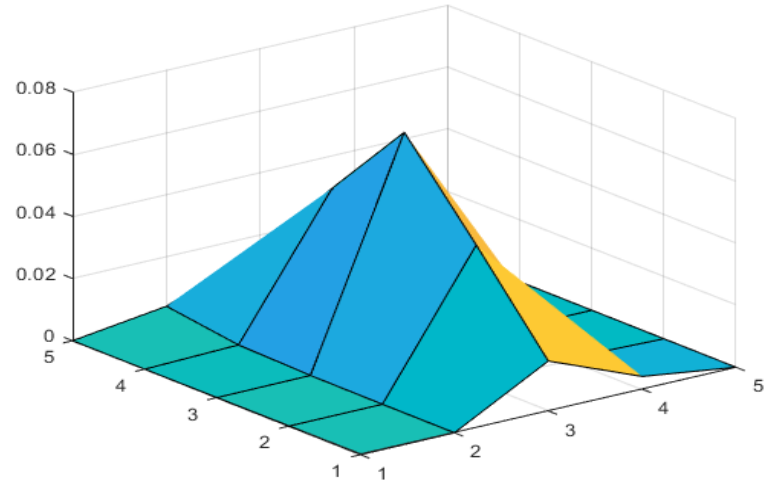


Figure 6e. Distribution of the air temperature in selected times (fifth step simulation)



5. Conclusions

The heat transfer problem from our point of view using multi-frequency heating is estimated under different methods. The effect of different frequencies of sinusoidal heating, along with the uniform heating, is discussed using simulations, numerical methods, and approximations. This is performed while using thermal boundary layers and headlines.

The numerical methods are used to solve the heat transfer problem, considering the methods and computational operations, approximation, and simulation model. The mean-temperature constraint is applied for enhancement analysis. We can say that it is possible with the help of numerical results to simplify problems in a way that is efficient for us to compute the result.

We can say that it is possible with the help of numerical results to simplify problems in a way that it is efficient for us to compute the result. Computers have limitations concerning the number of operations they can perform. Nevertheless, in most cases, we end up with a problem that requires many calculations.

In the next future, the different frequencies, amplitudes, and phase angles of sinusoidal heating will be investigated to understand their major impacts on heat transfer characteristics. The present research idea can usefully be extended to other multi-physical areas (nanofluids, magneto-hydrodynamics, etc.)

Theoretical analysis shows that each of the schemes is stable, its solution exists uniquely, and has temporal accuracy. The corresponding iteration number has the same order of accuracy and quadratic convergent speed. Numerical tests verify the conclusions and demonstrate the high accuracy and efficiency of the algorithms.

The numerical method provides theoretical and technical support to accelerate resolving heat diffusion, heat transport problems, approximation of solutions, and expected results. We develop a numerical model for analyzing the heat transfer problem using a heat source in a closed environment and how it transfers in the neighboring sections. The temperature distribution inside the space was compared and the results were shown in table data and in graphical form.

Future work

Based on the previous studies, we will study the efficient solving method for nonlinear convection-diffusion equations and convert the proper equation for the heat transfer problems. Of course, this would be a challenge for us, a difficult situation because of the combination of conservative diffusion and convection operator. Furthermore, the algorithm would be extended from first-order temporal accuracy to second-order temporal accuracy.

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