



Exploring the Factors Influencing the Maintenance and Decline of High-Level Cognitive Demand Tasks in Classroom Implementation

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Abstract

This study aims to examine which factors influence novice mathematics teachers' maintenance or decline in cognitive demands during the classroom implementation of high-level mathematical tasks. Based on the factors by Stein et al. (2006), the study provides an analysis of task implementation, identifying how and why shifts occur in practice. The participant is a mathematics teacher with two years of experience. In an ongoing project on the mathematical coaching of novice teachers, data are collected through classroom observations and transcripts. During the implementation of the first task on fractions, students reached conclusions by using standard algorithms rather than making estimations. This led the teacher to provide a direct explanation instead of fostering conceptual understanding. In the second task on patterns, students focused on developing mathematical understanding rather than reaching an exact answer. As the teacher was prepared for possible student responses, she could ask 'why' and 'how' questions purposefully to elicit students' thinking and reasoning. This instance aligns with the maintenance factor of "sustained press for justification, explanation, and meaning through teacher questioning, comments, or feedback," as the teacher maintained the cognitive demand of the high-level task. The findings indicate several aspects that influence the maintenance and decline of cognitive demand in tasks. Therefore, teachers must consider students' readiness and anticipate student responses.

Keywords: classroom implementation; cognitive demand; mathematical task; middle school mathematics; novice teachers

1. Introduction

Mathematical tasks are widely recognized as the cornerstone of mathematics education, as they shape not only what students learn but also how they engage with mathematical ideas (Stein et al., 1996; Watson & Ohtani, 2015). According to Smith and Stein (1998), mathematical tasks are classroom activities designed to focus students on a particular mathematical idea. Mathematical tasks help students learn mathematical concepts, participate in classroom discussions, and improve their reasoning skills (Breen & O'Shea, 2018; Clarke & Roche, 2018; Ni et al., 2018). Consequently, the nature and quality of mathematical tasks play an important role in students' learning.

In particular, the cognitive demand of mathematical tasks is one of the key elements that enables students to engage in meaningful learning. Levels of cognitive demand can be divided into two categories: lower-level and higher-level. Low-level cognitive tasks, including "memorization" and "procedures without connections," emphasize routine procedures and the obtaining of correct answers, which often limit students' opportunities to construct meaning and engage with mathematical ideas. In contrast, high-level cognitive tasks, such as "procedures with connections" and "doing mathematics," require students to make sense of mathematical relationships, justify their reasoning, and engage in non-algorithmic thinking. These tasks are important for fostering conceptual understanding, problem-solving abilities, and mathematical reasoning (Stein & Smith, 1998). The characteristics of these four levels are shown in Table 1 below.

Table 1. Characteristics of Cognitive Demands

Lower Level Demands	Higher Level Demands
Memorization: - Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory. - Cannot be solved using procedures because a procedure does not exist, or because the time frame in which the task is being completed is too short to use a procedure. - Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. - Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced. Procedures Without Connections: - Are algorithmic. Use of the procedure is specifically called for or is evident from prior instruction, experience, or placement of the task. - Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it. - Have no connection to the concepts or meaning that underlie the procedure being used.	Procedures With Connections: - Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. - Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. - Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning. - Require some degree of cognitive effort. Doing Mathematics: - Require complex and non-algorithmic thinking; a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example. - Require students to explore and understand the nature of mathematical concepts, processes, or relationships. - Demand self-monitoring or self-regulation of one's own cognitive processes.

- Are focused on producing correct answers instead of on developing mathematical understanding. - Require no explanations or explanations that focus solely on describing the procedure that was used.	- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. - Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. - Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.
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Adapted from Stein and Smith (1998)

Designing and implementing high cognitive level mathematical tasks can help students develop higher-order mathematical thinking and mathematical skills (Stein et al., 2000). Tasks designed at a high cognitive level provide students with opportunities to explore, justify, and reason; and foster the development of conceptual understanding (Stein & Smith, 1998). Although high-level cognitive tasks are known to be beneficial, maintaining their cognitive level in classroom implementations is critical for developing students' reasoning abilities (Parrish & Bryd, 2022). However, maintaining this cognitive demand throughout classroom implementation can be challenging because it is influenced by the dynamic and interactive nature of classroom environments. Factors such as time constraints, classroom management, students' prior knowledge, and, importantly, teachers' instructional decisions contribute to this challenge (Chapman, 2013; Henningsen & Stein, 1997; Stein et al., 1996). Recent studies on maintaining the cognitive demand of mathematical tasks also emphasize that high-level tasks often decline during classroom implementation when instruction becomes overly procedural or teacher directed. Research shows that cognitively demanding tasks are more successfully maintained when teachers encourage students to explain reasoning, explore multiple strategies, and engage in meaningful mathematical discourse rather than focusing solely on correct answers (Parrish & Byrd, 2022; Kaur, 2022). In addition, current research highlights that instructional support must balance guidance with productive struggle because excessive scaffolding can lower cognitive engagement, whereas carefully designed support can help students sustain complex reasoning (Prediger et al., 2024).

Teacher actions play an important role in maintaining the mathematical task's cognitive demand. Instructional moves such as providing direct explanations, emphasizing procedural correctness, or reducing the task complexity can unintentionally lower the cognitive demand of a mathematical task. Conversely, practices such as pressing for justification, encouraging multiple solution strategies, and scaffolding students' reasoning can support the maintenance of high-level cognitive demand of the task (Stein et al., 1997). Thus, the way a mathematical task is implemented becomes critical for maintaining or reducing its cognitive demand.

Although prior research has examined the classification and characteristics of mathematical tasks, there is still a need to better understand how these tasks are carried out in real classroom settings (Henningsen & Stein, 1997; Stein et al., 1996). This issue becomes more important in the context of novice teachers, who are still developing their pedagogical knowledge and instructional decision-making skills (Santagata & Yeh, 2015; Stahnke et al., 2016). Novice teachers often face challenges in anticipating students' responses, managing different classroom dynamics, and providing appropriate scaffolding without direct explanations or simplifying the task (Monarrez & Tchoshanov, 2020; Stein et al., 1996). Their limited teaching experience may lead them to make instructional choices that reduce the cognitive demand of tasks, even though the tasks are designed at high-level cognitive demand (Monarrez & Tchoshanov, 2020; Stein et al., 1996). Therefore, research that focuses on the practices of

novice teachers can support the development of more effective professional learning opportunities to improve high-level task implementation. In particular, studies that examine the classroom implementation can help explain how cognitive demand is maintained or reduced, considering the cognitive demand of the mathematical task.

In this regard, the present study aims to examine the classroom implementation of high-level cognitive demand tasks by focusing on the aspects that influence their decline and maintenance. Drawing on the factors proposed by Stein et al. (2006), which will be explained in detail in the following section, the study provides an analysis of task implementation, identifying how and why shifts occur in practice. Hence, the research question is *“What aspects influence novice mathematics teachers’ maintenance or decline of cognitive demands during the classroom implementation of high-level mathematical tasks?”* By addressing this question, the study aims to contribute to the literature by providing an analysis of the dynamic nature of task implementation, highlighting how cognitive demand shifts in practice and offering insights into the challenges faced by novice mathematics teachers and the instructional decisions that influence students’ mathematics learning. Finally, the current study offers implications for both research and practice in supporting teachers to maintain the demand for the mathematical task.

2. Methodology

In the current study, a qualitative case study research method was adopted (Creswell, 2014). This study was conducted with an in-service mathematics teacher with two years of experience. In an ongoing project on the mathematical coaching of novice teachers, the teacher has designed two different high-level cognitive tasks and implemented these tasks in a private school with the fifth-grade students.

The study was conducted in accordance with ethical research guidelines. Written informed consent was obtained from the participating teacher before data collection. In addition, permissions were obtained from the school administration and the parents of the participating students for classroom observations and video recordings. Participation in the study was voluntary.

The tasks were implemented in classes of approximately 15 students. Unlike traditional educational settings, the classroom environment was designed for group work, with 3-4 students per table. Both activities were planned to be completed within a single lesson, approximately half an hour, although the teacher was informed that the lesson time could be extended as needed. Therefore, considering class discussions and teacher feedback, the classroom implementation of both tasks took approximately one hour.

The first task as shown in figure 1 is designed to make estimations with fractions. In the task, students are required to compare the parts of a wall painted by different students, where the painted parts are given in different representations, such as percentage, decimal, and fraction. The students are expected to estimate who painted the most and the least, determine whether the painted parts have equal areas, and order the painted parts from smallest to largest. They are also required to compare these amounts with another student who painted a specific area of a wall ($\frac{7}{8}$) and represented the painted areas visually using two rectangular shapes, one of which was divided into 40 equal parts rather than 100.

Figure 1. First Task

The large wall at the school entrance will be painted for April 23rd! Each student has been given a specific area, and their teachers have told them they can freely design the given wall. When the painting is finished, the area painted by each student is given below:

- Rümeysa: $\frac{1}{22}$
- Ayşel: 0.3
- Mete: 0.24
- İlknur: $\frac{6}{25}$

Based on this information, answer the following questions by estimation without doing any calculations.

- 1) Who colored the most area?
- 2) Who colored the least area?
- 3) Are there any students who colored an equal amount? If so, who are they?

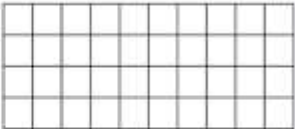
4) Rank the students from smallest to largest area they colored.

5) Alpcan wanted to contribute to this activity. He painted $\frac{7}{8}$ of the wall opposite. Accordingly, compare the areas painted by Alpcan, Rümeysa, Ayşel, Mete, and Rümeysa.

6) The rectangle on the side represents the entire wall. Show each student's painted area by coloring it with different colors.

- Rümeysa: blue
- Ayşel: yellow
- Mete: green
- İlknur: red

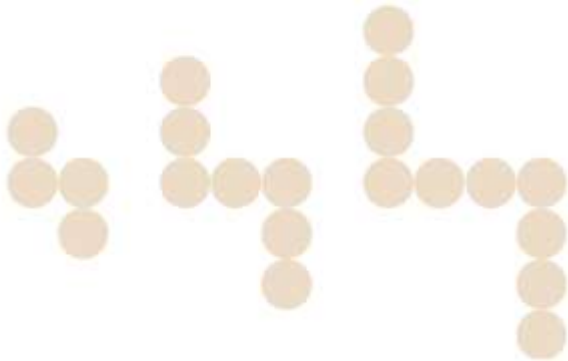
7) Show the areas painted by Rümeysa, Ayşel, Mete, and İlknur by modeling them as 40 separate parts.




The second task as shown in figure 2 is designed to identify the rule of a pattern. The task involves a factory producing three steps using shapes, and students are expected to determine the increment of each step and answer questions about the pattern. In the questions, students are required to identify the rule of the pattern, find terms at the desired steps in the pattern, and determine which term in the pattern corresponds to a given number.

Figure 2. Second Task

A robot in a robotics factory produces shapes that progress according to a specific pattern at each step. Below are the shapes the robot created in the first three steps:



Step 1 Step 2 Step 3

- 1) What might be the rule for this pattern?
- 2) According to this pattern, how many units of shape will have been produced in step 34?
- 3) Using 100 circles, what step of this shape pattern can the exploration robot create?

Data were collected through classroom observations, video recordings and lesson transcripts. Two classroom implementations were observed in detail, and each implementation lasted approximately one class hour, including whole-class discussions and group work processes. Video and audio recordings were transcribed to examine teacher-student interactions and instructional moves during task implementation.

An observation form was used during classroom observations. The form included open-ended questions in which the observer described the teacher's teaching actions regarding whether

they maintained the level of the task during its implementation, and the teacher's teaching actions in response to different student responses.

Data were analyzed through the framework "Factors associated with the decline and maintenance of high-level cognitive demands," as shown above in Table 2 (Stein et al., 1996). There are six factors associated with the decline of high-level cognitive demands and 7 factors associated with the maintenance of high-level cognitive demands. In classroom observation, data obtained from observation notes and transcripts were used to examine whether cognitive demand was declined or maintained during instruction when high-level tasks were applied, according to these factors. While analyzing data codes and sub-codes were created according to these factors in the framework, and classroom observation transcripts were coded according to these codes. For example, when a teacher refers to students' prior knowledge or has moves that assess students' prior knowledge during the task's implementation in the classroom, this is coded with the "draw conceptual connection" subcode within the maintenance of cognitive demands code.

Table 2. Factors Associated with the Decline and Maintenance of High-Level Cognitive Demands

Factors Associated with the Decline of High-Level Cognitive Demands	Factors Associated with the Maintenance of High-Level Cognitive Demands
Problematic aspects of the task become routinized.	Scaffolding of students' thinking and reasoning.
The teacher shifts the emphasis from meaning, concepts, or understanding to the correctness or completeness of the answer.	Students are provided with means of monitoring their own progress.
Not enough time is provided to wrestle with the demanding aspects of the task, or too much time is allowed, and students drift into off-task behavior.	Teacher or capable students model high-level performance.
Classroom management problems prevent sustained engagement in high-level cognitive activities.	Sustained press for justifications, explanations, and/or meaning through teacher questioning, comments, and/or feedback.
Inappropriateness of tasks for a given group of students.	Tasks build on students' prior knowledge.
Students are not held accountable for high-level products or processes.	Teacher draws frequent conceptual connections.
	Sufficient time to explore (not too little, not too much).

Adapted from Stein et al. (1996)

To ensure the trustworthiness of the study, several strategies were employed throughout the data collection and analysis processes (Creswell, 2014). First, triangulation was established through the use of multiple data sources, including classroom observations and lesson transcripts. In addition, the coding process and emerging interpretations were discussed among the researchers involved in the project to enhance consistency and reduce potential researcher bias. Furthermore, detailed descriptions of the classroom context and instructional episodes were provided to strengthen the credibility and transferability of the findings.

3. Findings

In the first task, students tended to reach conclusions using standard algorithms rather than making estimations. Although the teacher tried to guide the students toward estimation, the students argued that estimation is just the mental application of the standard algorithm, meaning it follows the same process as the standard algorithm and yields the same result, so there was no need to perform the estimation. They reached the result directly using the method of equalizing denominators without estimation, which contradicts the meaning of estimation. For example, one of the students said, *“I performed the calculation to be more certain... Even if we did these mentally, we would have reached the same result.”* This aligns with the factor “Inappropriateness of tasks for a given group of students,” since students are unsure how to make estimates, and the task expectations do not put students in the right cognitive space.

Since the task is not appropriate for the students, the teacher provides direct explanation and leads the students' thinking and reasoning to foster conceptual understanding. Her intervention prevented students from linking operations to underlying meanings. In addition, the teacher's failure to transition between calculation and estimation, and to use them sequentially, prevented students from focusing on estimation. The teacher's lack of guidance, additional questions, or use of materials for estimation hindered the transition between estimation and calculation. Therefore, the teacher's requirement for students to only perform estimation to conclusion, without any further instructional moves, caused the task to become routine. This instance aligns with the declining factor of “problematic aspects of the task becoming routinized,” as the teacher's actions shifted the high-level task to a low-level task.

In the second task, students tended to focus on developing mathematical understanding rather than reaching an exact answer. Instead of simply finding the rule of the pattern and leaving it at that, they were able to focus on how that pattern was formed. A dialogue from the lesson that illustrates this is shown below:

Teacher

Your friend said the number of steps multiplied by three plus one. Why plus one?

Student

Because if we consider the first step, three equals three, and since there are four here, plus one.

Teacher

Your friend said that he's applying this rule to the first step as well. 3 times 1 equals 3. But he said that he has 4 circles in the first step, so he should add 1. Does this hold true for the second step? Did we check? What do you think of your friend's rule?

And earlier, your friend did some grouping, how would you group them? What does that plus one represent? Where did that plus one come from? Let's think about that.

Student

There are four rounds, and according to the pattern rule, they're adding three at a time. There are four here. When I subtract the pattern rule from four, I get one... So, one is added.

Teacher

Did you think the plus one came from there? How would you do it for the second step?

Student

We'll subtract 2 from 7, which is 5, and subtract 4 from 5, which is 1.

Teacher

Do you think the plus one comes from there? Let's think about that. Could there be another, different rule besides this?

As the teacher was prepared for possible student responses, she was able to ask ‘why’ and ‘how’ questions more purposefully to get students to express their thinking and reasoning. This instance aligns with the maintenance factor of “sustained press for justification, explanation, and meaning through teacher questioning, comments, or feedback,” as the teacher maintained the cognitive demand of the high-level task.

Moreover, instead of stating the answer directly, the teacher allowed the student to reason by asking guiding questions. Also, she encouraged students to use different mathematical representations to help them understand the mathematical concept in depth. To illustrate, the teacher supported students in using different representations, such as formulas and diagrams, and asked questions that elicited students’ thinking, such as “*What is the meaning of ‘1’ here?*” *Let’s think about that. Could there be another, different rule besides this? Your friend said she grouped these in pairs... Would you like to draw it?*” After the teachers’ guidance, one of the students explained the meaning of ‘1,’ such as “*It starts with 4. Then it increases by 3 each time. It becomes 7, then 10. But it starts from 4. When we subtract 3 from 4, we find that it increases by 1 each time.*” This aligns with the factor ‘Scaffolding of students’ thinking and reasoning’ since the teacher helped students to build mathematical understanding step by step through questions and guidance.

4. Discussion

The findings of this study indicate that the cognitive demand of mathematical tasks can either be maintained or declined during classroom implementation, which depends on several factors. A comparison of the two classroom implementations shows a clear contrast: while the first task was characterized by a factor associated with the decline of cognitive demand, the second task demonstrated conditions that support the maintenance of cognitive demand.

One of the key factors that influences this difference is the alignment between the task and students’ prior knowledge. In the first task, students’ limited understanding of estimation led them to rely on familiar procedures, which shifted the focus from conceptual understanding of estimation to obtaining correct answers. This supports previous research that emphasizes that tasks that do not appropriately build on students’ prior knowledge are more likely to decline in cognitive demand (Chapman, 2013; Stein et al., 1996). When students are not adequately prepared for the demands of a task, they may turn to routine procedures; therefore, the opportunity for meaningful engagement is reduced.

In addition to task appropriateness, teacher actions play an important role in maintaining the cognitive demand of a task. In the first implementation, the teacher's decision to provide direct explanations in response to students' difficulties contributed to the decline of the cognitive demand of the task. This finding is consistent with the literature, which states that emphasizing procedures and correctness over reasoning can lower the cognitive demand of tasks (Stein et al., 1996). In contrast, during the second task, the teacher adopted instructional strategies that encouraged students to explain their thinking and justify their reasoning. Such actions align with the maintenance factors proposed by Stein et al. (1996), particularly sustained press for justification and meaningful questioning.

Moreover, the findings highlight the importance of scaffolding in maintaining cognitive demand. In the second task, the teacher supported students' thinking through guiding questions and encouraged the use of multiple representations. This type of scaffolding enabled students to engage more deeply with mathematical ideas. As suggested by Martino and Maher (1999), teacher questioning that promotes explanation and justification can support deeper mathematical reasoning. Similarly, effective scaffolding can support students' understanding while preserving the cognitive demand of the task (Stein et al., 1996).

All in all, the results suggest that maintaining high-level cognitive demand depends not only on task design but also on how the task is implemented in the classroom. Factors such as students' readiness, teachers' questioning strategies, and the use of scaffolding play an important role in this process. When these factors are carefully considered, teachers can create learning environments that support the maintenance of engagement with mathematical thinking. Conversely, when these factors are not adequately addressed, the cognitive demand of the mathematical tasks may decline during the implementation, even if they are well-designed.

5. Conclusion

The findings of this study offer important implications for teachers, curriculum developers, and educational administrators. Encouraging teachers by administrators to implement high-level tasks in their classrooms, giving more emphasis to high-level tasks in textbooks, or encouraging students to think and develop conceptual understanding while scaffolding them during lessons can be effective in terms of teaching mathematics. Because implementing high-level tasks in classrooms can provide students with opportunities to engage in mathematical thinking and develop deep conceptual understanding. Unlike routine procedures, such tasks can encourage reasoning and support students in becoming more cognitively active. Also, the findings show that several aspects influence both the maintenance and decline of cognitive demand in tasks. Therefore, it is critical for teachers to consider students' prior knowledge, interests, and experiences to maintain the level of cognitive demand of mathematical tasks as well as scaffold students to make deep mathematical reasoning and develop mathematical understanding. Overall, the study highlights the importance of both task design and classroom implementation in maintaining a high-level cognitive demand of mathematical tasks. Moreover, teachers' instructional moves are important in maintaining the cognitive demand level when high-level tasks are implemented. Teachers encouraging students to think mathematically, establish causal relationships, and develop conceptual understanding rather than procedural skills by asking purposeful questions may highly be effective in maintaining the cognitive levels of high-level tasks.

This study was conducted considering one teacher and two lesson applications; therefore, further research could be done with a larger sample size. Studies could be conducted with multiple teachers and classrooms, and different grade levels. Furthermore, this study in the field of mathematics education could also be applied to different teaching contexts.

Acknowledgment

This study was funded by the Scientific and Technological Research Council of Turkey (TUBITAK) ARDEB 1001 under Grant Number 223K526. Appreciation is also extended to the novice teachers who participated in this project.

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