Optimal Cross Futures Hedging Under Regime Switching Time-Varying Correlated Jump Dynamic

Hsiang-Tai Lee [1]

[1] Professor, Department of Banking and Finance, 1, University Rd., Puli, Nantou Hsien,

National Chi Nan University, Taiwan 54561

Abstract. A bivariate regime switching time-varying correlated jump model (RSTVCJ) is proposed for optimal hedging. Most of the existing discrete time hedging models does not consider simultaneously the jump risk and regime switching risk. Two recent articles (Lee 2009a, 2015) suggest jump switching models that possess both of these risks. These models, however, assume a common jump for spot and futures returns. This might be too restrictive because spot and futures returns are normally not perfectly correlated especially for the cases of cross hedging where the corresponding futures contract is not available for the underlying spot. In this paper, I release this common jump assumption and envision a regime switching time-varying correlated jump model for optimal hedging. In the proposed RSTVCJ, the spot and futures returns possess both self-owned and correlated jumps and the correlated jump is captured via a regime switching spillover factor. We illustrate the usefulness of RSTVCJ by performing the exercise of hedging jet fuel spot holding with crude oil futures. We compare the performance of RSTVCJ with its nested models including common jump (CJ), state-independent correlated jump (SICJ), regime switching correlated jump (RSCJ). Empirical results show that RSCJ has the best out-of-sample hedging performance. Regime switching correlated jump is superior to its state-independent or common jump counterparts. Adding time-varying dynamic to the jump process, however, does not further improve the hedging effectiveness.

Keywords: Regime switching; correlated time-varying jump; cross hedging

1. Introduction

The benefits of fitting the spot and futures returns with more flexible distribution or specifications in futures hedging are widely investigated in the past two decades. Apart from the conventional static hedging with ordinary least square (OLS) method, the most popular futures hedging strategies are implemented with a wide array of multivariate GARCH models to estimate the time-varying minimum variance hedge ratio (MVHR) (Baillie and Myers, 1991; Kroner and Sultan, 1993; Park and Switzer, 1995; Gagnon and Lypny, 1998; Brooks et al., 2002; Alexander and Barbosa, 2008; Ederington and Salas, 2008). These models, however, capture mainly the continuous variations in the covariance structures of the spot and futures returns.

To capture the sudden jump risk in the covariance structure of spot and futures returns, a number of studies apply jump models for futures hedging. Chan and Young (2006) proposed a bivariate GARCH-jump model with autoregressive jump intensity (Chan and Maheu, 2002) to capture the features of the joint distribution of copper cash and futures returns. Based on the results of within-sample and out-of-sample hedging exercises, they report significant gains of applying a time-varying optimal hedging strategy that incorporates the information from the common jump dynamics. Chan (2008) apply a dynamic hedging strategy based on a bivariate GARCH-jump model augmented with autoregressive jump intensity for hedging foreign currency jump risk. It finds significant common jump components in the currency spot rate and futures basis with jump sizes response asymmetrical to futures basis changes. Chan (2010) applies a bivariate GARCH model augmented with a common jump component to manage currency risk. It finds significant common jump components in the British pound spot and futures rates and shows that incorporating common jump dynamics substantially reduce daily and weekly portfolio risk. Generally speaking, incorporating common jump risks for spot and futures returns improves futures hedging effectiveness. Although these common jump models are more flexible than jump-free models, it ignores the property of regime switching (Alizadeh and Nomikos, 2004; Lee and Yoder, 2007a) and it assumes a common jump dynamic for both spot and futures returns which might be too restrictive because spot and futures returns are normally not perfectly correlated.[1]

Another strand in the line of optimal futures hedging literature focuses on the regime-shifting risk in the joint distribution of spot and futures returns. The existence of regime-shifting relationship between spot and futures data series is demonstrated in a series papers of Sarno and Valente (2000, 2005a, 2005b). Based on these findings, many studies start to estimate the so-called state-dependent time-varying minimum variance hedge ratio by proposing a wide array of multivariate Markov regime switching models (Alizadeh and Nomikos, 2004; Lee and Yoder, 2007a, 2007b; Alizadeh et al., 2008; Lee, 2010; Sheu and Lee, 2014). Lee and Yoder (2007a) apply a regime switching BEKK GARCH for estimating the state-dependent time-varying MVHR. Alizadeh et al. (2008) further suggest a regime switching BEKK GARCH with cointegration relationship between spot and futures returns for implementing energy futures hedging strategy. For explicitly modeling the correlation dynamic of spot and futures returns, Lee and Yoder (2007b) suggest a regime switching varying correlation GARCH, and Lee adopts a copula-based regime-switching GARCH model (2009b) and a regime switching dynamic conditional correlation GARCH (2010) for futures hedging. To release the assumption of common switching dynamic for spot and futures returns, Sheu and Lee (2014) suggest a multi-chain Markov regime switching GARCH model for futures hedging. A general finding is that incorporating regime switching effect improves futures hedging effectiveness.

Most of the existing hedging models to date consider the jump risk and regime-shifting risk separately. Two recent articles (Lee 2009a, 2015) suggest jump switching models that possess both of these risks.[2] (Lee 2009a) develops a Markov regime switching Generalized Orthogonal GARCH model with conditional jump dynamics (JSGO) for hedging FTSE 100 spot returns and finds significant hedging gains of JSGO compared to its jump-free and state-independent counterparts. (Lee 2015) suggests a multi-chain regime switching jump spillover GARCH (*MCRSJ*) model to investigate the spillover behavior from stock index futures to stock sectors. Cross hedging exercises show that taking account of volatility spillover and state-dependent jump dynamic improves futures hedging performance. These models, however, assume a common jump for spot and futures returns which might be too restrictive. For cross

[1] Chan (2003) suggests a correlated bivariate poisson jump model for foreign exchange. The model, however, is state-independent and is not applied to futures hedging.

[2] Although Chan and Young (2009) and Sheu and Lee (2012) also suggest jump switching models, these models are univariate and do not consider the correlated jump dynamic across asset returns.

hedging applications, such as cross hedging the crude oil risk of non-energy commodities (Sheu et al., 2015), hedging the single stock with ADR (Lee and Tsang, 2011), hedging the stock sectors price risk with stock index futures (Lee and Ko, 2010), hedging stock sector risk with credit default swap (Ratner and Chiu, 2013) and so on, the corresponding hedging asset has quite different price dynamic from the underlying spot and the assumption of common jump for both assets will be far away from realistic. In this paper, I attempt to release this common jump assumption and envision a regime switching time-varying correlated jump model for optimal hedging. The proposed *RSTVCJ* model allows the spot and futures returns to possess both self-owned and correlated jumps and the correlated jump is captured via a regime switching spillover factor. In this paper, cross hedging exercises on jet fuel spot holding with crude oil futures will be performed to justify the usefulness of *RSTVCJ*.

2. Regime switching time-varying correlated jump (*RSTVCJ*) model

The main feature of the proposed regime switching time-varying correlated jump model (*RSTVCJ*) that distinguishes itself from previous regime switching jump model is that different assets could possess simultaneously the self-owned jumps and correlated jumps and the correlated jump is captured via a regime switching spillover factor. The specification of *RSTVCJ* is given below:

Let $\mathbf{R}_t$ be a 2×1 vector of spot and futures returns with conditional mean equation

$$\mathbf{R}_t=\begin{bmatrix} r_{c,t} \\ r_{f,t} \end{bmatrix}=\boldsymbol{\mu}_{s_t}+\mathbf{v}_{t,s_t}=\boldsymbol{\mu}_{s_t}+\boldsymbol{\varepsilon}_{t,s_t}+\mathbf{J}_{t,s_t}\ , \tag{1}$$

where $r_{c,t}$ and $r_{f,t}$ are respectively the spot and futures returns and the subscription $s_t=\{1,\ 2\}$ stands for the state variable assumed to follow a first-order, two-state Markov process with a logistic state transition probabilities given by

$$\Pr(s_t=1\,|\,s_{t-1}=1)=\frac{\exp(p_0)}{1+\exp(p_0)}, \tag{2}$$

$$\Pr(s_t=2\,|\,s_{t-1}=2)=\frac{\exp(q_0)}{1+\exp(q_0)}, \tag{3}$$

where p_0 and q_0 are unconstrained parameters to be estimated along with system parameters.

$\boldsymbol{\mu}_{s_t}=\begin{bmatrix} \mu_{c,s_t} \\ \mu_{f,s_t} \end{bmatrix}$ is a vector of state-dependent unconditional mean returns, $\boldsymbol{\varepsilon}_{t,s_t}=\begin{bmatrix} \varepsilon_{c,t,s_t} \\ \varepsilon_{f,t,s_t} \end{bmatrix}$ is a bivariate state-dependent residual vector and $\mathbf{J}_{t,s_t}=\begin{bmatrix} J_{c,t,s_t} \\ J_{f,t,s_t} \end{bmatrix}$ is a bivariate state-dependent jump component vector. The state-dependent residual vector $\boldsymbol{\varepsilon}_{t,s_t}$ is assumed to be normally distributed given by

$$\boldsymbol{\varepsilon}_{t,s_t}\,|\,\psi_{t-1}\sim N\left(\mathbf{0},\tilde{\mathbf{H}}_{s_t}\right). \tag{4}$$

where ψ_{t-1} is the information set available at time $t-1$ and the state-dependent covariance matrix $\widetilde{\mathbf{H}}_{s_t}$ is given by

$$\widetilde{\mathbf{H}}_{s_t} = \begin{bmatrix} h_{c,t,s_t} & h_{cf,t,s_t} \\ h_{cf,t,s_t} & h_{f,t,s_t} \end{bmatrix} = \begin{bmatrix} \sqrt{h_{c,t,s_t}} & 0 \\ 0 & \sqrt{h_{f,t,s_t}} \end{bmatrix} \begin{bmatrix} 1 & \rho_{cf,s_t} \\ \rho_{cf,s_t} & 1 \end{bmatrix} \begin{bmatrix} \sqrt{h_{c,t,s_t}} & 0 \\ 0 & \sqrt{h_{f,t,s_t}} \end{bmatrix}$$

$$= \mathbf{D}_{t,s_t} \mathbf{\Gamma}_{t,s_t} \mathbf{D}_{t,s_t} . \tag{5}$$

In equation (5), h_{c,t,s_t} and h_{f,t,s_t} are respectively the state-dependent volatilities of spot and futures returns assumed to follow regime switching GARCH(1,1) processes defined as

$$h_{c,t,s_t} = \gamma_{c,s_t} + \alpha_{c,s_t} v^2_{c,t-1,s_t} + \beta_{c,s} h_{c,t-1,s_t} , \tag{6}$$

$$h_{f,t,s_t} = \gamma_{f,s_t} + \alpha_{f,s_t} v^2_{f,t-1,s_t} + \beta_{f,s} h_{f,t-1,s_t} . \tag{7}$$

As for the jump component, Lee (2009a) assumes a state-independent common jump for both spot and futures returns. Although Lee (2015) suggests a model with state-dependent jump, it still assumes a common jump for both spot and futures returns. The proposed *RSTVCJ* possesses both self-owned and correlated jumps with specification depicted below:

The jump component $\mathbf{J}_{t,s_t}$ is a 2-dimensioned state-dependent vector defined as

$$\mathbf{J}_{t,s_t} \mid \psi_{t-1} = \begin{bmatrix} J_{c,t,s_t} \\ J_{f,t,s_t} \end{bmatrix} \mid \psi_{t-1} \sim N\left(\mathbf{0}, \mathbf{\Sigma}_{J,s_t}\right), \tag{8}$$

where $\mathbf{\Sigma}_{J,s_t}$ is the state-dependent jump covariance matrix and the jump components are defined as $J_{c,t,s_t} = J^*_{c,t,s_t} + \eta_{s_t} J^*_{f,t,s_t}$ and $J_{f,t,s_t} = J^*_{f,t,s_t}$. The jump components J^*_{c,t,s_t} and J^*_{f,t,s_t} capture respectively the spot and futures self-owned jump dynamics and the term $\eta_{s_t} J^*_{f,t,s_t}$ captures the correlated jump via a state-dependent spillover factor η_{s_t}. These jump components enters the mean equation with an expected value of zero achieved by subtracting the expected values from the series of random jumps defined as

$$J^*_{c,t,s_t} = \sum_{k=1}^{n_{c,t,s_t}} Y_{k,c,t} - E\left[\sum_{k=1}^{n_{c,t,s_t}} Y_{k,c,t}\right], \tag{9}$$

$$J^*_{f,t,s_t} = \sum_{k=1}^{n_{f,t,s_t}} Y_{k,f,t} - E\left[\sum_{k=1}^{n_{f,t,s_t}} Y_{k,f,t}\right]. \tag{10}$$

The state-dependent discrete counting processes governing the number of jumps that arrives between time $t-1$ and t for the spot and futures returns are denoted respectively as n_{c,t,s_t} and n_{f,t,s_t} and are constructed as a sum of a series of random variables $Y's$ given by

$$\sum_{k=1}^{n_{c,t,s_t}} Y_{k,c,t} = Y_{1,c,t} + Y_{2,c,t} + \cdots + Y_{n_{c,t,s_t},c,t} , \tag{11}$$

$$\sum_{k=1}^{n_{f,t,s_t}} Y_{k,f,t} = Y_{1,f,t} + Y_{2,f,t} + \cdots + Y_{n_{f,t,s_t},f,t}, \tag{12}$$

where the random variable $Y's$ stand for jump size governed by a normal distribution with constant mean and constant volatility given by

$$Y_{c,t} \sim N\left(\tau_c, \delta_c^2\right), \tag{13}$$

$$Y_{f,t} \sim N\left(\tau_f, \delta_f^2\right). \tag{14}$$

The means and volatilities are assumed to remain the same across time but different across assets. The discrete counting processes n_{c,t,s_t} and n_{f,t,s_t} are distributed as a Poisson random variable with the jump intensity parameter $\lambda_t > 0$ and density

$$P\left(n_{c,t,s_t} = i \mid s_t = k, \psi_{t-1}\right) = \frac{\exp\left(-\lambda_{c,s_t}\right)\lambda_{c,s_t}^i}{i\,!}, \quad i = 0,1,\cdots. \tag{15}$$

$$P\left(n_{f,t,s_t} = j \mid s_t = k, \psi_{t-1}\right) = \frac{\exp\left(-\lambda_{f,s_t}\right)\lambda_{f,s_t}^j}{j\,!}, \quad i = 0,1,\cdots. \tag{16}$$

The jump intensities $\lambda_{i,t,s_t} = E\left[n_{i,t,s_t} \mid \psi_{t-1}\right]$, $i \in \{c, f\}$ are assumes to be time-varying given by

$$\lambda_{i,t,s_t} = \lambda_{i,0,s_t} + a_{i,s_t}\lambda_{t-1,s_t} + b_{i,s_t}\xi_{t-1,s_t}, \tag{17}$$

where ξ_{t-1,s_t} is the jump intensity residual defined as

$$\begin{aligned} \xi_{i,t-1,s_t} &= E\left[n_{i,t-1,s_t} \mid \psi_{t-1}\right] - \lambda_{i,t-1,s_t} \\ &= \sum_{i=0}^{\infty} iP\left(n_{i,t,s_{f,t-1}} = i \mid s_{t-1} = k, \psi_{t-1}\right) - \lambda_{i,t-1,s_t}. \end{aligned} \tag{18}$$

With these definitions, the return vector $\mathbf{R}_t$ is normally distributed with covariance matrix $\mathbf{H}_{t,s_t}$ given by

$$\mathbf{H}_{t,s_t} = \tilde{\mathbf{H}}_{s_t} + \mathbf{\Sigma}_{J,s_t}, \tag{19}$$

where the regular covariance matrix $\tilde{\mathbf{H}}_{s_t}$ is defined in equation (5), the jump covariance matrix $\mathbf{\Sigma}_{J,s_t}$ can be shown as

$$\mathbf{\Sigma}_{J_{t,s_t}} = \begin{bmatrix} \lambda_{c,s_t}\left(\delta_c^2 + \tau_c^2\right) + \eta_{s_t}^2\lambda_{f,s_t}\left(\delta_f^2 + \tau_f^2\right) & \eta_{s_t}\lambda_{f,s_t}\left(\delta_f^2 + \tau_f^2\right) \\ \eta_{s_t}\lambda_{f,s_t}\left(\delta_f^2 + \tau_f^2\right) & \lambda_{f,s_t}\left(\delta_f^2 + \tau_f^2\right) \end{bmatrix}, \tag{20}$$

and the state-dependent jump correlation coefficient of spot and futures returns ρ_{t,s_t} can be calculated as

$$\rho_{J,t,s_t} = \frac{\eta_{s_t}\lambda_{f,s_t}\left(\delta_f^2+\tau_f^2\right)}{\sqrt{\left[\lambda_{c,s_t}\left(\delta_c^2+\tau_c^2\right)+\eta_{s_t}^2\lambda_{f,s_t}\left(\delta_f^2+\tau_f^2\right)\right]\left[\lambda_{f,s_t}\left(\delta_f^2+\tau_f^2\right)\right]}}. \tag{21}$$

Accordingly, the state-dependent total correlation is given by

$$\rho_{t,s_t} = \frac{h_{cf,t,s_t}+\eta_{s_t}\lambda_{f,s_t}\left(\delta_f^2+\tau_f^2\right)}{\sqrt{\left[h_{c,t,s_t}+\lambda_{c,s_t}\left(\delta_c^2+\tau_c^2\right)+\eta_{s_t}^2\lambda_{f,s_t}\left(\delta_f^2+\tau_f^2\right)\right]\left[h_{f,t,s_t}+\lambda_{f,s_t}\left(\delta_f^2+\tau_f^2\right)\right]}}. \tag{22}$$

Equations (1)-(18) constitute the specification of *RSTVCJ*. The detail estimation procedure of *RSTVCJ* is depicted in the next section.

3. The estimation procedure of the regime switching time-varying correlated jump (*RSTVCJ*) model

The regime switching time-varying correlated jump (*RSTVCJ*) model is estimated by maximizing the following log-likelihood function

$$L(\Theta) = \sum_{t=1}^{T} \log\left[f\left(\mathbf{R}_t \mid \psi_{t-1}\right)\right], \tag{23}$$

where T is the total number of observations, Θ is a set of unknown parameters, $f\left(\mathbf{R}_t \mid \psi_{t-1}\right)$ is the mixture distribution weighted by the conditional joint probabilities of state variables. The conditional likelihood of each observation at time t is given by

$$f\left(\mathbf{R}_t \mid \psi_{t-1}\right) = \sum_{k=0}^{1}\sum_{j=0}^{\infty}\sum_{i=0}^{\infty} f\left(\mathbf{R}_t, n_{c,t,s_t}=i, n_{f,t,s_t}=j, s_t=k \mid \psi_{t-1}\right) \tag{24}$$

$$= \sum_{k=0}^{1}\sum_{j=0}^{\infty}\sum_{i=0}^{\infty} f\left(\mathbf{R}_t \mid n_{c,t,s_t}=i, n_{f,t,s_t}=j, s_t=k, \psi_{t-1}\right)\times P\left(n_{c,t,s_t}=i, n_{f,t,s_t}=j \mid s_t=k, \psi_{t-1}\right)\times P\left(s_t=k \mid \psi_{t-1}\right) \tag{25}$$

where $f\left(\mathbf{R}_t \mid n_{c,t,s_t}=i, n_{f,t,s_t}=j, s_t=k, \psi_{t-1}\right)$ denotes the conditional density of return vector given that i jumps occur in the spot return, j jumps occur in the futuers return and state variable is in the k state, $P\left(n_{c,t,s_t}=i, n_{f,t,s_t}=j \mid s_t=k, \psi_{t-1}\right)$ is the joint distribution of two discrete counting processes n_{c,t,s_t} and n_{f,t,s_t} and $P\left(s_t=k \mid \psi_{t-1}\right)$ is the conditional regime probability of being in state k. To complete the likelihood of *RSTVCJ*, a jump switching algorithm has to be applied. Lee's filter (Lee, 2009a, Sheu and Lee, 2012) is an intervening filter of Hamilton filter (Hamilton, 1989) and Chan and Maheu's filter (Chan and Maheu, 2002). The steps of generalized filtering algorithm for *RSTVCJ* with correlated jump components are depicted below:

(1) projects both the state and jump probabilities

$$P\left(s_t=i \mid \psi_{t-1}\right) = \sum_{j=0}^{1} P\left(s_t=i \mid s_{t-1}=j\right)\times P\left(s_{t-1}=j \mid \psi_{t-1}\right), \tag{26}$$

$$P\left(n_{c,t,s_t}=i,n_{f,t,s_t}=j\mid s_t=k,\psi_{t-1}\right)=P\left(n_{c,t,s_t}=i\mid s_t=k,\psi_{t-1}\right)\times P\left(n_{f,t,s_t}=j\mid s_t=k,\psi_{t-1}\right), \quad (27)$$

where $P(s_t=i\mid s_{t-1}=j)$ is the regime transition probability defined in equations (2) and (3) and $P\left(n_{c,t,s_t}=i\mid s_t=k,\psi_{t-1}\right)$ and $P\left(n_{f,t,s_t}=j\mid s_t=k,\psi_{t-1}\right)$ are Poisson distributed state-dependent probabilities defined in equations (15) and (16).

(2) Evaluate the jump and regime dependent likelihood

The assumptions in (4) and (8) imply that the joint distribution of spot and futures returns conditional on the most recent information set, state k, i spot jumps and j futures jumps is normally distributed given by

$$f\left(\mathbf{R}_t\mid n_{c,t,s_t}=i,n_{f,t,s_t}=j,s_t=k,\psi_{t-1}\right)=\frac{1}{2\pi}\left|\mathbf{H}_{t,i,j,k}\right|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\mathbf{v}_{t,i,j,k}^T\mathbf{H}_{t,i,j,k}^{-1}\mathbf{v}_{t,i,j,k}\right\}$$

(28)

$$=\frac{1}{2\pi}\left|\tilde{\mathbf{H}}_{t,k}+\mathbf{\Sigma}_{J_{t,i,j,k}}\right|^{-\frac{1}{2}}\times\exp\left\{-\frac{1}{2}\mathbf{v}_{t,i,j,k}^T\left[\tilde{\mathbf{H}}_{t,k}+\mathbf{\Sigma}_{J_{t,i,j,k}}\right]^{-1}\mathbf{v}_{t,i,j,k}\right\}, \quad (29)$$

where the jump and regime dependent residual vector can be shown as

$$\mathbf{v}_{t,i,j,k}=\begin{bmatrix} r_{c,t}-\mu_{c,k}-i\tau_c+\lambda_{c,k}\tau_c-\eta_k\left(j\tau_f-\lambda_{f,k}\tau_f\right) \\ r_{f,t}-\mu_{f,k}-j\tau_f+\lambda_{f,k}\tau_f \end{bmatrix}, \quad (30)$$

and the jump and regime dependent covariance matrix is given by

$$\mathbf{\Sigma}_{J_{t,i,j,k}}=\begin{bmatrix} i\delta_c^2+\eta_k^2 j\delta_f^2 & \eta_k j\delta_f^2 \\ \eta_k j\delta_f^2 & j\delta_f^2 \end{bmatrix}. \quad (31)$$

(3) Evaluate the mixture likelihood

The mixture likelihood is given by

$$f\left(\mathbf{R}_t\mid\psi_{t-1}\right)=\sum_{k=0}^{1}\sum_{j=0}^{\infty}\sum_{i=0}^{\infty}f\left(\mathbf{R}_t,n_{c,t,s_t}=i,n_{f,t,s_t}=j,s_t=k\mid\psi_{t-1}\right), \quad (32)$$

where $f\left(\mathbf{R}_t,n_{c,t,s_t}=i,n_{f,t,s_t}=j,s_t=k\mid\psi_{t-1}\right)$ is the joint distribution of return vector, spot jump, futures jump and state variable. [3]

(4) Update both state and jump probabilities

$$P(s_t=k\mid\psi_t)=\frac{f\left(\mathbf{R}_t\mid s_t=k,\psi_{t-1}\right)\times P(s_t=k\mid\psi_{t-1})}{f\left(\mathbf{R}_t\mid\psi_{t-1}\right)}, \quad (33)$$

$$P\left(n_{c,t,s_t}=i,n_{f,t,s_t}=j\mid s_t=k,\psi_t\right)$$

[3] Equation (32) involves an infinite sum over the possible number of jumps n_t which is not feasible in estimation. According to Lee (2009a), in practice, the maximum number of jumps η is truncated at a large enough value such that the probability of η or more jumps to the machine precision is 0 and the likelihood and the parameter estimate do not change.

$$= \frac{f\left(R_t \mid n_{c,t,s_t}=i, n_{f,t,s_t}=j, s_t=i, \psi_{t-1}\right) \times P\left(n_{c,t,s_t}=i \mid s_t=k, \psi_{t-1}\right) \times P\left(n_{f,t,s_t}=j \mid s_t=k, \psi_{t-1}\right)}{f\left(R_t \mid s_t=i, \psi_{t-1}\right)} \quad (34)$$

where $f\left(R_t \mid s_t=i, \psi_{t-1}\right)$ can be shown as

$$f\left(R_t \mid s_t=i, \psi_{t-1}\right)$$
$$= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} f\left(R_t \mid s_t=i, n_{c,t,s_t}=i, n_{f,t,s_t}=j, \psi_{t-1}\right) \times P\left(n_{c,t,s_t}=i \mid s_t=k, \psi_{t-1}\right) \times P\left(n_{f,t,s_t}=j \mid s_t=k, \psi_{t-1}\right) \quad (35)$$

(5) Calculate the weighted average residuals, volatilities, covariance, jump intensity and spillover factor for feasible estimation. The procedure of recombining is depicted below.
(6) Iterate (1) to (5) until the end of the sample and the likelihood is obtained as given in equation (23).

When a recursive process is subject to regime switching, the recursive nature of the process makes the model intractable (Cai, 1994; Hamilton and Susmel, 1994; Gray, 1996, Lee et al., 2006, Lee and Yoder, 2007a). To complete the likelihood in (23), step (5) requires a recombining procedure to solve this path-dependency problem to make the *RSTVCJ* tractable. The residuals of the spot return $v_{c,t}$ and futures return $v_{f,t}$ can be recombined using the following equations:

$$v_{c,t} = r_{c,t} - E\left[r_{c,t} \mid \psi_{t-1}\right] = r_{c,t} - \left[p_{1t}\left(\mu_{c,1}\right) + \left(1-p_{1t}\right)\left(\mu_{c,2}\right)\right] \quad (36)$$

$$v_{f,t} = r_{f,t} - E\left[r_{f,t} \mid \psi_{t-1}\right] = r_{f,t} - \left[p_{1t}\left(\mu_{f,1}\right) + \left(1-p_{1t}\right)\left(\mu_{f,2}\right)\right] \quad (37)$$

where p_{1t} is the conditional probability of being in regime 1 at time t. The conditional variance of futures return can be recombined using the following equation:

$$h_{f,t|t-1} = Var(\varepsilon_{f,t} \mid \psi_{t-1}) = Var(r_{f,t} \mid \psi_{t-1}) - Var\left(J_{f,t} \mid \psi_{t-1}\right),$$
$$= p_{1t}\left(\mu_{c,1}^2 + h_{c,t,1}\right) + \left(1-p_{1t}\right)\left(\mu_{c,2}^2 + h_{c,t,2}\right) - \left[p_{1t}\left(\mu_{f,1}\right) + \left(1-p_{1t}\right)\left(\mu_{f,2}\right)\right]^2$$

(38)

The conditional variance of spot return can be recombined using the following equation:

$$h_{c,t|t-1} = Var(\varepsilon_{c,t} \mid \psi_{t-1}) = Var(r_{c,t} \mid \psi_{t-1}) - Var\left(J^*_{c,t,s_t} + \eta_{s_t} J^*_{f,t,s_t} \mid \psi_{t-1}\right),$$
$$= p_{1t}\left(\mu_{c,1}^2 + h_{c,t,1}\right) + \left(1-p_{1t}\right)\left(\mu_{c,2}^2 + h_{c,t,2}\right) .$$

(39)

As for the recombination of the covariance of spot and futures returns, the following weighted average is applied:

$$h_{cf,t} = Cov\left(r_{c,t}, r_{f,t} \mid \psi_{t-1}\right) = E\left[r_{c,t} r_{f,t} \mid \psi_{t-1}\right] - E\left[r_{c,t} \mid \psi_{t-1}\right] E\left[r_{f,t} \mid \psi_{t-1}\right] ,$$

(40)

where

$$E\left[r_{c,t} \mid \psi_{t-1}\right] = \left[p_{1t}\left(\mu_{c,1}\right) + \left(1 - p_{1t}\right)\left(\mu_{c,2}\right)\right] ,$$
(41)

$$E\left[r_{f,t} \mid \psi_{t-1}\right] = \left[p_{1t}\left(\mu_{f,1}\right) + \left(1 - p_{1t}\right)\left(\mu_{f,2}\right)\right] ,$$
(42)

and

$$E\left[r_{c,t} r_{f,t} \mid \psi_{t-1}\right] = p_{1t}\left[\mu_{c,1}\mu_{f,1} + h_{cf,t,1} + \eta_{c,1}\lambda_{f,1}\left(\delta_f^2 + \tau_f^2\right)\right] + \left(1 - p_{1t}\right)\left[\mu_{c,2}\mu_{f,2} + h_{cf,t,2} + \eta_{c,2}\lambda_{f,2}\left(\delta_f^2 + \tau_f^2\right)\right] . \quad (43)$$

The time-varying jump intensity $\lambda_{i,t}$ and $\eta_{i,t}$ can be recombined as

$$\lambda_{i,t} = p_{1t}\lambda_{i,t,1} + \left(1 - p_{1t}\right)\lambda_{i,t,2} , \; i \in \{c, f\},$$
(44)

$$\eta_t = p_{1t}\eta_{t,1} + \left(1 - p_{1t}\right)\eta_{t,2} .$$
(45)

The unknown parameters in *RSTVCJ* are $\mathbf{\Theta} = \{p_0 , q_0 , \mu_{c,s_{c,t}} , \mu_{f,s_{f,t}} , \gamma_{c,s_{c,t}} , \gamma_{f,s_{f,t}} , \alpha_{c,s_{c,t}} , \alpha_{f,s_{f,t}} , \beta_{c,s_{c,t}} , \beta_{f,s_{f,t}} , \tau_c , \delta_c , \tau_f , \delta_f , \lambda_{c,0,s_t} , a_{c,s_t} , b_{c,s_t} , \lambda_{f,0,s_t} , a_{f,s_t} , b_{f,s_t} , \rho_{s_t} \}$ for $s_t \in \{1,2\}$, which is estimated via maximum likelihood estimation.

4. Measuring hedging performance, data description and empirical results

The proposed bivariate regime switching time-varying correlated jump model (*RSTVCJ*) is applied for optimal futures hedging. The focus of this paper will be applications of cross hedging, where common jump is unlikely to be realistic and modeling simultaneously the self-owned and correlated jump might be beneficial. To implement futures hedging strategy, the *RSTVCJ* estimates of time-varying minimum variance hedge ratio denoted as $\chi_{t|t-1}$ is given by

$$\chi_t \mid \psi_{t-1} = \frac{Cov\left(r_{c,t}, r_{f,t} \mid \psi_{t-1}\right)}{Var\left(r_{f,t} \mid \psi_{t-1}\right)} = \frac{h_{cf,t} + \eta \; \lambda_f \left(\delta_f^2 + \tau_f^2\right)}{h_{f,t,s_t} + \lambda_f \left(\delta_f^2 + \tau_f^2\right)} \quad (46)$$

where $h_{f,t}$ and $h_{cf,t}$ are the conditional variance of futures returns and conditional covariance of spot and futures returns calculated respectively with equations (38) and (40), and the spillover factor and time-varying jump intensity are calculated respectively with equations (44) and (45). The hedger chooses a hedging strategy that minimizes the variance of hedged portfolio return $r_{p,t}$ measured with the hedging effectiveness (HE) given by

$$HE = \frac{Var\left(r_{c,t}\right) - Var\left(r_{p,t}\right)}{Var\left(r_{c,t}\right)} \times 100 , \quad (47)$$

where $r_{p,t} = r_{c,t} - \hat{\chi}_{t|t-1} r_{f,t}$ is the return on hedged portfolio, $\hat{\chi}_{t|t-1}$ is the estimated hedge ratio estimated at time $t-1$ and to be held over the time period $[t-1, t]$, $Var(r_{p,t})$ and $Var(r_{c,t})$ are respectively the variances of hedged portfolio and unhedged spot position. Further taking account of the hedged portfolio return, hedger will choose a hedging strategy that maximizes the expected utility. The mean-variance expected utility function (Kroner and Sultan, 1993, Alizadeh and Nomikos, 2004, Lee, 2010, Sheu and Lee, 2014) is applied in this paper to compare the hedging performance of alternatively models:

$$E[U(r_{p,t}) | \psi_{t-1}] = E[r_{p,t} | \psi_{t-1}] - \kappa \, Var(r_{p,t} | \psi_{t-1}), \tag{48}$$

where κ stands for the coefficient of absolute risk aversion.

The usefulness of the proposed *RSTVCJ* is illustrated with the exercise of cross hedging jet fuel spot holding with crude oil futures. Weekly jet fuel prices and nearby futures prices of crude oil for the period of January 1991 to December 2016 obtained from Datastream is applied. The returns are computed as the changes in the natural logarithms of prices multiplied by 100. We use the data from January 1991 to December 2015 for in-sample parameters estimation and the remaining data are used for out-of-sample analysis. Table I shows the summary statistics of the level and returns series of jet fuel spot and crude oil futures. Both jet fuel spot and crude oil futures have the same return of 0.02% which is positive but quite small. The jet fuel spot return has a volatility of 5.02% and crude oil futures return has a volatility of 4.47%. According to the Skewness, leptokurtosis, and significant Jarque-Bera statistics, unconditional distribution of jet fuel spot and crude oil futures returns is asymmetric, fat-tailed, and non-Gaussian. This shows the importance of applying a more flexible regime switching correlated jump model for capturing the dynamic of jet fuel spot and crude oil futures returns.

Table II shows the estimates of unknown parameters of regime switching time-varying correlated jump model (*RSTVCJ*) and two its nested models, regime switching correlated jump model (*RSCJ*) without autoregressive jump intensity dynamic and state-independent correlated jump model (*SICJ*) with no regime switching effect in the correlated jump dynamic. To estimate the jump dynamic, the maximum number of jumps $\eta = 10$ was selected as the truncation points. In the volatility equation, $\alpha's$, $\beta's$ and $\gamma's$ are the GARCH(1,1) parameters of regular volatility dynamic. For a given α and β, higher γ indicates a higher steady state volatility. Based on the estimates of *RSTVCJ*, γ has a higher value in state 2 for both spot and futures returns. $\gamma's$ are respectively equal to 0.212 and 5.022 in state 1 and state 2 for spot return and are respectively equal to 0.925 and 2.697 in state 1 and state 2 for futures return. State 2 is the higher volatility state. The term $\alpha + \beta$ in the volatility equation measures the volatility persistence. The $\alpha + \beta$ for spot return in state 1 and 2 estimated with *RSTVCJ*, are respectively equal to 0.902 and 0.814 and $\alpha + \beta$ for futures return in state 1 and 2 estimated with *RSTVCJ* are respectively equal to 0.884 and 0.923. Figure 1 shows the state-dependent jump volatility of jet fuel returns estimated with *RSTVCJ*. State 2 has higher jump volatility than state 1. The regular correlations, namely the correlation without considering the jump component, of spot and futures returns in state 1 and 2 estimated with *RSTVCJ* are equal to 0.930 and 0.786, respectively. Figure 2 shows the state-dependent correlation with *RSTVCJ*. State 1 has higher correlation than state 2. Figure 3 is the regime probability of being in state 1 estimated with *RSTVCJ*.

The jump intensity is measured with the variable of $\lambda's$. For a given $a+b$, higher λ implies higher expected number of jumps. The $\lambda's$ estimated with *RSTVCJ* for jet fuel spot are equal to 0.009 and 0.110 for state 1 and 2, respectively and the $\lambda's$ estimated with *RSTVCJ* for crude oil futures are equal to 0.349 and 0.351 for state 1 and 2, respectively. Figure 4 shows the state-dependent jump intensity of jet fuel returns estimated with *RSTVCJ*. State 2 has higher mean expected jump for jet fuel returns.

Table III presents the out-of-sample hedging effectiveness of alternative models including ordinary least square (*OLS*), constant correlation model (*CC*, Bollerslev, 1988), the common jump model (*CJ*) which has a jump dynamic restricted to be the same for both spot and futures returns, the state-independent correlated jump model (*SICJ*) with no regime switching effect in the correlated jump dynamic, the regime switching correlated jump model (*RSCJ*) without autoregressive jump intensity dynamic and the regime switching time-varying correlated jump (*RSTVCJ*) model. We find that *RSCJ* exhibits superior out-of-sample hedging performance. Figure 5 shows the estimates of hedge ratios estimated with dynamic *RSTVCJ* model and static OLS. method

The unhedged out-of-sample variance is 41.333. If hedger applies static OLS hedging, the variance is reduced to 6.273 or an 84.75% variance reduction. Without considering the jump dynamic, the percentage variance reductions is 84.37% for *CC* hedging strategies. When we take account of the common jumps between spot and futures return and assume a time-invariant jump dynamic, *CJ* has a percentage variance reduction of 84.16%%. We find that *CC* and *CJ* do not provide superior performance compared with the static OLS hedging. We consider three correlated jump models in this paper. The percentage variance reduction of *SICJ* is equal to 84.96%. Allowing the jumps to be correlated improves the crude oil futures hedging effectiveness. When we further incorporate regime switching dynamic into the jump dynamic, the *RSCJ* hedging has an 85.67% variance reduction. Taking account of the regime switching effect improves cross hedging effectiveness. If we allow the jump dynamic to be time-varying with an autoregressive jump intensity as shown in equation (17), the *RSTVCJ* has a percentage variance reduction of 85.04%. Taking the time-varying jump intensity dynamic into consideration does not further increase the hedging performance. *RSCJ* is the best performer and the improvements of *RSCJ* over *CC*, *CJ*, *SICJ* and *RSTVCJ* are 1.3%, 1.51%, 0.71% and 0.63%, respectively. Overall, a hedging model taking account of both regime switching and correlated jumps improves crude oil futures hedging effectiveness.

To better understand the economic significance of the superiority of *RSTVCJ* over alternative models, he expected utility gains is also reported in Table III. In line with previous research (Alizadeh and Nomikos, 2004; Lee 2009, 2010; Sheu and Lee, 2014), the hedger is assumed to have an expected utility function given by equation (48) with the coefficient of absolute risk aversion κ equal to 4. If a hedger uses *OLS* hedging, the average weekly utility is $U_{OLS} = 0.155 - 4(6.273) \approx -24.938$. With *RSCJ* hedging, the average weekly utility is $U_{RSCJ} = 0.095 - 4(5.894) \approx -23.481$. The hedger's net benefit from using *RSCJ* hedging over

OLS hedging is equal to $U_{RSCJ} - U_{OLS} - C = 1.457 - C$, where C stands for the extra transaction cost from dynamic rebalancing using *RSCJ* hedging over *OLS* hedging. A mean-variance expected utility-maximizing hedger will adopt *RSCJ* hedging if the rebalancing cost is less than the utility gain of switching from *OLS* hedging to *RSCJ* hedging. Because a typical round trip transaction cost is around 0.02% to 0.04% (Lee, 2010) and the utility gains of *RSCJ* hedging over all alternative models are all positive and much higher than the typical round trip transaction cost, hedger will adopt the *RSCJ* hedging.

5. CONCLUSIONS

The main contribution of this paper is to propose a bivariate regime switching time-varying correlated jump model (*RSTVCJ*) for cross futures hedging. Most of the existing discrete time hedging models assume a common jump for both spot and futures returns which might be too restrictive because spot and futures returns are normally not perfectly correlated. The propose *RSTVCJ* release this common jump assumption and allow the spot and futures returns to possess both self-owned and correlated jumps via a regime switching spillover factor. The *RSTVCJ* allows us to analyze the covariance structure of spot and futures returns by considering simultaneously many time-series features including regime switching variance and correlations, regime switching jump intensity and time-varying autoregressive jump intensity dynamic.

The usefulness of *RSTVCJ* is illustrated by performing an exercise on hedging jet fuel spot holding with crude oil futures. The hedging performance of *RSTVCJ* is compared with three nested models including the common jump model (*CJ*) which has a jump dynamic restricted to be the same for both spot and futures returns, the state-independent correlated jump model (*SICJ*) with no regime switching effect in the correlated jump dynamic and the regime switching correlated jump model (*RSCJ*) without autoregressive jump intensity dynamic. Empirical results show that *RSCJ* has the best out-of-sample hedging performance. *RSCJ* is the best performer and the improvements of *RSCJ* over *CJ*, *SICJ* and *RSTVCJ* are 1.51%, 0.71% and 0.63%, respectively. Generally speaking, a hedging model taking account of both regime switching and correlated jumps improves the crude oil futures hedging effectiveness. Adding time-varying autoregressive jump intensity dynamic to the jump process, however, does not further improve the futures hedging effectiveness. Results on expected utility gains of *RSCJ* over alternative models also show that a mean-variance expected utility-maximizing hedger will adopt *RSCJ* hedging even after taking account of the transaction costs.

Table 1. Summary statistics for weekly returns of jet fuel spot and crude oil futures

	Jet Fuel		*Crude Oil Futures*	
	Level	*Log Return*	*Level*	*Log Return*
Mean	138.78	0.02	48.97	0.02
Maximum	416.38	27.70	144.26	21.94
Minimum	29.88	-29.54	9.98	-38.45
Std. Dev.	96.14	5.02	35.26	4.47
Skewness	0.75	-0.33	0.78	-0.74
Kurtosis	2.19	6.74	2.19	8.80
Jarque-Bera	156.68***	784.54***	167.39***	1948.85***

Note: *** indicates significance at the 1% level and returns are calculated as the differences in the logarithm of prices multiplied by 100.

Table 2. Estimates of unknown parameters for *RSTVCJ*, *RSCJ* and *SICJ* models

	SICJ	*RSCJ*	*RSTVCJ*		*SICJ*	*RSCJ*	*RSTVCJ*
		Volatility Equation				*Jump Intensity*	
γ_{c1}	0.000	0.251	0.212	λ_{c1}	1.072	0.000	0.009
	(0.023)[1]	(0.500)	(0.156)*		(0.694)*	(0.009)	(0.020)
γ_{c2}	9.034	4.869	5.022	λ_{c2}		0.076	0.110
	(8.850)	(1.208)***	(3.016)*			(0.050)*	(0.108)
γ_{f1}	0.764	1.258	0.925	λ_{f1}	1.939	1.697	0.349
	(0.690)	(0.462)***	(0.718)*		(1.818)	(2.292)	(0.565)
γ_{f2}	3.558	1.393	2.697	λ_{f2}		0.641	0.351
	(3.496)	(0.696)**	(1.609)**			(0.778)	(0.364)
α_{c1}	0.085	0.070	0.081			*Time-Varying Jump Dynamic*	
	(0.031)***	(0.031)**	(0.060)*	a_{c1}			0.342
α_{c2}	0.181	0.114	0.065				(0.426)
	(0.065)***	(0.034)***	(0.069)	a_{c2}			0.747
α_{f1}	0.202	0.178	0.197				(0.243)***
	(0.061)***	(0.033)***	(0.056)***	b_{c1}			0.342
α_{f2}	0.017	0.021	0.028				(0.603)
	(0.043)	(0.033)	(0.058)	b_{c2}			0.747
β_{c1}	0.803	0.860	0.821				(0.413)**
	(0.048)***	(0.065)***	(0.111)***	a_{f1}			0.906
β_{c2}	0.589	0.707	0.749				(0.324)***
	(0.203)***	(0.065)***	(0.215)***	a_{f2}			0.416
β_{f1}	0.692	0.715	0.687				(0.432)
	(0.048)***	(0.054)***	(0.061)***	b_{f1}			0.906
β_{f2}	0.907	0.951	0.895				(0.993)
	(0.077)***	(0.048)***	(0.120)***	b_{f2}			0.136
		Correlation Equation					(0.079)**
ρ_1	0.944	0.914	0.930			*Spillover factors*	
	(0.054)***	(0.015)***	(0.019)***	η_1	0.719	-1.182	-0.686
ρ_2	0.634	0.712	0.786		(0.273)***	(1.323)	(1.335)
	(0.082)***	(0.030)***	(0.037)***	η_2		0.384	0.053
		Jump Distribution				(0.378)	(0.047)

τ_c	0.229	0.226	0.373
	(0.240)	(0.806)	(0.535)
δ_c	1.206	9.013	4.601
	(0.312)***	(2.839)***	(0.752)***
τ_f	0.720	0.190	0.381
	(0.444)*	(0.170)	(0.241)*
δ_f	0.010	0.000	0.003
	(0.018)	(0.028)	(0.012)
LL[2]	-6827.1405	-6807.122	-6803.56

1. Figures in parentheses are standard errors and *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.
2. *LL* stands for the log likelihood value.

Table3. Out-of-sample hedging effectiveness

	Variance of Hedged Portfolio Return	Percentage Variance Reduction[1]	Improvement of *RSCJ* over Other Models[2]	Hedged Portfolio Return	Expected Weekly Utility[3]	Utility Gain of *RSCJ* over Other Models [4]
Unhedged	41.133					
OLS	6.273	84.75%	0.92%	0.155	-24.938	1.457
CC	6.427	84.37%	1.30%	0.165	-25.543	2.063
CJ	6.516	84.16%	1.51%	0.146	-25.918	2.437
SICJ	6.186	84.96%	0.71%	0.106	-24.637	1.156
RSCJ	5.894	85.67%		0.095	-23.481	
RSTVCJ	6.154	85.04%	0.63%	0.148	-24.468	0.987

1. Percentage variance reductions are calculated as the differences of the variance of unhedged position and the estimated variances of alterative models over the variance of unhedged position multiplied by 100.
2. Improvement of *RSCJ* over other hedging strategies is defined as the differences of the percentage variance reduction of *RSCJ* and the percentage variance reduction of alternative models.
3. Expected weekly utility is calculated based on equation (48)

Utility gains of *RSCJ* over other hedging strategies are defined as the differences of the expected utility of *RSCJ* and the expected utilities of alternative models.

Figure1. State dependent jump volatility of jet uel returns estimated with RSTVCJ

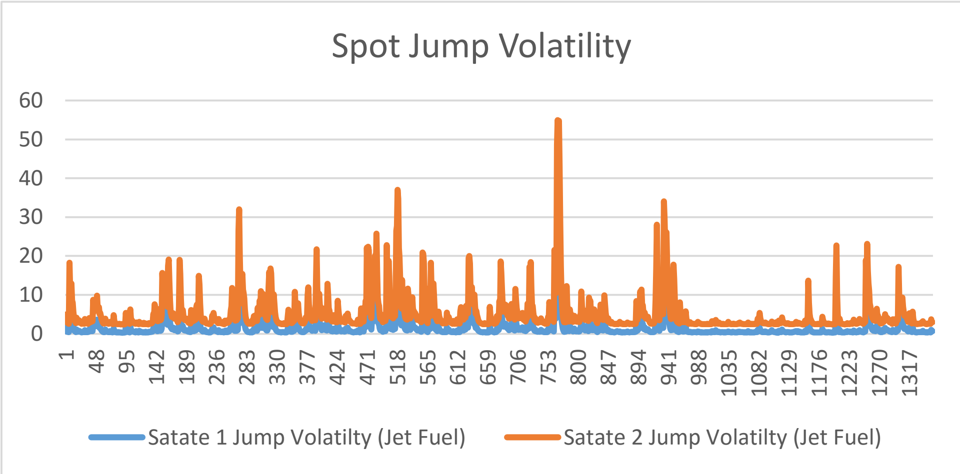


Figure 2. State-dependent correlation estimated with RSTVCJ

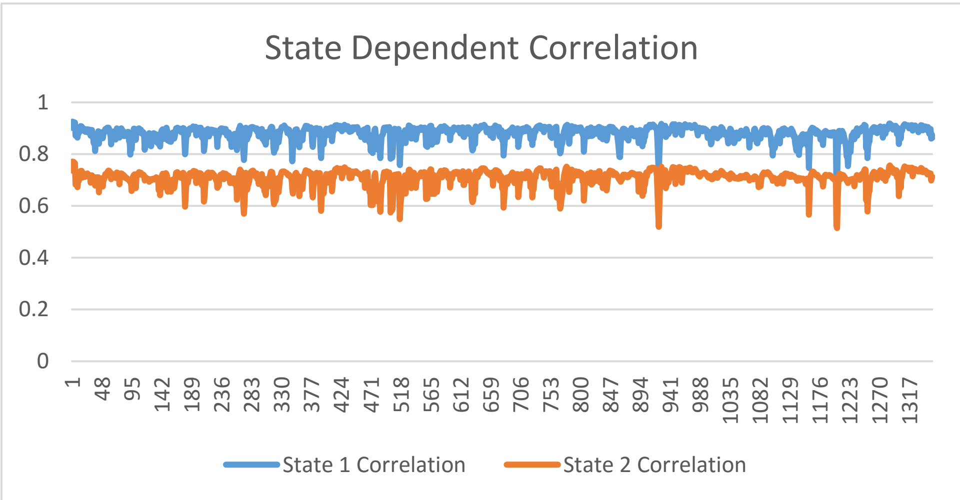


Figure 3. Regime probability of being in state 1 estimated with RSTVCJ

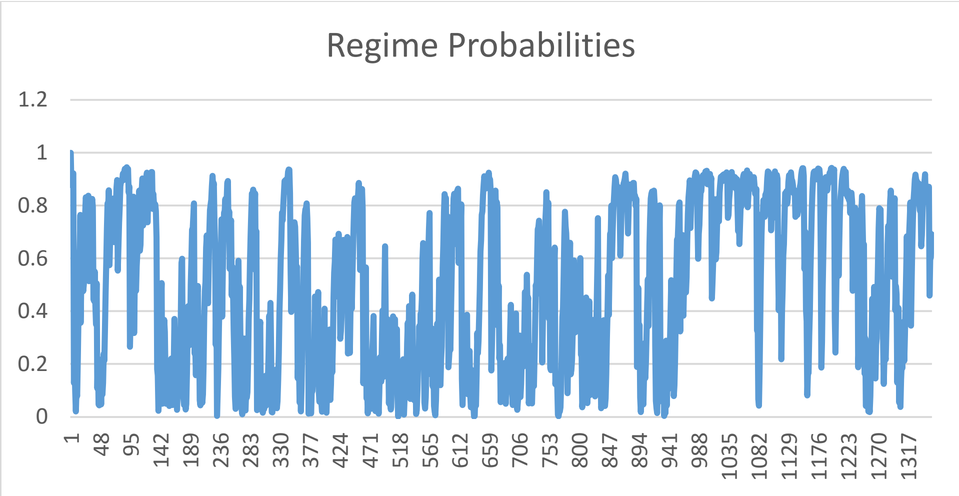


Figure 4. State dependent jump intensity of jet fuel returns estimated with RSTVCJ

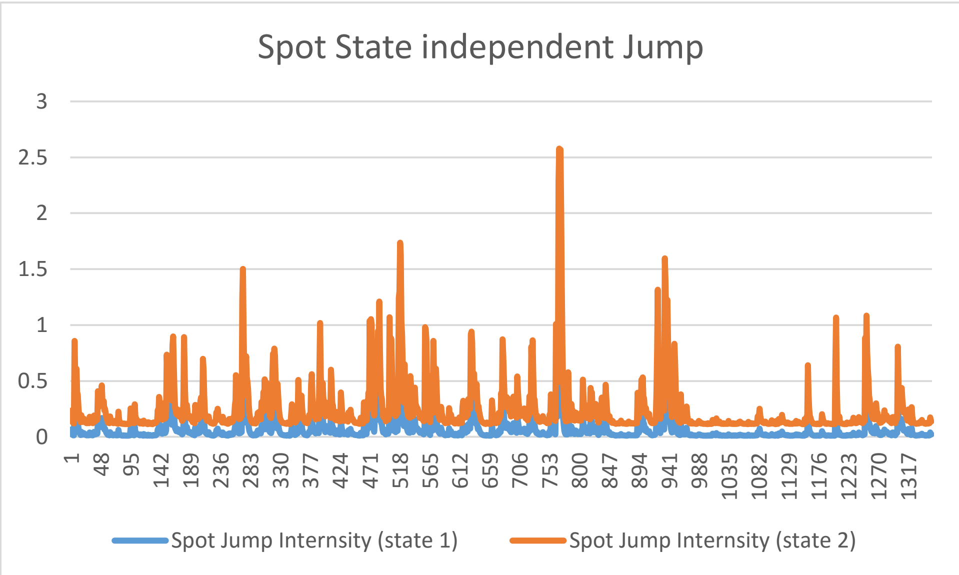
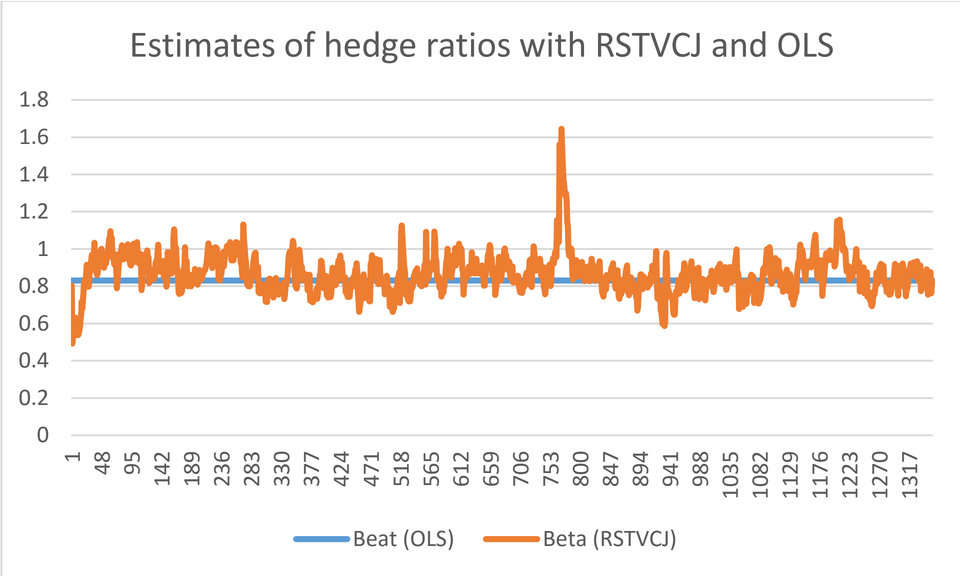


Figure 5. Hedge ratio estimated with OLS and RSTVCJ



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